On the Measurement of Intergenerational Mobility in Germany and the United States

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Abstract

This study is a cross-country comparison of intergenerational mobility through a novel welfare-based approach. We utilize the concept of multivariate risk aversion, also known as correlation aversion, to compare the interdependence between parents' income and their children in Germany and the United States. This new approach is a generalization of the Atkinson framework. We use data from the German Socio-Economic Panel and Panel Study of Income Dynamics to illustrate the new welfare measure of intergenerational mobility. Our results are compared with prior literature findings based on the standard intergenerational elasticities. Our findings indicate higher intergenerational mobility in the United States challenging standard literature results.

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Introduction

Atkinson (1970) links the literature on the measurement of inequality and risk using an analogue of the certainty equivalent and the proportional risk premium for the introduction of an index of income inequality. In this one-dimensional approach, the proposed index can be considered as the proportion of welfare that a society is willing to waste for an equally distributed income — the prefect equality. The Atkinson index is the most popular welfare-based measure of income inequality. Knowing the dynamics of inequality over the horizon of a working lifetime is certainly important, but we also need to know what happens for a longer horizon. As Corak (2013) emphasizes we need to investigate how inequality transmits from parents to their children and whether families pass on their economic status to their children. Our focus is on the study of changes of inequality from generation to generation for a better understanding of intergenerational mobility. We generalize Atkinson framework based on multivariate risk aversion.

Multivariate risk aversion is a concept used in the literature of economics of risk. Richard (1975) is one of the first papers regarding multivariate risk aversion where the notion of risk aversion is extended to bivariate utilities. Compared with other early papers on this subject, his approach is closer to the idea of utility premium. We extend Richard's (1975) framework to multivariate utility functions with more than two attributes and suggest its application for the study of intergenerational mobility. The concept of multivariate risk aversion is used to propose a new measure of intergenerational mobility.

Continuing the line of Richard (1975), Epstein and Tanny (1980) discuss further the question "when is a multidimensional random variable Y riskier or more variable than another random variable X," and change the name of the concept from multivariate risk aversion to correlation aversion (CA). A correlation-averse agent is described through his/her preference for a lottery with two negatively correlated attributes than one with two positively correlated attributes. The utility function of such an agent is characterized by a negative second cross derivative. Second cross-derivative functions have been used for a lot of different definitions, within a utility function framework, such as the characterizations of two commodities as Auspitz-Lieben-Edgeworth-Pareto (ALEP) complimentary.

One of the most often used statistic in this literature is named "intergenerational elasticity in earnings", representing the percentage difference in earnings in the child's generation associated with the percentage difference in the parental generation. This is an indicator of how correlated the two variables are. Even for the multidimensional inequality the correlation between the attributes that account for the multiple facets of an individual's well-being matters, as suggested by Stiglitz et al. (2009). Atkinson and Bourguignon (1982) use the concept of elementary correlation increasing transformation as introduced by Epstein and Tanny (1980) while they discuss different assumptions about the sign of the second cross derivative. The correlation increasing transformation is an extension of the notion of a mean-preserving risk from one to two variables. Tsui (1995, 1999) moved on to the axiomatic approach to inequality indices to achieve complete orderings. Tsui (1999) also includes the concept of correlation-increasing transformation on his framework by introducing a new majorization criterion in the context of multidimensional inequality. Correlation increasing majorization (CIM) is actually a sequence correlation-increasing transfer as an exchange of all attributes between two individuals after which one individual is left with the lowest endowment and the other with the maximum endowment of each attribute. By concentrating attributes, this type of transfer leads to a distribution that is less socially preferable than the original one.

The new proposed measure of intergenerational mobility generalizes the one-dimensional Atkinson approach by defining inequality from generation to generation to be the fraction of total welfare gained by avoiding perfectly correlated variables of earnings between parents and children. The attributes of the welfare function represent the income of parents and children on different periods allowing us to shed light to how income inequality evolves over a longer time horizon than lifetime. Our approach is based on a submodular welfare function and a measure of the intensity of (CA) generalized for multivariate utility functions.

Bommier (2007) discusses the notion of CA in the context of an inter-temporally, non-separable bivariate utility function and introduces a measure of the intensity of CA. The attributes represent consumption at different moments in time, and the measure is given with respect to two attributes. In the same direction, a generalized definition of CA for an inter-temporal model with multi-attribute utility functions was discussed by Lichtendahl et al. (2012), where all the attributes represent income at different moments in time. Without loss of generality, Crainich et al. (2017) propose a measure of absolute CA under the assumption that the first attribute could be the subject to a monetary payment while the second attribute is nonpecuniary. This assumption allows us to include attributes in the welfare function that are not monetary. Despite the fact that Bommier (2007) uses a framework with continuous distribution while Crainich et al. (2017) uses discrete distributions, the two indices are similar under the assumption that only the first attribute could be the subject to a monetary payment. In this paper we generalize this approach for multivariate utility functions with more than two attributes that can be either pecuniary or nonpecuniary.

This study is organized as follows. In the first section, we generalize the index of absolute CA for utility functions with two or more attributes. The new index allows the introduction of a concept that we call intergenerational immobility aversion. In the second section, we present a new measurement of intergenerational mobility. Finally, an illustration of the index is presented based on household data from the German Socio-Economic Panel (SOEP) and Panel of Income Dynamics (PSID).

Intergenerational Immobility Aversion

Starting with the bivariate case, we consider the case of preferences over two attributes measured by the variables x and y. As discussed in the literature, bivariate risk aversion (correlation aversion) can also be defined as follows:

Definition 1. The individual is correlation averse if and only if, for all x, y, and $l > 0$ and $k > 0$, the lottery A is preferred over lottery B

(i) Lottery A gives the decision maker a 0.5 probability of receiving (x-l, y-k) and a 0.5 probability of receiving (x, y).

(ii) Lottery B gives the decision maker a 0.5 probability of receiving (x-l, y) and a 0.5 probability of receiving (x, y-k).

Correlation premium (m) can be defined as the payment that makes an individual indifferent between the two lotteries. Crainich et al. (2017) create the index of correlation aversion by answering the question: "What is the amount of money (correlation premium) to be paid to the decision maker who initially faces lottery A so that he becomes indifferent between A and B?" This amount of money (m) will be paid only if the outcome of the lottery is the ordered pair that involves the high levels of variables X and Y. By following Pratt (1964), Crainich et al. (2017) prove that the payment is given as below:

$$m \approx -\frac{lkU_{12}(x,y)}{U_1(x,y)} < 0.$$

The proposed index is given by the fraction

$$C(x,y) = -\frac{lkU_{12}(x,y)}{U_1(x,y)} \qquad (1)$$

In this section, we generalize the index $I(x,y)$ for utility functions with more than two attributes. This generalization helps us to measure the intensity of multivariate risk aversion. We propose the characterization of multivariate risk aversion to be referred as intergenerational immobility aversion for welfare functions. Thus submodular (supermodular) welfare functions exhibit multidimensional intergenerational immobility (seeking). This index summarizes information about the correlation among all the attributes.

Continuing the line of the bivariate case, we consider a twice differential utility function $U(x,y,z)$ that has three attributes. Additionally, we introduce the following operations between two vectors. Based on two randomly given vectors $v_0 = (v_{01}, v_{02}, v_{03})$ and $v_1 = (v_{11}, v_{12}, v_{13})$, we denote

$v_0 \vee v_1 = (max\{v_{01}, v_{11}\}, max\{v_{02}, v_{12}\}, max\{v_{03}, v_{13}\})$ while $v_0 \wedge v_1$ is denoted as $v_0 \wedge v_1 = (min\{v_{01}, v_{11}\}, min\{v_{02}, v_{12}\}, min\{v_{03}, v_{13}\})$. Without loss of generality, consider the following order for the attributes of two given vectors $v_{01} > v_{11}$, $v_{02} < v_{12}$ and $v_{03} > v_{13}$. We create two lotteries where lottery B gives the decision maker a 0.5 probability of receiving v_0 and 0.5 probability of receiving v_1. Meanwhile, lottery A gives the decision maker a 0.5 probability of receiving $v_0 \vee v_1$ and 0.5 probability of receiving $v_0 \wedge v_1$ as shown in Figure 4.1.

Figure 4.1 Correlation aversion in a multi-attribute setting (two binary lotteries), where v_0 and $v_1 \in \mathbb{R}^n, n > 2$

To the best of our knowledge, this is the first time that CA in a multi-attribute setting has been introduced in this manner. The definition based on lotteries A and B help us to introduce a generalization of the index of absolute CA, $I(x, y)$ for utility functions with more than two attributes. This index summarizes information about the correlation among all the attributes. Lottery B offers to the decision maker a chance to get all high-level or all low-level values for all the attributes with the same probability. On the other hand, lottery A offers two outcomes with some high-level and some low-level values. Without loss of generality assume that the values for the first and the last attribute of v_0 are higher than v_1. Set $v_0 = (x, y - l_2, z)$ and $v_1 = (x - l_1, y, z - l_3)$, where l_1, l_2 and l_3 are positive real numbers. For a decision maker the difference between expected utility for lottery A and B is given as

$$Eu(A) - Eu(B) = 0.5\big(U(v_0) + u(v_1)\big) - 0.5\big(U(v_0 \vee v_1) + U(v_0 \wedge v_1)\big) =$$

$$0.5\big(U(x, y - l_2, z) + U(x - l_1, y, z - l_3)\big) - 0.5\big(U(x, y, z) + U(x - l_1, y - l_2, z - l_3)\big)$$

By expanding each term up to the second order around $U(x, y)$, we get

$$EU(A) - EU(B) = 0.5\big(l_1 l_2 U_{12}(x, y, z) + l_2 l_3 U_{23}(x, y, z)\big) < 0 \qquad (3)$$

As we show in proposition 1, a decision maker with a utility where all the second cross derivatives are negative would prefer A to B.

Following the same process as in the previous section, we can show that a payment (p), which is the maximum amount of money to be paid by the decision maker to avoid a perfectly positively correlated loss is given by Equation (4). This is a payment from the first attribute that applies to both states of lottery A in Figure 4.1.

$$\mathrm{p} = -\frac{\sum_{i=1}^{3} l_i U_{ii}(v_0 \vee v_1) + \sum_{i=1}^{3}\sum_{j=1}^{3} 2 l_i l_j U_{ij}(v_0 \vee v_1)}{4U_1(v_0 \vee v_1)} \qquad (4)$$

Without loss of generality, we assume the same example and set $v_0 = (x, y - l_2, z)$ and $v_1 = (x - l_1, y, z - l_3)$, where l_1, l_2 and l_3 are positive real numbers. If the decision maker is indifferent between lottery A and B in Figure 4.1 then

$$U(x, y, z) - (p + l_1)U_1(x, y, z) - l_3 U_3(x, y, z) + U(x, y, z) - pU_1(x, y, z) - l_2 U_2(x, y, z) =$$

$$= U(x,y,z) - l_1 U_1(x,y,z) - l_2 U_2(x,y,z) - l_3 U_3(x,y,z) + \frac{1}{2} l_1^2 U_{11}(x,y,z) + \frac{1}{2} l_2^2 U_{22}(x,y,z) + \frac{1}{2} l_3^2 U_{33}(x,y,z)$$

Solving for the payment p, we can derive that

$$p = -\frac{l_i^2 \sum_{i=1}^{3} U_{ii}(\cdot) + l_i l_j \sum_{i=1}^{3} \sum_{j=1}^{3} 2U_{ij}(\cdot)}{4U_1(\cdot)}.$$

Moreover, by assuming small losses similar to Arrow-Pratt's methodology, we assume that the index plays the role of multivariate CA when $n = 3$.

$$\mathrm{I} = -\frac{\sum_{\mathrm{i}=1}^{3} \mathrm{U}_{\mathrm{ii}}(\cdot) + \sum_{\mathrm{i}=1}^{3} \sum_{\mathrm{j}=1}^{3} 2\mathrm{U}_{\mathrm{ij}}(\cdot)}{4\mathrm{U}_1(\cdot)} \tag{5}$$

The payment as defined in Equation (4) can be applied to all equivalent frameworks for CA in a multi-attribute setting. Assume two vectors v_0 and v_1 from $\mathbb{R}^n$, where n ≥ 3 and a utility function with n attributes. We also denote the decision maker's utility function as $U: \mathbb{R}^n \to \mathbb{R}$ which exists, and it is twice partially differentiable. Lichtendahl et al. (2012) define CA in a multi-attribute setting as follows.

Definition 2. A decision maker with utility function U, is correlation-averse if for all v_0 and v_1 and any $q_L < q_H$, he prefers lottery L(q_L) to lottery L(q_H), where lottery L(q) is defined in Figure 4.2 .

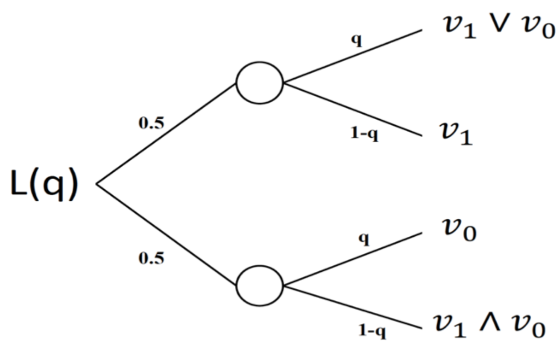

Figure 4.2 Correlation aversion in a multi-attribute setting (one lottery), *where* v_0 *and* $v_1 \in \mathbb{R}^n, n > 2$

The vectors can also represent consumption or income at different time periods. For example, the vector $v_0 = (v_{01}, v_{02}, v_{03}, \dots, v_{0n})$ can be interpreted as the n-period lifetime consumption stream where the decision maker consumes v_{0i} over the period time $i - 1$ to i. The utility comparison that follows from a preference for lottery $L(q_L)$ to lottery $L(q_H)$ can reduce the definition to the condition $U(v_0 \vee v_1) + U(v_0 \wedge v_1) \geq U(v_0) + U(v_1)$. Thus, a correlation averse decision maker, as proposed above, always prefers lottery A than lottery B in Figure 4.1. In other words, the definitions of CA based on Figure 4.1 and 4.2 are equivalent. The payment (p), as denoted in Equation (4), in the setting of Figure 4.2 can be interpreted in the following way. If the lottery realization is either v_0 or v_1, the decision maker will subtract m units from the first attribute. Thus, m is the maximum amount of money to be paid by the decision maker today at branches v_0 and v_1 to become indifferent about the value of q. We generalize all the previous results for the multivariate case.

Proposition 1. A decision maker, where all the second cross derivatives are negative, is multivariate risk averse.

Proof

Suppose there are vectors $v_k \in \mathbb{R}^n, k = 1, \dots m$, where their distribution is described by F_A, and a utility function $U(x_1, \dots, x_n): \mathbb{R}^n \to \mathbb{R}$. When $m = 2$, F_A can be described by the lottery A of Figure 4.1. When all the pairwise operations $v_k \vee v_l, \forall k, l, and\ k \neq l$ are performed, the new distribution is denoted by F_B. The marginals distributions are unaffected by the described operations, but the pairwise correlation is increased. Using a second-order Taylor approximation around $(\bar{x}_1, \dots, \bar{x}_n)$, where $\bar{x}_i$ denotes the average value of the marginal distributions for dimension i, the difference in expected utilities:

$$\sum \dots \sum U(x_1, \dots, x_n)\,(f_B(\cdot) - f_A(\cdot)) \approx \sum_{\substack{i,j \\ i \neq j}} U_{ij}(\bar{x}_1, \dots, \bar{x}_n)\, \Delta cov(x_i, x_j)$$

Since $\Delta cov(x_i, x_j) \geq 0$, and $U_{ij}(\bar{x}_1, \dots, \bar{x}_n) < 0, , \forall i, j,$ then

$$\sum_{i,j} U_{ij}(\bar{x}_1, \dots, \bar{x}_n)\, \Delta cov(x_i, x_j) < 0$$

So, F_A is preferred.

Although the continuous case is beyond the scope of this article, after a finite number of operations, we can show easily that the process is similar for continuous distributions F_B and F_A, and the result remain the same.

$$\int \int U(x_1, \dots, x_n) d(F_B - F_A) \approx \sum_{\substack{i,j \\ i \neq j}} U_{ij}(\bar{x}_1, \dots, \bar{x}_n) \int \int (x_i - \bar{x}_i)(x_j - \bar{x}_j) d(F_B - F_A)$$

$$= \sum_{\substack{i,j \\ i \neq j}} U_{ij}(\bar{x}_1, \dots, \bar{x}_n)\, \Delta cov(x_i, x_j)$$

Since $\Delta cov\left(x_i, x_j\right) \geq 0$, and $U_{ij}(\bar{x}_1, \dots, \bar{x}_n) < 0, , \forall i, j,$ then

$$\sum_{i,j} U_{ij}(\bar{x}_1, \dots, \bar{x}_n)\ \Delta cov\left(x_i, x_j\right) < 0$$

∎

By following similar arguments, from a payment (p) that is related only to the first attribute, the following equation gives the index of multivariate risk aversion

$$(change\ the\ notation) I = -\frac{\sum_{i=1}^{n} U_{ii}(\cdot) + \sum_{i=1}^{n} \sum_{j=1}^{n} 2U_{ij}(\cdot)}{4U_1(\cdot)} \tag{6}$$

Although the payment (p) is related to the first attribute, the index above includes all the second cross derivatives of the utility functions, U_{ij}, where $0 < i, j < n, i \neq j, n \geq 3$. Moreover, the index above leads to the index proposed in Equation (2) when n=2. This is related to the matrix of local risk aversion as defined by Karni (1977).

As indicated above, these indices are related to payments that make an individual indifferent between discrete lotteries and can be applied to lotteries A and B with continuous density functions $f_A(x, y)$ and $f_B(x, y)$, respectively. For all these cases, the measurement of the intention of an agent to prefer the lottery with the lower level of interdependence gives the degree of CA. Since the formula in Equation (6) works for preferences with two or more attributes, this generalization allows application of the index to various settings in welfare or financial economics. A good example is an intertemporal model in which preferences depend on the lifetime path of consumption and the decision maker faces various trade-offs between the levels and riskiness of consumption in successive periods.

In this study, we will discuss an application to the study of inequality from generation to generation. So, we first define the social evaluation function and then introduce the index for the measurement of the degree of intergenerational immobility aversion.

Suppose there are k generations from n families. Each distribution can be represented by a matrix in $\mathbb{R}_+$. The normative evaluation of inequality from generation to generation will be based on an underlying social evaluation function. Consider a utility function that is given as $U(x_1, x_2, \ldots, x_k)$. The attributes represent earnings from each generation. Without loss of generality, we can assume that x_1 represents the current wealth of an individual. Consider two lotteries A and B with probability functions $f_A(x_1, x_2, \ldots, x_k)$ and $f_B(x_1, x_2, \ldots, x_k)$ respectively. The univariate margins are the same but the correlations between the attributes differ. Correlation premium (m) can be defined as the payment that makes an individual indifferent between the two lotteries (the distributions can be discrete or continuous).

The social evaluation ordering $\underline{R}$ is defined as a reflexive, transitive, and complete binary relation on the set of distributions. We write $f_A \, \underline{R} \, f_B$when f_A is socially preferred to f_B, and $f_A \, \underline{I} \, f_B$ for indifference. The notation for the strict social preference is $f_A \, \underline{P} \, f_B$.

A social welfare function $W: \mathbb{R}_+^{m \times n} \to \mathbb{R}$ allows us to analyze conveniently the social evaluation ordering. The framework used in this study can be considered as a direct generalization of the Atkinson's approach. A society would rank social distributions that are represented by $f_A(x_1, x_2, \ldots, x_k)$ according to the following social welfare function;

$$W = \int_0^{\chi_k} \ldots \int_0^{\chi_1} U(x_1, x_2, \ldots, x_k) \cdot f_A(x_1, x_2, \ldots, x_k) dx_1 \ldots dx_n$$

where, $0 \leq x_1 \leq \chi_1$, $0 \leq x_2 \leq \chi_2$,…,$0 \leq x_k \leq \chi_k$.

This is an additively separable social welfare function across individuals. In a special case where the social distributions are discrete, implying a finite population of individuals, social distributions can be simply represented by matrices in $\mathbb{R}_+^{m \times n}$.

Definition 3. f_A is socially preferred to f_B iff

$$\int_0^{\chi_k} \dots \int_0^{\chi_1} U(x_1, x_2, \dots, x_k) \cdot f_A(x_1, x_2, \dots, x_k) dx_1 \dots dx_n$$

$$> \int_0^{\chi_k} \dots \int_0^{\chi_1} U(x_1, x_2, \dots, x_k) \cdot f_B(x_1, x_2, \dots, x_k) dx_1 \dots dx_n$$

The specification of the function $U(\cdot)$ is important for the rankings of the distributions.

Definition 4. A function $U(\cdot)$ where all the second cross derivatives of the are negative, exhibits intergenerational immobility aversion.

The index, as defined in Equation (6) acts as a measurement of intergenerational immobility aversion. for a society with a specific social evaluation function. So, we summarize this as follows:

Definition 5. An index of intergenerational immobility aversion for preferences with more than two attributes is given by

$$I = -\frac{\sum_{i=1}^{n} U_{ii}(\cdot) + \sum_{i=1}^{n} \sum_{j=1}^{n} 2U_{ij}(\cdot)}{4U_1(\cdot)}$$

In this next section we discuss a new measurement of intergenerational immobility that will contribute to the problem of comparing distributions of many attributes.

A Measurement of Mobility

When we need to compare multivariate distributions, as in the literature of well-being inequality (Kolm (1997), Tsui (1997), Atkinson and Bourguignon (1982), Delancq and Lugo (2012)), there are two basic multidimensional distributional concerns.

The first one, called uniform majorization (UM) principle, is a generalization of the Pigou-Dalton transfer principle. The idea for the Pigou-Dalton transfer can be expressed as follows. If a strictly positive transfer is made from a richer person to a poorer person, then the new distribution is more socially preferred. Hence, in a multidimensional setting, if a uniform mean preserving averaging is carried in all dimensions, the resulting distribution is socially preferred to the original one. In a discrete case, the UM principle can be defined as follows.

Uniform majorization principle (UM). For all distribution matrices X and Y in $\mathbb{R}_{++}^{n\times m}$, if Y = BX for some $n \times m$ bistochastic matrix B, $Y \neq X$, and Y is not a permutation of X.

A sequence of uniform mean-preserving averaging in all dimensions can result in a social state where there are no differences for the values of the variables of income within families. The other concern is related to the idea that society is averse to correlation-increasing transfers. The correlation increasing transfer was introduced by Epstein and Tanny (1980). Equivalently to definition 1, a utility function is correlation-averse iff it is reduced by a correlation increasing transformation. The definition of CIT is given below.

Definition 6. For all $X, Y \in \mathbb{R}_{++}^{n\times m}$. Y is obtained from X through a CIT if $Y \neq X$, Y is not a permutation of X, and there are two individuals, k and l in N such that, $y^i = x^i$ for all i∉{k,l}

Now we can introduce the second multidimensional distributional concern as defined in the literature of well-being inequality.

Correlation increasing majorization principle (CIM). For all distribution matrices X and Y in $\mathbb{R}_{++}^{n\times m}$, if Y is obtained from X by a correlation increasing transfer (CIT).

These two multidimensional distributional concerns are related to the degree of correlation aversion and risk (inequality) aversion in all dimensions for the society. The correlation-increasing principle and UM principle are addressed as independent in the literature of multidimensional inequality. One reason, these principles seem to present different aspects of multidimensional inequality is that UM comes from an idea in one dimension while CIM cannot be easily defined in one dimension. Based on definition 2, we can show that CIM in one dimension is similar to UM. Our motivation comes from the following example.

Example. Suppose a society has four different social classes over a period of time that extends beyond one generation. (0.5-π) percent of the families belong to the first class, where $\pi \in (0, 0.5)$. π percent of the families belong to the second class and observe a vector of income over generations v_o, while π percent of the families to the third class with a vector of income over generations that represented by v_1. The rest of the families belong to the fourth social class. The vector of income over generations for the first class is given as $v_o \vee v_1$, and the vector of income for fourth class as $v_o \wedge v_1$.

Table 1 Initial social distribution

Class	Proportion
Old money	$0.5 - \pi$
Upper middle	π
Lower middle	π
Poor	$0.5 - \pi$

The families in the first class enjoy the maximum value of income over generations and can be considered "old money", while those of the fourth class enjoy the minimum value and can be called as "poor". By assuming that there is an indicator that subsumes all income levels over generations, we can separate the rest of the families in the distribution as upper middle and lower middle. Since we consider that the attributes represent income in different time periods, in that case, the indicator is the present value of the income stream. Thus, the second class of families[2] (upper middle) enjoys a higher level of the discounted income stream than the third class (lower middle). The initial social distribution is indicated in Table 1. Then we apply a sequence of CIT among 50% of the families in the upper middle class and 50% of those in the lower middle class. By replacing 50% of the pairs v_o and v_1 by $v_o \wedge v_1$ and $v_o \vee v_1$, we get a new distribution that has more rich and poor families than before, as presented in Table 2.

[2] The vector v_o can represent families where some of the incomes over time are relatively high. For example, new money or old rich families who have not maintained their initial level of wealth.

Table 2 New social distribution

Class	Proportion
Old money	$0.5-\frac{\pi}{2}$
Upper middle	$\frac{\pi}{2}$
Lower middle	$\frac{\pi}{2}$
Poor	$0.5-\frac{\pi}{2}$

The new distribution is less socially preferred than the initial one and comes as a result of a four-point mean-preserving spread, as defined by Rothschild and Stiglitz (1970). This can be considered as a result opposite to that of Pigou-Dalton transfer from poor (individuals with a low-level indicator of well-being) to rich. A similar result has been shown in Bosmans et al. (2009) and is summarized in the following proposition.

Proposition 2. In one dimension, the CIT can be considered as a mean-preserving spread.

In a multidimensional setting, as a sequence of UM transfers would have resulted in a threshold social distribution, where all people in the society enjoy the same level for each variable, a sequence of CI transfers would have resulted in another threshold social distribution with highly correlated attributes without changes for the marginal distributions. In this situation, one family is top-ranked in all dimensions, another second-ranked, and so on. This can be characterized as the perfect inequality. Thus, we propose that intergenerational mobility can be measured as follows.

Definition 7. The degree of intergenerational mobility is measured as the proportion of welfare that a society is willing to receive additionally for perfectly correlated attributes among the individuals.

Based on the initial threshold social distribution, the standard normative approach defines the degree of multidimensional inequality as a scalar that represents the fraction of the aggregate amount of each dimension that can be destroyed if every dimension of the matrix is equalized while keeping the resulting matrix socially indifferent to the original matrix. Definition 7 does not present an indexing dilemma but rather, is a complement approach that should be used, especially for the study of many types of multidimensional inequality.

The following type of social welfare function offers a plethora of attractive properties including monotonicity, symmetry, and separability. Hence, the proposed $U(x_1, x_2, \ldots, x_k)$ is multiplicatively separable across the generations and given as

$$U(x_1, \ldots x_n) = -\frac{x_1^{-a_1}}{a_1} \ldots \frac{x_n^{-a_n}}{a_b}$$

Thus, $U(x_1, \ldots x_n)$ is a constant relative inequality aversion dimension by dimension and exhibits CA for each pair of (x_i, x_j), i$\neq j$.

The index of intergenerational mobility is given by[3]

$$MI = 1 + \frac{p}{\bar{x}} \tag{7}$$

[3] Add a general equation

where p (or m) is the correlation premium, and $\bar{x}$ is the average wealth of the society today. For the empirical illustration in the next section, we consider a bivariate utility function with $a_1 = a_2 = 0.5$. The index MI is well-defined if $\underline{R}$ is continuous and monotonic (Weymark 2006).

Since this measure captures the correlation among different variables through a supermodular order, it can be more easily related to the study of the inequality from generation to generation (intergenerational mobility), where the attributes of welfare are quantitative variables. In the literature, the study of intergenerational mobility and cross-country comparisons are usually established on the ordinary least square estimation of intergenerational earnings elasticities in standard empirical models of father-and-son earnings. Certainly, the empirical approach changes accordingly, depending on the data available for a specific country. In this study we suggest a welfare-based measurement for cross country comparisons where its order may be stronger when we include more dimensions in the multivariate distributions than just the earnings of father and son. In the following section, we use the proposed index for multivariate distributions.

Empirical Illustration and Discussion

The data used in this research is derived from the Panel Study of Income Dynamics (PSID). PSID data is selected for the comprehensive coverage of income dynamics and socio-economic factors in the United States. The PSID is a longitudinal household survey, which was conducted annually from 1968 to 1997 and biannually from 1997 onward. It provides rich data including demographic and economic information, allowing for the analysis of inequality of opportunity. The variables used in our analysis are age, gender, income, and the number of years in the son's income average. These variables were chosen to align with the research framework presented in the Stockhausen paper, allowing for a direct comparison of our results with those of Stockhausen. Age is a continuous variable representing the age of the individual at the time of the survey. It is used as a control variable in our analysis, as age can significantly influence income levels and career progression. Gender is a binary variable indicating whether the individual is male or female. It is included in our analysis to account for potential gender disparities in income levels and career progression, which can impact intergenerational income mobility. Income is a continuous variable representing the individual's annual income. In our analysis, we use the logarithm of income to account for the skewed distribution of income data. This variable is central to our analysis, as it allows us to measure intergenerational income mobility. The number of years in the son's income average is a continuous variable that represents the number of years over which the son's income is averaged. This variable is used to account for potential fluctuations in income over time, which can impact measures of intergenerational income mobility. Our analysis follows the regression framework presented in Stockhausen (2021). This framework allows us to estimate the intergenerational income elasticity, which is a standard measure of intergenerational income mobility. In addition, we propose a novel measure of intergenerational mobility, which we compare to the standard intergenerational income elasticities.

Our research design, with its focus on the same variables and regression framework as the Stockhausen paper, allows us to provide a robust comparison of intergenerational income mobility in Germany and the United States. The variable of interest is a proxy for lifetime income. We develop a dataset of fathers and sons with their corresponding proxy lifetime incomes. The dataset is constructed by first identifying all household heads that are male. Following the work of others, we exclude female household members from our analysis because changes in the labor market over the period of interest in both countries experienced social changes that likely influence the relationship of interest for women (women join the labor market in the United States during this period, for example). The PSID provides a relationship matrix mapping the connection between parents and their adult children participating in the sample. The male heads of household are then linked by this father-son relationship. To construct the corresponding lifetime income proxies, we take income data for these individuals over a decade. Individuals are only included if they are in their prime earning years. Our analysis starts from the broadest definition of this, typically ages 30 to 50 years old, but alternate definitions have support and can be explored at a tradeoff to sample size. The sample is further subset by the number of observations of incomes an individual has. Any father with 5 or more income observations is retained and an average of their income over the period is constructed. Any son with 3 or more income observations is retained and an average of their income over the period is constructed. (The literature recommends as many years as possible, up to 15 years (e.g., Mazumder, 2018). We use 3 for sons because there are fewer years available in the decade of interest due to the biannual nature of PSID data in later years. The 5-year requirement can be met but dramatically decreases the sample size. Several solutions are available to treat this, if necessary). The years chosen for analysis follow the logic of cohort development presented in the online appendix of Stackhausen (2021). Our resulting dataset contains fathers' lifetime incomes and sons' lifetime incomes, which we use to investigate the transmission of income from father to son.

Based on equation (7) we calculate the index for the two longitudinal datasets.

Table 3 Index illustration

Country	Intergenerational Mobility
USA (PSID)	1.16
Germany (SOEP)	1.02

Our Welfare index indicates a higher intergenerational mobility in the United States in contrast with the standard results in the literature. This is an interesting result given that supermodular order in stronger than concordance (Müller & Scarsini, 2000). Schnitzlein (2016) finds weak evidence for higher intergenerational mobility in Germany compared to the US. The robustness of our results is examined for different level of welfare function's intergenerational immobility aversion.

Conclusion

This study proposes a new framework for analyzing multidimensional inequality. We introduce a new concept called intergenerational immobility aversion, which is parallel to multivariate risk aversion. A new measurement for intergenerational mobility is proposed and illustrated using the German Socio-Economic Panel and Panel Study of Income Dynamics. The results highlight once again the need for more cross-country studies in international comparisons of intergenerational mobility based on different aspects.

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