Debt and Growth: Friends or Foes? A Political Economy Approach

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Abstract: In this paper I discuss the relationship between government debt and growth in small open economies using an overlapping generations' model with two generations and probabilistic voting. In this context, intergenerational conflict over government policy drives the results of the model. A calibration of the model using data from 20 western democracies verifies that the model is consistent with growth and debt in these countries during the period 1980-2010. The results of the calibration imply that an increase in the political power of the old decreases growth and increases debt both in the short and in the long run. The same is true for an increase in government corruption. Moreover, the calibration indicates a negative relationship between debt to output and growth in the short run. However, shifts in the steady state or differences in parameters between countries might make the identification of this relationship difficult.

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1 Introduction

This paper discusses the relationship between government debt and growth in developed democracies. This issue has gained a lot of attention lately because of the significant increase of debt in these countries (see figure 1 below). Moreover, this increase in debt has generated fears regarding the effect it will have on growth and welfare. In this respect Reinhart et al. (2012) offer a very informative discussion.

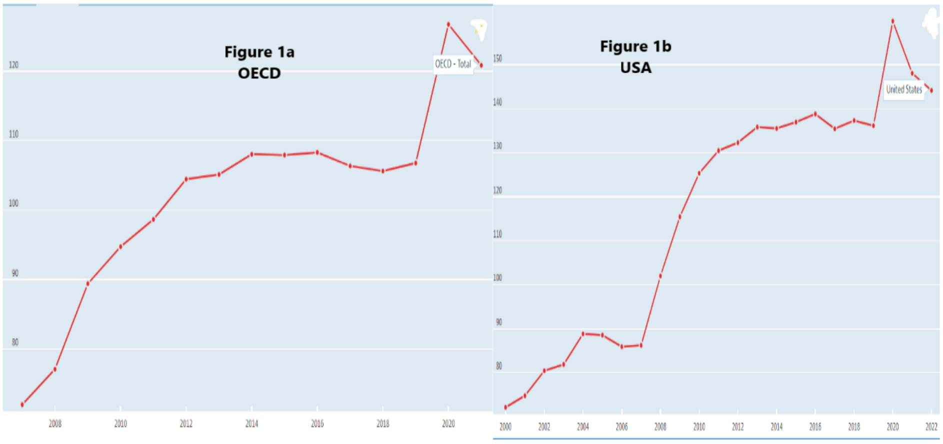

Figure 1: Figures 1a and 1b respectively portray the evolution of debt to output after 2000 in OECD and the USA countries.

Fears for growth are in line with standard economic theory which predicts a negative relationship between growth and debt. In particular, the argument goes as follows. A high level of debt induces high interest payments, which in turn absorb government and private resources, the latter through increased taxation. This diversion of resources to interest payments inevitably reduces investment and growth. Moreover, if taxes are distortive, their increase causes individuals to reduce their economic activities, thus adversely affecting growth. This observation is especially true for the most productive individuals who typically pay more taxes. Additionally, an excessive debt may instigate doubts regarding the ability of governments to service it. These doubts can cause an increase in interest rates which again crowds out investment. Again Reinhart et al. (2012) provide an excellent summary of this discussion.

However, despite the overwhelming theoretical arguments, empirical research fails to offer an equally conclusive picture. Specifically, some researchers manage to pin down a negative relationship between debt and growth (Chudik et al. (2017)), while others either fail to do so (Ash et al. (2017)), or conclude that this relationship emerges only for high levels of debt (Reinhart and Rogoff (2010)).

In this respect, a richer theoretical framework than the one available so far, might help to better understand these mixed empirical findings. This paper aims to contribute to such a richer framework by building on the ideas of Song et al. (2012), who investigate the role of institutional and political factors on the evolution of debt. In this respect, I maintain that increases in corruption or in the political power of the old have two effects. First, they increase debt as argued by Song et al. (2012) and second, they increase government transfers (e.g. pensions) vis-a-vis public investment (e.g. spending on education). The second effect apparently also reduces growth. These ideas are consistent with data from western developed democracies. In particular, I consider the 24 countries that have been OECD members since 1980, with the exception of Turkey. For these countries, the correlation of the average per capita growth, in the period 2008-2022, with the current Corruption Perception Index (CPI) is 0.42 (0.07), while with the current median age is -0.63 (0.003). Likewise, the correlation of debt to GDP in 2022, with the CPI and median age is -0.65 (0.002) and 0.67 (0.001) respectively[1].

In order to investigate these ideas further, I follow Song et al. (2012) and consider an overlapping generations model of a small open economy, with time-consistent government policy. In this context, I model government decisions using probabilistic voting. In particular, there are two groups of voters, the young and the old, which are the two generations of the model. The outcome of probabilistic voting decides government policy, which amounts to two things: First, the allocation of government revenues between taxes and new debt. Second, the size of government spending and its allocation between transfers and public investment.

Besides allocating government resources, probabilistic voting also has an implicit function: it provides a disciplining effect for the government with regard to debt. In particular, young voters typically vote against undertaking excessive debt, because they worry that such an action will undermine the provision of public goods in the future.

Following these assumptions, the model in this paper can be solved analytically and its equilibrium determines simultaneously debt to output and growth. Moreover, there are two steady states. An interior one, in which equilibrium debt to output and growth take positive values and an "immiseration" equilibrium, in which debt grows until the interest payments absorb all the resources of the economy, thus depleting growth. Moreover, it turns out that one of the two steady states is always stable.

[1]p-values in parenthesis. The CPI index decreases with corruption

Also, the model exhibits transitional dynamics featuring an explicit relationship between growth and the debt-to-output ratio. However, whether this relationship is negative or not depends on the value of the parameters. This is because an increase in debt has two opposing effects. On the one hand, it increases future interest payments, thereby crowding out investment and reducing growth. On the other hand, it can be used to finance an increase in public investment, which increases the productivity of the economy and thus growth. Which one of these two effects dominates the other is an empirical question. Additionally, it is an empirical question whether the interior steady state is stable and what the signs of various derivatives of interest are.

In order to resolve the empirical questions paused above, in a manner which is consistent with key economic variables of western developed democracies, I proceed with a calibration of the model. I carry out the calibration using data from 20 out of the 24 countries that have been members of the OECD since 1980. These 20 countries qualify as small open economies and are both democratic and developed. The results of the calibration indicate that the interior steady state matches the targeted economic features of these 20 economies and that for these values it is stable. Moreover, the transitional dynamics of the calibrated model imply a negative relationship between debt to output and growth along the transition path. Finally, comparative statics also give intuitive results. First, an increase in the political power of the young decreases government spending and increases the ratio of public investment to transfers, thus reducing debt to output and increasing growth. Second, an increase in the preferences for public goods, often viewed as a proxy for government corruption, also decreases debt to output and increases growth. Third, an increase in the firms' efficiency parameter increases both debt to output and growth, indicating that more efficient countries can also sustain greater debt.

These results have a significant implication. In particular, although the model predicts a negative relationship between debt to output and growth in the short run, cross country empirical research might fail to identify this relationship, since differences in parameters across countries might obscure it. Also, even if an empirical study tracks the behavior of governments across time, results may still be inconclusive, since the negative relationship between debt and growth might not be identified if there is a shift in the steady state.

Let me now briefly discuss the relevant literature. My paper mostly relates to Song et al. (2012) and Arai et al. (2018). I share with the first the basic characteristics of the model (small open economy, political economy perspective, focus on debt) and the details of the calibration (time period and countries used). However, there is a difference in the the relationship between debt and output. Specifically, in Song et al. (2012) an increase in debt, increases distortive taxation thus causing the young to substitute market labor supply with home production. In this respect, Song et al. (2012) do not deliver an explicit relationship between debt and growth. On the contrary, in the present paper, output is determined by the inter-generational conflict over the allocation of government budget, between public investment and transfers. The political process resolves this conflict, along with the similar conflict for the financing of government budget, between taxes and new debt. In turn, this political resolution simultaneously determines debt to output and growth. This approach delivers an explicit relationship between debt and growth in the short run and an implicit relationship between the two, through the parameters that affect both in the long run.

On the other hand Arai et al. (2018) discuss a model in which, like in the paper at hand, debt to output and growth are simultaneously determined in equilibrium. However, unlike this paper they consider a closed economy with no transitional dynamics. Thus, they don't find an explicit relationship between debt and growth. Also, in contrast to my results, they find that debt to output is independent of the political power of the old.

Other papers that follow a similar methodology are Hassler et al. (2005), Gonzalez-Eiras and Niepelt (2012), Lancia and Russo (2016), Müller et al. (2016) and Ono and Uchida (2018). These papers also develop an overlapping generations' model with probabilistic voting and time consistent government policy. They use the distribution of political power among generations to analyze political outcomes. However, none of these papers explicitly associates debt to output and growth.

The rest of the paper is organized as follows: Section 2 describes the model, section 3 describes the equilibrium and section 4 discusses the solution of the model. Section 5 discusses a calibrated version of the model and section 6 concludes. Appendix A contains a graphical analysis of the stability of steady states.

2 Model

2.1 Overview and Notation

Overview

I consider an overlapping generations' model of a small open economy, with discrete time, infinite horizon and two generations of individuals. Besides individuals, two more types of agents also populate this economy: firms and government. In this model individuals live for two periods (in the first they are young and in the second they get old), while governments live only for one period. Firms don't face any inter-temporal decisions so their lifespan is irrelevant.

Notation

In what follows, I suppress indices for individuals and time, in order to simplify notation. Instead, I use the subscripts y and o for the representative young and old respectively and the standard recursive notation to identify next period variables (for any variable ζ, the expression ζ' signifies next period's ζ). I follow the same notation throughout the paper.

Let me first describe individuals.

2.2 Indivduals

Preferences.

Both age groups consume a private good (c) and a generation specific public good (X for the young and Φ for the old)[2]. For tractability, I represent preferences using a standard logarithmic utility function. Thus, the lifetime utility of a young individual is:

$$u_y = \ln c_y + \lambda \ln X + \beta(\ln c_o' + \theta \ln \Phi') \tag{1}$$

where $\beta \in (0,1)$ is the time discount factor and θ, $\lambda > 0$ capture the preference weights on the two public goods. In accordance with the lifetime utility of the young, the utility of the current old is:

$$u_o = \ln c_o + \theta \ln \Phi \tag{2}$$

Budget constraints.

In each period, each young individual supplies inelastically one unit of labor. The income from this unit of labor (w) is taxed by a tax t and is the sole source of income for the young. Moreover, the young allocate their disposable income between private good consumption and savings (S). Thus, their budget constraint is:

$$w - t = c_y + S \tag{3}$$

The young invest their savings in the international government bond and capital markets.

On the other hand, the old do not work and do not pay taxes. They live off their investments. Thus, their budget constraint is:

$$c_o = R'S \tag{4}$$

[2] I realize that I abuse the term public good when I assume that it can be generation specific. I apologize for this misuse.

This budget constraint implies that the old do not make any decisions. Their choices in the first period, along with political and market outcomes, determine their income.

Let me now turn to firms.

2.3 Firms

In each period, a large number of identical firms produce the unique consumption good. These firms are price takers, hire labor locally but also have access to an international market for capital.

Firms use the following production technology:

$$y = A\left(\frac{G}{L}\right)^{(1-a)} k^{a} l^{(1-a)} \tag{5}$$

where y is the output of the representative firm, κ, l are the private capital (henceforth capital) and labor used by each firm and $\frac{G}{L}$ is the ratio of public capital to total labor in the economy. Capital is perfectly mobile and depreciates fully after each period.

The public capital forms according to the following rule:

$$G' = (1-\delta)G + X \tag{6}$$

where $\delta \in [0,1]$ is the discount rate of the public capital and X is the public good of the young.

I think of public capital as the intangible characteristics of an economy that increase productivity and are the result of government effort in the past. For example, public health, literacy e.t.c.

As a motivation for the above formulation think of a government vaccination program for kids. This program provides utility to young parents because their children don't get sick. However, in the long run systematic vaccinations improve public health and therefore increase productivity.

Furthermore, once public health or literacy reach a certain level, they become almost self sustained and require little or no government funding. This observation motivates the smaller discount rate for G relative to k.

Given the above characteristics of the public good of the young (X), from now on I will refer to it as public investment. In contrast, I will refer to the public good used by the old (Φ) as transfer.

Next I consider the government.

2.4 Government

Budget.
The budget constraint of the government is

$$b' = Rb + X + \Phi - tN_y \tag{7}$$

where b is the government debt and N_y is the size of the young generation.
Political decision process
The government determines policy. Specifically, it decides on the provision of the two public goods and the level of taxation. These decisions subsequently determine government debt.

I model these decisions, using the probabilistic voting model. According to this model, equilibrium policy maximizes the following political objective function $V = \varphi u_y + (1-\varphi)u_o$, where φ and $1-\varphi$ represent the political power of young and old voters respectively. These parameters incorporate the political characteristics of the two age groups. In particular, $\varphi = \frac{\omega_y \lambda_y}{\omega_y \lambda_y + \omega_o \lambda_o}$ where ω_y and ω_o are the shares of young and old voters in the electorate, while λ_y and λ_o capture the number of swing voters in each age group[3]. In order to simplify algebra, I consider the following equivalent political objective function:

$$V = \mu u_y + u_o \tag{8}$$

where $\mu = \frac{\varphi}{(1-\varphi)}$.

Next section discusses the equilibrium of the model.

3 Equilibrium

3.1 Sequence of events

In each period, government policy is determined first. Then, the firms choose the amount of capital and labor they employ. Finally, young individuals decide on the allocation of their resources.

3.2 Private sector equilibrium

Utility maximization
The young decide on the allocation of their disposable income between consumption and savings, so as to maximize their inter-temporal utility, subject

[3]See pages 52-62 in Persson and Tabellini (2002). The model in Persson and Tabellini (2002) was introduced by Lindbeck and Weibull (1987).

to the two individual budget constraints. In order to determine this allocation, I solve the two individual budget constraints, which are (3) and (4), with respect c_y and c_o' respectively, substitute them in the lifetime utility of the young (1) and maximize the function I derive with respect to S. The first order condition for this problem yields:

$$\frac{c_o'}{c_y} = \beta R' \tag{9}$$

Then, solving the last equation with respect S, I get:

$$S = \frac{\beta}{(1+\beta)}(w - t) \tag{10}$$

Profit maximization

Firms choose capital and labor, so as to maximize their profit. Since capital depreciates fully in each period, profit maximization implies that the marginal productivity of capital equals the international interest rate. Furthermore, due to the production technology in (5) this condition yields

$$R = a\frac{y}{k}$$

Solving the above equation with respect k, substituting the result in (5) and solving with respect to y, yields:

$$y = EG\frac{l}{L}$$

where $E = \left(\frac{a}{R}\right)^{\frac{a}{1-a}} A^{\frac{1}{1-a}}$. This equation implies that the total output Y equals:

$$Y = EG \tag{11}$$

Moreover, the standard profit maximizing condition for the choice of labor implies that the marginal product of labor equals the wage. In this case, this condition becomes

$$w = (1-a)\frac{y}{l}$$

and because of (11) above and the fact that only the young work it follows that:

$$wN_y = (1-a)EG \tag{12}$$

where wN_y is the total labor income of the economy or equivalently the total income of the young.

Next I turn to the decision process of the government.

3.3 Political equilibrium

3.3.1 Preliminaries

In order to concentrate on a single small open economy, I assume that the rest of the world is constantly at the steady state and that all parameters are fixed (there are no exogenous variables). This assumption implies that the interest rate is always constant at its steady state value and therefore $R' = R$.

Using the budget constraint of the young and the saving rule, which are equations (3) and (10), we get that $c_y = \frac{w-t}{1+\beta}$. Then, the utility of the young along with (9) imply that $u_y = (1+\beta)\ln(w-t) + \lambda \ln X + \beta\theta \ln \Phi' + Con$, where $Con = \ln \beta R - \ln(1+\beta)$. Finally, using the equilibrium wage and the utility of the old, the political objective function takes the following form:

$$V = \mu \left[(1+\beta) \ln \left[\frac{(1-a)EG}{N_y} - t \right] + \lambda \ln X + \beta\theta \ln \Phi' + Con \right] + \ln RS + \theta \ln \Phi \tag{13}$$

The equation above contains Φ'. This fact implies that the objective of the current government depends on the actions of the next government (choice of Φ'). As a result, the choice of policy is essentially a game between successive governments.

Typically, such a game has many equilibria. Here, I follow Krusell et al. (1997) and solve for a differentiable Markov perfect equilibrium[4]. This equilibrium depends only on the current values of payoff relevant state variables. There are only two such variables in my model: the public capital G and the government debt b, since private wealth (S) does not affect government policy.

The aforementioned strategic interaction between successive governments provides a disciplining effect which allows for a finite debt in equilibrium. Without it, a government has the motive to run an infinite deficit, in order to provide infinite public goods, free of taxes. However, fear of what the next government will do can harness this maverick policy. Specifically, the young might vote against a higher debt, if they believe that it will decrease public good provision in the future.

Now I proceed with the definition of equilibrium.

[4] Other papers following this approach include Grossman and Helpman (1998), Krusell and Rios-Rull (1999) and recently Song et al. (2012) and Lancia and Russo (2016).

3.3.2 Definition of Equilibrium

Let $T = (t, X, \Phi)$ be the government policy profile, $W = (b, G)$ be the profile of state variables and $\pi = (a, A, \beta, \lambda, \theta, \mu, R)$ be the profile of parameters and international interest rate. Using these definitions, I can rewrite the political objective and the two dynamic constraints that govern the state variables, which are (6) and (7), in a more compact way:

$$\begin{aligned} V &= V(T, T'; \pi) \\ W' &= W(T, T'; W, \pi) \end{aligned}$$

I can describe informally the derivation of equilibrium in the following way: First the current government guesses the policy function of the next government. Call this guess $T^*(\cdot)$. Function $T^*(\cdot)$ determines the expectation of future policy as a function of the future value of the state variables, so $T^{*\prime} = T^*(W')$. Then, the current government maximizes its objective function under the two dynamic constraints and the aforementioned guess. This operation determines current policy as a function of the current value of the state variables. Define this outcome as function $T(\cdot)$, so $T = T(W)$. If $T^*(\cdot)$ and $T(\cdot)$ coincide, then $T(\cdot)$ is an equilibrium of the game between successive governments. This equilibrium describes a notion of stationarity, in the sense that successive governments respond in the same way to the same conditions.

Let me now proceed with a formal definition of the equilibrium. In what follows I suppress π.

Definition. *Political equilibrium.*
Let $W \in \mathbb{R}_+^2$. A function $T : \mathbb{R}_+^2 \to \mathbb{R}_+^3$, such that $T = T(W) = (t(b, G), X(b, G), \Phi(b, G))$ is a political equilibrium, if it maximizes the political objective function $V = V(T, T'; W)$, with respect to T, subject to $T' = T(W')$ and $W' = W(T, T'; W)$.

Next, I proceed with the derivation of the political equilibrium.

3.3.3 Derivation of equilibrium

Since only one future policy variable (Φ') appears in the political objective, the current government need only form a guess function for this variable. Call this guess function $\Phi(\cdot)$, so $\Phi' = \Phi(b', G')$. Then, the maximization problem in the definition above is:

$$\max_{wr\,t,X,\Phi}\{\mu\left[(1+\beta)\ln\left[\frac{(1-a)EG}{N_y}-t\right]+\lambda\ln X+\beta\theta\ln\Phi'+Con\right]+\ln RS+\theta\ln\Phi\} \tag{14}$$

s.t.

$$\begin{aligned}\Phi' &= \Phi(b', G')\\ G' &= (1-\delta)G+X\\ b' &= Rb+X+\Phi-tN_y\end{aligned}$$

The first order conditions to this problem with respect to t, X and Φ respectively are:

$$\begin{aligned}\frac{-(1+\beta)}{[\frac{(1-a)EG}{N_y}-t]}+\frac{\beta\theta}{\Phi'}\frac{\partial\Phi(b',G')}{\partial b'}\frac{\partial b'}{\partial t} &= 0\\ \frac{\lambda}{X}+\frac{\beta\theta}{\Phi'}\left(\frac{\partial\Phi(b',G')}{\partial b'}\frac{\partial b'}{\partial X}+\frac{\partial\Phi(b',G')}{\partial G'}\frac{\partial G'}{\partial X}\right) &= 0\\ \frac{\theta}{\Phi}+\frac{\beta\theta\mu}{\Phi'}\frac{\partial\Phi(b',G')}{\partial b'}\frac{\partial b'}{\partial\Phi} &= 0\end{aligned}$$

These conditions are also known in the literature as Generalized Euler Equations (GEE). In order to proceed, I guess and verify a linear equilibrium policy rule[5]. In particular, my guess is that:

$$\Phi' = \Phi(b', G') = \omega_1 b' + \omega_2 G' \tag{15}$$

where $\omega_1, \omega_2 \in \mathbb{R}$. Following this assumption the GEE take the form:

$$\begin{aligned}\frac{(1+\beta)}{[\frac{(1-a)EG}{N_y}-t]}+\frac{\beta\theta}{\Phi'}\omega_1 N_y &= 0\\ \frac{\lambda}{X}+\frac{\beta\theta}{\Phi'}(\omega_1+\omega_2) &= 0\\ \frac{\theta}{\Phi}+\frac{\beta\theta\mu}{\Phi'}\omega_1 &= 0\end{aligned}$$

Solving this system with respect to X, tN_y and Φ' yields:

$$X = \frac{\lambda\mu}{\theta}\frac{\omega_1}{\omega_1+\omega_2}\Phi \tag{16}$$

$$tN_y = (1-a)EG-\frac{(1+\beta)\mu}{\theta}\Phi \tag{17}$$

$$\Phi' = -\omega_1\beta\mu\Phi \tag{18}$$

[5] See for example Song et al. (2012).

Then, using the guess function, $\Phi' = \Phi(b', G') = \omega_1 b' + \omega_2 G'$ and substituting into it, b' and G' from (6) and (7) and X, tN_y and Φ' from (16)-(18) above, yields an equation in Φ. Solving this equation I find:

$$\Phi = -\Lambda R b + \left[(1-a)E - \frac{\omega_2}{\omega_1}(1-\delta)\right]\Lambda G \tag{19}$$

where $\Lambda = \frac{\theta}{\mu[1+\beta(1+\theta)+\lambda]+\theta}$. As a result, $\omega_1 = -\Lambda R$ and ω_2 satisfies $\omega_2 = \Lambda(1-a)E + \omega_2 \frac{1-\delta}{R}$, which implies in turn that $\omega_2 = \frac{\Lambda R(1-a)E}{R-(1-\delta)}$. Thus, the political equilibrium policy is

$$\Phi = \Lambda R\left[-b + \frac{(1-a)E}{R-(1-\delta)}G\right] \tag{20}$$

Solving for X and tN_y with the help of (16) and (17) yields:

$$\begin{aligned}
X &= \frac{\lambda\mu}{\theta}\Lambda R\left[\frac{R-(1-\delta)}{R-(1-\delta)-(1-a)E}\right]\left[-b + \frac{(1-a)E}{R-(1-\delta)}G\right] \\
tN_y &= \frac{(1+\beta)\mu}{\theta}\Lambda R b + \left[1 - \frac{(1+\beta)\mu}{\theta}\frac{\Lambda R}{R-(1-\delta)}\right](1-a)EG
\end{aligned}$$

Substituting the equations above in the equations for b' and G', which are (6) and (7) yields:

$$G' = -\frac{\lambda\mu}{\theta}\Lambda R\left[\frac{R-(1-\delta)}{R-(1-\delta)-(1-a)E}\right]b + \left[(1-\delta) + \frac{\frac{\lambda\mu}{\theta}\Lambda R(1-a)E}{R-(1-\delta)-(1-a)E}\right]G \tag{21}$$

and

$$b' = (1 - \Xi\Lambda)Rb + \left(\frac{\Xi\Lambda R}{R-(1-\delta)} - 1\right)(1-a)EG \tag{22}$$

Where $\Xi = \frac{\lambda\mu}{\theta}\frac{R-(1-\delta)}{R-(1-\delta)-(1-a)E} + 1 + \frac{1+\beta}{\theta}\mu$

4 Steady states and transitional dynamics

4.1 Steady states

In general there is no theorem for the uniqueness of Markov Perfect equilibria. Thus, another non linear function $\Phi' = \Phi(b', G')$ besides (20) might exist that satisfies the definition of equilibrium. However, given (20) we can fully characterize the steady states and transitional dynamics of this model.

I start by defining the growth rate of output $g = \frac{Y'}{Y}$ and by noticing that because output is linear in public capital $g = \frac{G'}{G}$. Then, I divide (21) by G and (22) by Y' in order to get:

$$
\begin{aligned}
g &= -\frac{\lambda\mu}{\theta}\Lambda RE\left[\frac{R-(1-\delta)}{R-(1-\delta)-(1-a)E}\right]\frac{b}{Y}+\left[(1-\delta)+\frac{\frac{\lambda\mu}{\theta}\Lambda R(1-a)E}{R-(1-\delta)-(1-a)E}\right]\\
\left(\frac{b}{Y}\right)' &= \frac{1}{g}\left[(1-\Xi\Lambda)R\frac{b}{Y}+\left(\frac{\Xi\Lambda R}{R-(1-\delta)}-1\right)(1-a)\right]
\end{aligned}
$$

The two equations above fully describe the dynamics of the model with respect to output growth g and debt to output $\frac{b}{Y}$. In order to simplify notation, I combine the parameters in the equations above as implied by the following expressions:

$$
\begin{aligned}
g &= \Omega_1\frac{b}{Y}+\Omega_2\\
\left(\frac{b}{Y}\right)' &= \frac{1}{g}\left(\Omega_3\frac{b}{Y}+\Omega_4\right)
\end{aligned}
$$

Combining the last two equations yields:

$$
\left(\frac{b}{Y}\right)' = \frac{\Omega_3\frac{b}{Y}+\Omega_4}{\Omega_1\frac{b}{Y}+\Omega_2} \tag{23}
$$

In order to find the steady state I set $\left(\frac{b}{Y}\right)' = \frac{b}{Y}$ and solve with respect to $\frac{b}{Y}$. This operation yields two steady states:

$$
\begin{aligned}
g &= \frac{\Omega_3+\Omega_2\pm\sqrt{\Delta}}{2}\\
\frac{b}{Y} &= \frac{\Omega_3-\Omega_2\pm\sqrt{\Delta}}{2\Omega_2}
\end{aligned}
$$

where $\Delta = (\Omega_2-\Omega_3)^2+4\Omega_1\Omega_4$.

In one of these steady states it turns out that $\frac{b}{y} = \frac{1-a}{R-(1-\delta)}$ and $g = 1-\delta$. Using these values, I infer that in this steady state there is no government spending ($\Phi, X = 0$) and that all labor income is taxed away ($\frac{tN_y}{Y} = 1-a$), in order to finance interest payments on the outstanding debt. Thus, equilibrium debt reaches its natural sealing, which is the present value of all future labor income, given that there is no public investment. Consequently, output shrinks at the depreciation rate of the public capital (δ).

I call this steady state *immiseration equilibrium*. In real life situations, this equilibrium can't be reached, since there will be a debt default before that. In this respect, I think of this equilibrium as a proxy for government debt defaults.

In the other steady state, debt to output and growth are a complex function of all the parameters. I call this steady state *interior equilibrium*. I will discuss this equilibrium further in the context of the next section, which considers a calibration of the model.

4.2 Stability and transitional dynamics

Appendix A discusses graphically the issue of steady state stability. As it turns out, one of the above steady states is stable and one not. However, which of the two steady states is stable depends on the values of the parameters. Moreover, the values of the parameters determine the steady state with the greatest debt to output.

In this respect, I can identify four situations of interest: First, the interior steady state is stable and the corresponding equilibrium debt to output is smaller than in the immiseration steady state. Second, the interior steady state is stable and the corresponding equilibrium debt to output is greater than in the immiseration steady state. Third, the immiseration steady state is stable and the corresponding equilibrium debt to output is smaller than in the interior steady state. Fourth, the immiseration steady state is stable and the corresponding equilibrium debt to output is greater than in the interior steady state. Clearly, the first situation is the one which is empirical relevant in most cases, while the third and especially the fourth might be relevant in incidents of debt default.

The details of the transitional dynamics also depend on the value of the parameters. This is also true for the sign of Ω_1, which is of particular interest, given the empirical discussion on the relationship between government debt and growth. The ambiguous sign of Ω_1 is due to the dual role of debt in the model. Specifically, on the one hand an extra unit of debt increases interest payments. Consequently, it deprives the government from resources that can finance public investment and therefore enhance growth. On the other hand, an extra unit of debt can be used to increase public investment and thus boost growth.

I continue the discussion on the issues of stability and transitional dynamics in the next section, in which I consider a calibrated version of the model.

5 Quantative Analysis

This section shows that a reasonably calibrated version of my model delivers intuitive results that match key economic features of developed western democracies. I use the outcome of this calibration, in order to characterize the corresponding transitional dynamics and asses the effects of key parameters on equilibrium debt and growth. Then, I discuss some implications of my findings. This empirical exercise follows similar work by Song et al. (2012), Gonzalez-Eiras and Niepelt (2012) and Lancia and Russo (2016) among others.

In order to proceed with the calibration, I assume one period in the model corresponds to 30 years in the data. Furthermore, I assume that the capital's share of output is $a = 0.3$, that the annual interest rate is 5% ($R = 1.05^{30}$), that the annual time discount rate is 0.5% ($\beta = 0.995^{30}$) and that the public capital discount rate (δ) is zero. In accordance with the model, I also assume that the world interest rate is constant. Finally, I set the relative political power of the two generations μ equal to 3. This value corresponds to a political power of the old equal to 0.25, as in Song et al. (2012). Thus, three parameters remain undetermined: the preferences for public goods θ and λ and production efficiency A. I calibrate these parameters in order to fit available data from developed western democracies in the period 1980-2010. In particular, I let $\frac{X+\Phi}{Y}$ in the steady state match actual government spending to GDP, g in the steady state match the actual growth rate and $\frac{b}{Y}$ in the steady state match the actual debt to GDP ratio.

The data I use for the calibration come from a synthetic economy consisting of countries that have been OECD members for the entire 1980-2010 period. In particular, from the 24 countries that make up this particular group, I exclude USA, Japan, Germany and Turkey. The remaining 20 countries[6] form a homogeneous set of small open economies that have been democratic and developed throughout the entire period between 1980 and 2010.

In order to carry out the necessary calculations, I set the debt and government spending to GDP equal to their respective values in 1995, which is the middle of the period of interest and the output growth equal to the actual GDP growth between 1980 and 2010. The following table summarizes the key details of the calibration.

TABLE 1[7]

[6]These 20 countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweeden, Switzerland and United Kingdom.

[7]Data source: International Monetary Fund

Calibration overview

Preset Parameters			
Time Disc. (β)	0.995^{30} =0.8604	Pol. Power (μ)	3
Int. Rate (R)	1.05^{30} =4.3219	Cap. sh. (a)	0.3
Pub. Cap. Disc. (δ)	0		
Targets			
Debt to GDP (b/Y)	0.0239= 0.717/30	gsp/gdp ($\frac{X+\Phi}{Y}$)	0.4927
Growth (g)	1.8950		
Calibrated Parameters			
Pref.-o (θ)	3.3475	Pref.-y (λ)	0.72
Effic. par. (A)	5.0708		
Calculated Parameters			
Ω_1	-4.7909	Ω_2	2.3399
Ω_3	0.8578	Ω_4	0.0444
E	3.2414	immis. b/Y	0.2107 (632%)

The above results indicate that the targeted steady state is stable in the calibration and that the corresponding debt to output is smaller than in the immiseration state. In this respect the results of the calibration are intuitive. Moreover, I find that preferences for public goods are stronger for the old than for the young, an outcome this model shares with Song et al. (2012).

The fact that Ω_1 is negative implies that on the transition path to the steady state, debt to output and growth have a negative relation. In order to better describe the transitional dynamics of the calibration let me follow Song et al. (2012) and consider an unexpected event (e.g. a war), which increases debt. In particular, consider an increase by 100 percentage points from 72% on an annual basis to 172%. As a result, the growth rate drops by 8.5%, which on an annual basis amounts to a drop from 2.2% to 1.9%. Then, as debt to output decreases, in order to revert to its steady state level, the growth rate gradually increases. In this respect, the calibration indicates that debt to output covers half of the distance to the steady state, in the first period after the initial shift, while growth requires an additional period to achieve that. The figure below depicts this exercise.

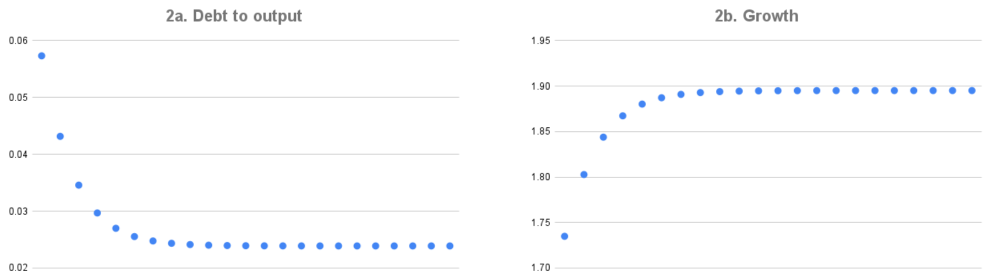

Figure 2: Figures 2a and 2b respectively portray the path of debt to output and growth after an exogenous increase in debt to output by a hundred percentage points on an annual basis.

The next table considers a comparative statics exercise. I calculate the effect of an 1% increase in the value of a parameter on debt to output, growth and government spending, both in the steady state (LR) and in the immediate period after the shift (SR).

TABLE 2
Comparative Statics I

Elasticities	**b/Y SR**	**b/Y LR**	**g SR**	**g LR**
λ	4.2	9	0.4	-0.1
θ	-5.4	-11.3	-0.3	0.4
$\theta + \lambda$	-1.2	-2.3	0.1	0.3
μ	-0.8	-1.5	0.1	0.2
A	16.5	34.9	2.2	0

The table above indicates that μ has the expected effect. Specifically, an increase in the political power of the young decreases debt and increases growth, both in the short and in the long run. Also, the sum $\theta+\lambda$ displays the effect of an 1% increase on both θ and λ. I interpret an increase in this sum, as an increase in the trust of the citizens to the government, or a decrease in government corruption[8]. In this respect, such an increase also has the expected effect, since it decreases debt to output and increases growth. On the other hand, the isolated effects of the preference parameters are mixed, while an increase in the efficiency parameter (A) increases both debt to output and growth, at least in the short run. Thus, certain shifts in parameters can have effects that are inconsistent with the transitional dynamics.

However, countries typically differ in parameters. Therefore, a cross country empirical investigation might not be able to identify a direct negative relationship between debt and growth. This fact can possibly explain why empirical findings on the relationship between growth and debt are in certain cases weak or inconclusive [9] .

I turn now on the effects of the parameters on government spending. The following table summarizes these results.

TABLE 3
Comparative Statics II

Elasticities	$\frac{X+\Phi}{Y}$ SR	$\frac{X+\Phi}{Y}$ LR	X/Φ SR	X/Φ LR
λ	0.7	0.9	1	1
θ	-0.5	-0.8	-1	-1
$\theta + \lambda$	0.2	0.1	0	0
μ	-0.2	-0.2	1	1
A	1.8	3.6	3.1	3.2

The table above shows that an increase in the political power of the young (μ) has the expected effect since it decreases government spending and increases the ratio of public investment to transfers. Also, an increase in production efficiency (A) increases government spending along with the share of public investment. This is a standard substitution effect. As A increases, the return of public investment also increases. Thus, the democratic government opts to transfer resources from consumption to public investment.

Moreover, the effect of the two preference parameters (θ and λ) on the ratio of the two public goods is intuitive. However, the negative effect of θ on government spending requires explaining. An increase in the preferences for transfers has two opposing effects. On the one hand it induces government to

[8]See Song et al. (2012) for a similar interpretation.

[9]See for example Reinhart and Rogoff (2010) along with Herndon et al. (2014)

increase spending in order to satisfy the current old. On the other hand, this increase intensifies the disciplining effect of future transfers. In particular, young voters worry that an increase in spending will also increase debt and thus undermine the provision of public goods in the future. If this disciplining effect is strong, θ can have a negative effect on government spending.

Finally, an increase in the trust to the government $\theta + \lambda$ increases government spending. However, table 2 indicates that this increase co-occurs with a smaller debt. In this respect the model is consistent with the Scandinavian type of economy, which exhibits high government spending but low debt and corruption.

6 Conclusion

This paper considers a political economy model which investigates the relationship between debt and growth. This model conveys the inter-generational conflict underpinning the allocation of government resources between transfers and investment. A calibration of the model indicates that corruption and population aging adversely affect both growth and debt accumulation. Moreover, the calibrated model implies a negative relationship between growth and debt in the short run. However, this negative relationship might fail to appear in cross country empirical research, since in the long run it might not exist, while in the short run, differences in parameters between countries might hide it.

These results emerge from a calibration based on data collected from developed western democracies, which is a group that best fits the assumptions of the model. Whether these results extend beyond this important, but nevertheless small, group of countries is an open question.

Appendix A

Graphical analysis of the stability of steady states

Figure A1 below depicts the relationship between $(b/Y)'$ and b/Y as portrayed by (23) in the main text. Specifically Figure A1 indicates that the graph of (23) has two distinct branches (A and B).

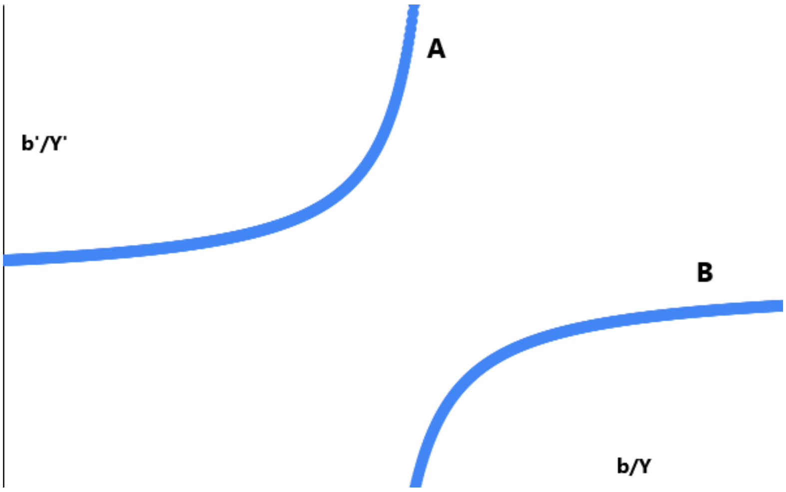

Figure A1

In this respect, the steady states lie on the intersections of the above graph with the 45 degree line. Depending on the value of the parameters, these intersections can take place either on branch A or on branch B. Moreover, the shape of both branches implies that in any case there will be two such intersections. Figure A2 below depicts these possible intersections.

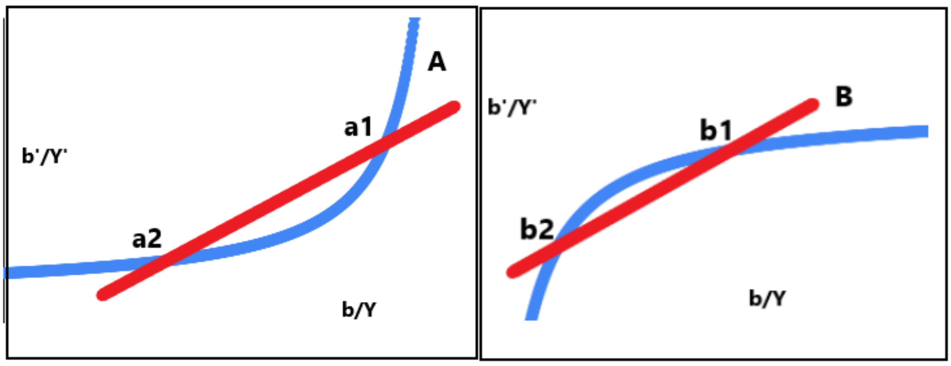

Figure A2: The red line depicts the 45 degree line.

Points $a1, a2, b1$ and $b2$ depict possible steady states. In each branch there is one stable and one unstable steady state. Specifically, if the steady states are in branch A, then the steady state depicted by $a2$ is stable and the one depicted by $a1$ is unstable. On the other hand if the steady states are in branch B, the steady state corresponding to $b1$, is stable while the one corresponding to $b2$ is unstable.

Moreover, following the analysis in 4.1, in any brunch, one finds the immiseration and the interior steady state. However, whether in branch A the immiseration steady state corresponds to $a1$ or $a2$ depends on the parameters and likewise for branch B. Thus, the four situations of interest discussed in 4.2 arise. In this respect the intuitive situation is when $a2$ depicts the interior steady state and $a1$ depicts the immiseration steady state. In this case the interior steady state is stable and exhibits smaller debt to output than the immiseration steady state. The calibration of the model yields intuitive results in this respect.

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