Versioning by Outsourcing

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Abstract

In this study, we examine the concept of versioning in the context of outsourcing either the high-end or low-end version of a product. Our findings indicate that outsourcing the high-end version leads to an improvement in the quality of the low-end version. This improvement is attributed to a bargaining effect, where a higher quality of the low-end version enhances the firm's bargaining power with its contractor, resulting in a lower wholesale price for the high-end version. In situations involving asymmetric information, this effect can help alleviate the downward distortion of the low-end quality, thereby enhancing overall welfare. However, outsourcing may also result in a level of low-end quality that surpasses the socially optimal level. On the other hand, outsourcing the low-end version is less likely to contribute to social welfare improvement, as it diminishes the quality of the in-house, high-end version.

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1 Introduction

Versioning is a retail strategy that allows companies to differentiate their products or services based on unobservable characteristics of consumers. This strategy involves releasing multiple versions of a product, quality levels, and pricing options. By doing so, companies can target different consumer segments and extract more value from each segment. The key idea behind versioning is to induce self-selection among consumers: consumers are led to choose the version of the product or service that best aligns with their own characteristics and preferences. Through this self-selection process, consumers reveal information about themselves that is otherwise unobservable to the company.

For example, restaurants use menu pricing strategies to implement versioning. They offer different menu items at different price points, allowing customers to choose based on their preferences and budget. Another example is the segmentation of passengers into "classes" on transportation services such as cruises, trains, and flights. Technology companies also employ versioning strategies, such as offering different versions of smartphones or software. By varying the features and functionalities, these companies can target different customer segments with different pricing options. Lastly, the freemium business model is another form of versioning. It offers a basic version of a product or service for free, while charging for premium features or additional functionalities. This allows companies to attract a wider audience with the free version and then upsell to paying customers based on their needs and willingness to pay.

It is important to note that versioning is closely related to the concept of second-degree price discrimination, where pricing is based on characteristics or behavior that are not directly observable, as opposed to third-degree price discrimination, which is based on observable characteristics such as age, gender, or location. Another widely used method of self-selection involves setting prices that fluctuate non-linearly based on the quantity being purchased.

Versioning in retail markets may involve various levels of the supply chain. In the case of airlines, versioning strategies initially focused on differentiating classes based on the level of in-flight services provided. However, with the introduction of separate cabins in the mid-1950s, aircraft designers and manufacturers became more involved in the versioning strategies of airlines. Nowadays, such strategies involves multiple players, including airlines, their suppliers (such as aircraft manufacturers and cabin designers), and contractors (such as catering services providers).

Despite the significant role of the supply chain in versioning, versioning strategies have so far been studied independently without considering vertical relations. This paper aims to bridge that gap by studying versioning strategies when either the high-end or low-end version of a product is outsourced. In some cases, retailers have advantages in producing high-quality versions of products and may choose to produce them in-house while outsourcing the lower-end versions. This can be seen in the fashion industry, where brands often offer both in-house, custom-made *haute couture* and outsourced, factory-made *prÃªt-Ã -porter* options. On the other hand, firms may need to outsource to provide higher-end products. This can be observed in supermarkets that sell both private labels and national brands, or in co-branding partnerships between sportswear and luxury brands.

An extensive literature in management science and operations research has thoroughly explored the potential effects of outsourcing on various aspects of quality. These effects include contractor opportunism (**?**), imitation (**?**), brand image (**?**), liability issues (**?**), and disruptions in the supply chain (**?**). In this paper, we approach the topic from a complementary perspective by examining how outsourcing impacts the choice of quality within the framework of microeconomic models of versioning.

Our analysis reveals that outsourcing the production of the high-end version of a product may alleviate the downward distortion in quality for the low-end version compared to when the firm produces both versions in-house. This result can be attributed to the fact that, with outsourcing, the negative impact of low-end quality on high-end profits is no longer fully internalized by the firm. Moreover, if outsourcing leads to improvements in cost-efficiency, the quality of the high-end version tends to increase, resulting in overall welfare gains. This outcome highlights the potential positive implications of outsourcing on quality and demonstrates that it can be Pareto-improving, particularly if the firm possesses sufficient bargaining power in its outsourcing arrangements. However, outsourcing may also result in a level of low-end quality that surpasses the socially optimal level, in which case its overall impact on social welfare is ambiguous.

On the other hand, outsourcing the production of the low-end version is less likely to enhance social welfare. This is due to the fact that it reduces the quality of the in-house, high-end version. Consequently, the decision to outsource this particular version should be carefully evaluated, taking into account the potential negative impact on overall product quality and welfare.

By exploring the relationship between outsourcing and quality choice within the framework of microeconomic models of versioning, our study offers valuable insights into the complex dynamics involved in outsourcing decisions. It sheds light on the potential benefits and drawbacks of outsourcing in terms of quality, allowing firms to make better-informed decisions regarding their outsourcing strategies. It also has important implications for regulations related to discrimination and quality levels.

Related literature. Since the seminal work of **?**, an extensive literature has studied how to induce self-selection through versioning, that is, how to adequately set the prices and quality levels of multiple versions of a product so as to prompt purchase decisions that reveal consumers' unobservable characteristics.[1] A well-know versioning principle is that the quality of low-end products is distorted downward, because higher-quality low-end products tend to raise the information rent of high-type consumers and, therefore, reduce the profits earned on high-end products. Following **?**, the literature has also studied when it is optimal to induce self-selection and when pooling, that is, offering a single version

[1] See **?** for recent developments of the concept of versioning, **?** for an application to privacy, and **?** and **?** for applications to freemium menu pricing.

of a product, is preferable.[2] We reexamine these fundamental questions in the context of outsourcing.[3]

Although versioning has been studied in isolation from vertical relations, the study of market segmentation in vertically related markets is not entirely new to the literature. Indeed, a large body of work has build upon the early contributions of **?** and **?** and studied price discrimination in input markets.[4] This literature analyzes third-degree price discrimination, that is, discrimination based on observable characteristics of groups of consumers, with the notable exception of a study by **?**, which consider discrimination based on unobserved characteristics and, thereby, resembles our paper. Besides, this literature researches price discrimination at the upstream level of the supply chain, whereas we consider discrimination at the downstream level, in relation to outsourcing and pricing at the upstream level.

A last strand of literature our paper is related to is the work on make-or-buy decisions and investment. For instance, **?** studies how the equilibrium quality of an integrated monopolist compares to that of a chain of monopolists; **?** conduct similar analyses for platforms; **?** study cost-reduction investment when a retailer may integrate with suppliers operating in a competitive upstream market. A fundamental difference between this literature and our work is that the former considers single-product firms, whereas versioning entail multi-production.

Finally, to understand the contribution of our paper, it is important to highlight the distinction between asymmetric information within the supply chain and asymmetric information between the supply chain and consumers. Since our goal is to study versioning strategies as implemented by the supply chain (instead a single integrated firm), we focus on the latter type of information asymmetry and assume perfect information within the supply chain. However, exploring how information asymmetry between different supply

[2] See **?**, **?**, **?**, and **?** for more recent contributions to the pooling vs. versioning problem.

[3] The research on versioning in a competitive market environment is very limited and, like the vast majority of the literature, we consider versioning by a monopolist.

[4] See **?** for an overview of the recent contributions.

chain partners affects the design of versioning strategies is an interesting avenue for future research.

The remainder of the paper is organized as follows. Section 2 sets up our baseline model, where the high-end version of a product may be outsourced. In Section 3, we derive the equilibrium versioning strategy under both integration and outsourcing, and study the welfare implications of the latter. In Section 4, we extend our baseline model and consider irrecoverable costs, double-marginalization, and outsourcing of the low-end version; We also discuss when versioning is more profitable than pooling. We provide some concluding remarks in Section 5.

2 The model

We set up a model where a firm, D, faces two groups of consumers, group i where $i \in \{\ell, h\}$, with different willingness to pay, v_i, for quality s. Without loss of generality, we consider $v_h > v_\ell$, so that group h values quality more than group ℓ. Consumers have unit demands and obtain utility u_i when consuming a product of quality s by paying a product price p:

$$u_i = sv_i - p, \ i \in \{\ell, h\}. \tag{1}$$

The firm may be vertically integrated and produce two versions of its product in-house by choosing a high quality s_h for the high-end version and a low quality s_ℓ for the low-end version. The per unit cost of quality s_i is $c(s_i)$, $i \in \{\ell, h\}$ with $c' > 0, c'' > 0$. In this case the profits of the firm are:

$$\pi = n_\ell \left(p_\ell - c(s_\ell)\right) + n_h \left(p_h - c(s_h)\right), \tag{2}$$

where n_i is the number of consumers in group i and p_i is the price of version i of the product. The firm initially chooses qualities s_ℓ, s_h and then sets the product prices p_ℓ, p_h.

Alternatively, the firm may outsource the production of the high-end version to an independent contractor, U, and pay a per unit price w_h to procure the high-end version from the contractor. The independent contractor may be more ($e > 0$) or less ($e < 0$) cost-effective compared to the firm. In this case the profits of the independent contractor and the firm are:

$$\begin{aligned} \pi_U &= n_h \left[w_h - (1-e)\, c\,(s_h)\right] \text{ with } e < 1, \\ \pi_D &= n_\ell \,(p_\ell - c\,(s_\ell)) + n_h \,(p_h - w_h)\,. \end{aligned} \tag{3}$$

The timing of the game, under outsourcing, is as follows. In the first stage, the firm commits to the level of the low quality s_ℓ, while the contractor to the level of the high quality s_h. In the second stage, the firm and the contractor engage in Nash bargaining over the contract term w_h. The bargaining weights (i.e., exogenous Nash parameters) of the firm and the contractor are β and $1-\beta$, respectively, with $\beta \in (0,1)$. In the third stage, the firm sets the retail prices p_ℓ, p_h for the two versions of the product and all costs are paid. We study the subgame perfect Nash equilibrium of this game.

Initially, we solve for the full information benchmark, that is, the case where v_ℓ and v_h are common knowledge. Then we solve for the asymmetric information case, that is, the case where v_ℓ and v_h are only known to the consumers. In-house production and outsourcing are compared under both full and asymmetric information. Our goal is to examine how outsourcing affects quality, profits and welfare, and to study the role of bargaining in such a vertical contracting setting with endogenous versioning.

3 Equilibrium versioning strategies and welfare implications of outsourcing

Initially, we study the benchmark case where the willingness to pay v_ℓ, v_h of the two different groups of consumers is common knowledge. Then we introduce asymmetric information, thus, the high value consumers can extract an information rent by the firm.

3.1 Full information benchmark

First we consider the case where the firm produces both versions of the product in-house. In such case, pricing issues at the wholesale level do not arise. Then we modify the model to consider outsourcing of the high-end version to an independent contractor.

In-house production

In this case, the firm knows exactly the willingness to pay of each group of consumers and sets each product price at such a level to extract all consumers' surplus, i.e. $u_i = 0$ or

$$p_i = s_i v_i,\ i \in \{\ell, h\}. \tag{4}$$

By substituting (4) into (2) and maximizing with respect to s_i, we obtain the optimum s_i by

$$c'(s_i) = v_i \Leftrightarrow s_i^* = \left(c'\right)^{-1}(v_i). \tag{5}$$

After substituting the optimum level of qualities and the product prices into (2), we get the equilibrium profits, π^*. The equilibrium level of total welfare W^* is equal to the firm's profits π^* since consumers' surplus is zero (it is extracted by the firm).

Outsourcing

Under outsourcing, at the third stage of the game product prices are again given by $p_i = s_i v_i$.

At the second stage of the game, the firm and the contractor engage in Nash bargaining over the contract term w_h by maximizing

$$M = \left(\pi_D - \pi_D^0\right)^{\beta} \left(\pi_U - \pi_U^0\right)^{1-\beta}. \tag{6}$$

If negotiations fail, the high-end version is not provided and only the low quality version appears in the market. The disagreement payoff of the independent contractor, π_U^0, is zero, while the disagreement payoff of the firm, π_D^0, depends on whether the low-end version is sold either to all consumers or only the high-type consumers. The firm chooses the former option if if $n_\ell/n_h \geq r(s_\ell)$, where $r(s_\ell) \equiv s_\ell (v_h - v_\ell) / (v_\ell s_\ell - c(s_\ell))$, $r'(s_\ell) > 0$, and the latter otherwise:

$$\pi_D^0 = \begin{cases} \pi_{D1}^0 = (n_\ell + n_h)(v_\ell s_\ell - c(s_\ell)) & \text{if} \quad n_\ell/n_h \geq r(s_\ell) \\ \pi_{D2}^0 = n_h (v_h s_\ell - c(s_\ell)) & \text{otherwise.} \end{cases} \tag{7}$$

It is important to note that the firm does not behave strategically when choosing a disagreement payoff: the more profitable disagreement payoff automatically results in a lower wholesale price and, therefore, higher equilibrium profits.

Nash product maximization $dM/dw_h = 0$ gives

$$\beta \left(\pi_U - \pi_U^0\right) \frac{d\pi_D}{dw_h} + (1-\beta)\left(\pi_D - \pi_D^0\right) \frac{d\pi_U}{dw_h} = 0. \tag{8}$$

We distinguish the two outsourcing subcases depending on π_D^0.

All consumers served if negotiations fail By substituting the product prices and the relevant expression for the disagreement payoff (i.e., $\pi_D^0 = \pi_{D1}^0$) into (8) and solving, we obtain the wholesale price as function of the product qualities:

$$\begin{aligned} w_{h1}(s_\ell, s_h) &= (1-\beta)(c(s_\ell) - s_\ell v_\ell + s_h v_h) + \beta (1-e) c(s_h) \\ &= (1-e) c(s_h) + (1-\beta)\left[(s_h v_h - (1-e) c(s_h)) - (s_\ell v_\ell - c(s_\ell))\right] \end{aligned} \tag{9}$$

The wholesale price equals the per unit cost of the high-end version plus a premium given to U (i.e. $(1-\beta)\left[(s_h v_h - (1-e) c(s_h)) - (s_\ell v_\ell - c(s_\ell))\right]$). This premium increases with the extra surplus extracted by the firms when the high-end version is introduced compared to the low-end version (i.e. $(s_h v_h - (1-e)\,c(s_h)) - (s_\ell v_\ell - c(s_\ell))$). Additionally, the premium and, thus, w_{h1} increases with the bargaining power of the independent contractor (i.e. $1-\beta$).

By direct differentiation of (9), we have that $dw_{h1}(s_\ell, s_h)/ds_\ell = -(1-\beta)(v_\ell - c'(s_\ell))$, $dw_{h1}(s_\ell, s_h)/ds_h = (1-\beta)\,v_h + \beta(1-e)\,c'(s_h) > 0$. As the high quality produced by the contractor, increases, the cost of selling this version to the retailer also increases.

At the first stage of the game, the firm and the contractor simultaneously and independently choose qualities s_ℓ and s_h, respectively, by maximizing their own profits:

$$\begin{aligned}\pi_D(s_\ell, s_h) &= n_\ell(p_\ell - c(s_\ell)) + n_h(p_h - w_h(s_\ell, s_h)),\\ \pi_U(s_\ell, s_h) &= n_h(w_h(s_\ell, s_h) - (1-e)\,c(s_h)),\end{aligned}$$

where $w_h(s_\ell, s_h)$ is given by (9).

By the first order condition (FOC) with respect to s_ℓ, $d\pi_D(s_\ell, s_h)/ds_\ell = 0$, we have:

$$\begin{aligned}\underbrace{n_\ell v_\ell}_{(+)\text{ price effect}} \underbrace{- n_\ell c'(s_\ell)}_{(\text{-})\text{ cost effect}} \underbrace{- n_h \partial w_h(s_\ell, s_h)/\partial s_\ell}_{(+/\text{-})\text{ bargaining effect}} &= 0 \\ (n_\ell + n_h(1-\beta))\left(v_\ell - c'(s_\ell)\right) &= 0.\end{aligned} \tag{10}$$

The price effect is positive, the cost effect is negative, while the bargaining effect is either positive or negative.

The optimum level of quality s_ℓ is given by

$$c'(s_\ell) = v_\ell \Leftrightarrow s^*_{\ell 1} = \left(c'\right)^{-1}(v_\ell). \tag{11}$$

Note that in this subcase of outsourcing, the level of quality s_ℓ is the same as under in-house production (see expression (5)).

By the FOC with respect to s_h, $d\pi_U(s_\ell, s_h)/ds_h = 0$, we have:

$$\begin{aligned}
\underbrace{n_h\ dw_h(s_\ell, s_h)/ds_h}_{\text{(+) bargaining effect}} \underbrace{- n_h\ (1-e)\,c'(s_h)}_{\text{(-) cost effect}} &= 0 \qquad (12)\\
\underbrace{n_h\ (1-\beta)\,v_h}_{\text{(+) share of the extra CS}} + \underbrace{n_h\ \beta(1-e)\,c'(s_h)}_{\text{(+) cost discount effect}} \underbrace{- n_h\ (1-e)\,c'(s_h)}_{\text{(-) cost effect}} &= 0\\
\underbrace{n_h\ (1-\beta)\,v_h}_{\text{share of the extra CS}} \underbrace{- n_h\ (1-\beta)(1-e)\,c'(s_h)}_{\text{share of the extra cost effect}} &= 0\\
n_h(1-\beta)\big(v_h - (1-e)\,c'(s_h)\big) &= 0
\end{aligned}$$

The contractor captures only a share of the extra consumers' surplus (CS) via w_h but also bears a share of the cost to produce quality s_h or, equivalently, there is a cost discount effect. The share of the production cost effectively paid by the contractor corresponds to the share of the extra CS it captures. The optimum level of quality s_h is given by

$$c'(s_h) = \frac{v_h}{1-e} \Leftrightarrow s_{h1}^* = (c')^{-1}\left(\frac{v_h}{1-e}\right). \qquad (13)$$

Note that whenever $v_h/(1-e) > v_h$, i.e. $e > 0$, the level of quality s_h is higher compared to the in-house production case. Whenever the independent contractor is more cost-efficient compared to the firm, then the contractor sets a higher quality for the high-end version than the one set by the firm under in-house production, where there is no outsourcing.

Only the high-type consumers served if negotiations fail Now $\pi_D^0 = \pi_{D2}^0$. Following the same steps as in the previous subcase, we get: $w_{h2}(s_\ell, s_h) = (1/n_h) \times$

$$(1-\beta)(n_\ell(s_\ell v_\ell - c(s_\ell)) - n_h(s_\ell v_h - c(s_\ell)) + n_h s_h v_h) + \beta n_h(1-e)\,c(s_h) \qquad (14)$$

with

$$
\begin{aligned}
dw_{h2}(s_\ell, s_h)/ds_\ell &= (1-\beta)\left(n_\ell\left(v_\ell - c'(s_\ell)\right) - n_h\left(v_h - c'(s_\ell)\right)\right)/n_h, \\
dw_{h2}(s_\ell, s_h)/ds_h &= (1-\beta)v_h + \beta(1-e)c'(s_h) > 0.
\end{aligned}
$$

Now the bargaining effect $dw_{h2}(s_\ell, s_h)/ds_\ell$ is reduced by $n_h(v_h - c'(s_\ell))$, given that $v_h > c'(s_\ell)$, compared to the previous subcase. The reduced $dw_{h2}(s_\ell, s_h)/ds_\ell$ gives a higher incentive for the firm to increase quality s_ℓ. Therefore, by proceeding to the first stage of the game, optimum quality levels are given by

$$
\begin{aligned}
c'(s_\ell) &= \frac{(1-\beta)n_h v_h + \beta n_\ell v_\ell}{(1-\beta)n_h + \beta n_\ell} \Leftrightarrow s_{\ell 2}^* = \left(c'\right)^{-1}\left(\frac{(1-\beta)n_h v_h + \beta n_\ell v_\ell}{(1-\beta)n_h + \beta n_\ell}\right), \\
c'(s_h) &= \frac{v_h}{1-e} \Leftrightarrow s_{h2}^* = \left(c'\right)^{-1}\left(\frac{v_h}{1-e}\right).
\end{aligned}
\tag{15}
$$

After comparing (15) to (5), we obtain that now the equilibrium quality s_ℓ is higher than under in-house production (since $((1-\beta)n_h v_h + \beta n_\ell v_\ell)/((1-\beta)n_h + \beta n_\ell) > v_\ell$ and $c' > 0$); the incentives for a higher quality are reinforced by the fact that only the high-type consumers will be served in the event of a negotiation breakdown. The equilibrium quality s_h is higher than under in-house production whenever $e > 0$, as in the previous subcase.

Subgame perfect equilibrium When setting the low-end quality s_ℓ in the first stage of the game, the firm selects the equilibrium that will prevail in the bargaining subgame.

Lemma 1. *There exist a threshold $\widetilde{r} \in [r(s_{\ell 1}^*), r(s_{\ell 2}^*))$ such that the equilibrium low-end quality is $s_{\ell 2}^*$ if $n_\ell/n_h < \widetilde{r}$ and $s_{\ell 1}^*$ otherwise.*

Proof. Since $s_{\ell 1}^* < s_{\ell 2}^*$ and $r'(s_\ell) > 0$, we obtain $r(s_{\ell 1}^*) < r(s_{\ell 2}^*)$, which implies that the equilibrium where the low-end quality is $s_{\ell 1}^*$ exists for all $n_\ell/n_h \geq r(s_{\ell 1}^*)$, while the equilibrium where the low-end quality is $s_{\ell 2}^*$ exists for all $n_\ell/n_h < r(s_{\ell 2}^*)$. Therefore, the equilibrium is $s_{\ell 2}^*$ if $n_\ell/n_h < r(s_{\ell 1}^*)$, $s_{\ell 1}^*$ if $n_\ell/n_h \geq r(s_{\ell 2}^*)$, and either $s_{\ell 1}^*$ or $s_{\ell 2}^*$ if $n_\ell/n_h \in$

$[r(s^*_{\ell 1}), r(s^*_{\ell 2}))$. We find that $\partial(\pi_D(w_{h1}) - \pi_D(w_{h2}))/\partial n_\ell = (1-\beta)(s_\ell v_\ell - c(s_\ell)) > 0$ and $\partial(\pi_D(w_{h1}) - \pi_D(w_{h2}))/\partial n_h = -(1-\beta)s_\ell(v_h - v_\ell) < 0$. Since $\pi_{D2}(s^*_{\ell 2}) > \pi_{D1}(s^*_{\ell 1})$, $\forall n_\ell/n_h < r(s^*_{\ell 1})$, and $\pi_{D2}(s^*_{\ell 2}) < \pi_{D1}$, $(s^*_{\ell 1}) \forall n_\ell/n_h \geq r(s^*_{\ell 2})$, it follows by continuity that there exists a threshold $\widetilde{r}$ such that $\pi_{D2}(s^*_{\ell 2}) > \pi_{D1}(s^*_{\ell 1})$ if $n_\ell/n_h < \widetilde{r}$ and $\pi_{D2}(s^*_{\ell 2}) \leq \pi_{D1}(s^*_{\ell 1})$, and that $\widetilde{r} \in [r(s^*_{\ell 1}), r(s^*_{\ell 2}))$. □

Lemma 1 indicates that the firm selects the equilibrium where only the high-type consumers are served in the event of a negotiation breakdown if the proportion of such consumers is sufficiently large. Likewise, it chooses the other equilibrium, where all consumers are served in the event of a negotiation breakdown, if the proportion of low-type consumers is large enough. The following result ensues.

Proposition 1. *Under full information, outsourcing yields a higher quality s_ℓ and a (strictly) higher quality s_h whenever the independent contractor is (strictly) more cost-efficient.*

Proof. After direct comparison of (11) and (15) to (5), we obtain that $s^*_{\ell 1} = s^*_\ell$ and $s^*_{\ell 2} > s^*_\ell$. After direct comparison of (13) and (15) to (5), we obtain that $s^*_{h1} = s^*_{h2} > s^*_h$ if and only if $e > 0$. □

The presence of the contractor boosts the quality level for the low-end version set by the firm, independently of whether the production of the high-end version is more or less cost-efficient. This is explained by a bargaining effect: the firm's disagreement payoff where it serves only the high-type consumers increases more with the quality of the low-end version than its profits when bargaining succeeds. Hence, a higher low-end quality improves the bargaining position of the firm, thereby reducing the wholesale price of the high-end version. By contrast, the bargaining effect is null when all consumers are served in the event of a negotiation breakdown.

Finally, after substituting the optimum levels of quality, the wholesale price and the product prices into (3), we get the equilibrium profits of the firm and the contractor for the two outsourcing subcases. Since consumers' surplus is zero, the equilibrium level of social welfare is equal to the sum of the firms' profits. The subsequent result occurs.

Proposition 2. *i) For all* $i \in \{1,2\}$*, there exists a threshold* $\widetilde{e}_i(\beta)$*, where* $\widetilde{e}_i(\beta) > 0$ *and* $\widetilde{e}'_i(\beta) < 0$*, such that outsourcing occurs if and only if* $e > \widetilde{e}_i(\beta)$ *ii) When outsourcing enhances cost-efficiency (*$e \geq 0$*), it improves social welfare if* $n_\ell/n_h \geq \widetilde{r}$*.*

Proof. i) For all $i \in \{1,2\}$, $d\pi^*_{Di}/d\beta = (\partial\pi_{Di}/\partial w_{hi})(\partial w_{hi}/\partial\beta) > 0$, $d\pi^*_{Di}/de = (c')^{-1}(v_h) > 0$, $\pi^* > \pi^*_{Di}$ if $\beta = 0$, and $\pi^*_{Di} > \pi^*$ if $\beta = 1$ and $e > 0$. Therefore, by continuity, there exists a threshold $\widetilde{e}_i(\beta) > 0$, where $\widetilde{e}'_i(\beta) = -(d\pi^*_{Di}/d\beta)/(d\pi^*_{Di}/de) < 0$ such that $\pi^*_{Di} \geq \pi^*$ if $e \geq \widetilde{e}(\beta)$ and $\pi^*_{Di} < \pi^*$ otherwise. ii) The high-end quality is set at the socially optimal level $s^*_{h1} = s^*_{h2} = s^*_h = (c')^{-1}(v_h)$. When $n_\ell/n_h \geq \widetilde{r}$, the low-end quality is also set at the socially optimal level, that is, $s^*_{\ell 1} = s^*_\ell = (c')^{-1}(v_\ell)$. Then, $W(s^*_{\ell 1}, s^*_{h1}, e) \geq W(s^*_\ell, s^*_h, 0)$ if and only if $e \geq 0$. □

The firm finds it profitable to outsource the production of the high-end version when it has sufficient bargaining power in its outsourcing arrangements or, equivalently, when the degree of cost-efficiency improvement is sufficiently high, given its bargaining power.

Under full in-house production, the firm chooses the levels of quality that maximize welfare, given its degree of cost-efficiency. It follows that outsourcing unambiguously improves social welfare when its sole effect is to enhance cost efficiency. By contrast, when outsourcing involves a bargaining effect that raises the quality of the low-end end version above the socially optimal level, its overall welfare implications are ambiguous if cost-efficiency is improved and negative otherwise.

Proposition 2 indicates that outsourcing does not always lead to an improvement in overall welfare. When it yields sufficient cost-efficiency gains ($e > \widetilde{e}_i(\beta)$) but raises the quality of the low-end version above the socially optimal level ($n_\ell/n_h < \widetilde{r}$), outsourcing is profitable for the firm but potentially detrimental to welfare. Additionally, the proposition highlights that even when outsourcing would improve welfare, it may not necessarily be profitable. This is the case when outsourcing leaves the low-end quality unchanged compared to full in-house production ($n_\ell/n_h \geq \widetilde{r}$) and yields limited cost-efficiency gains ($0 < e < \widetilde{e}_i(\beta)$).

3.2 Asymmetric information

Thus far we have examined the benchmark case of full information. Now we turn to the case where consumers' valuations v_i is private information, thus, the firm cannot directly distinguish between the two groups of consumers. The firm extracts all CS from the low-type consumers (participation constraint) and pays an information rent (incentive compatibility constraint) to the high-type consumers. Below, we first analyze the case where production is fully in-house and that where the high-end version of the product is outsourced.

In-house production

The firm sets its price for the low-end version such consumers with the lower willingness are left with zero rents, that is, $u_\ell = 0$. For the high-end version, the price set is such that the consumers with the higher willingness to pay have no incentive to mimic that they are the consumers with the lower willingness to pay, that is, $v_h s_h - p_h = v_h s_\ell - p_\ell$. Therefore, prices are given by

$$p_\ell = v_\ell s_\ell, \ p_h = s_h v_h - s_\ell (v_h - v_\ell). \tag{16}$$

It should be noted that under asymmetric information an increase in the quality of the low-end version leads to a decrease in the price paid for the high-end version. As the difference in qualities between the two versions narrows, high-end consumers find it more appealing to purchase the low-end version, which strengthens their information rent. The firm internalizes this negative impact of the lower-end quality on the high-end profits, causing a downward distortion in the quality of the low-end version. Indeed, by substituting

(16) into (2) and maximizing with respect to s_ℓ and s_h, we get

$$s_\ell^{**} = \left(c'\right)^{-1}\left(\frac{n_\ell v_\ell - n_h\left(v_h - v_\ell\right)}{n_\ell}\right) < s_\ell^*, s_h^{**} = s_h^* = \left(c'\right)^{-1}\left(v_h\right) \tag{17}$$

For the low-end version product to be offered, s_ℓ has to be non negative, that is, v_h has to be equal or lower than a threshold $\overline{v}_h \equiv \left(n_\ell + n_h\right) v_\ell / n_h$.

After substituting the optimum level of qualities and the product prices into (2), we get the equilibrium profits, π^{**}. The equilibrium level of consumer's surplus is positive due to the information rent extracted by the consumers with the high willingness to pay for high quality, i.e., $CS^{**} = n_h\left(v_h - v_\ell\right) s_\ell^{**}$. Total welfare W^{**} is equal to the firm's profits π^{**} plus consumer's surplus CS^{**}.

Outsourcing

Under outsourcing, at the third stage of the game product prices are given by (16). At the second stage of the game, the firm and its contractor engage in Nash bargaining over the contract term w_h by maximizing (6). Under a negotiation failure, the high-end version is not provided. The disagreement payoff of the independent contractor, π_U^0, is zero, while the disagreement payoff of the firm, π_D^0 is given by (7). We distinguish the two outsourcing subcases depending on π_D^0.

All consumers served if negotiations fail By maximizing (6) given the product prices (16) and the fact that $\pi_D^0 = \pi_{D1}^0$, we obtain the wholesale price of the high-end version as function of the product qualities:

$$w_{h1}\left(s_\ell, s_h\right) = \beta\left(1 - e\right) c\left(s_h\right) + \left(1 - \beta\right) c\left(s_\ell\right) + \left(1 - \beta\right) v_h\left(s_h - s_\ell\right), \tag{18}$$

with $dw_{h1}\left(s_\ell, s_h\right)/ds_\ell = -\left(1-\beta\right)\left(v_h - c'\left(s_\ell\right)\right) < 0$ for $v_h > c'\left(s_\ell\right)$, $dw_{h1}\left(s_\ell, s_h\right)/ds_h = \beta\left(1-e\right) c'\left(s_h\right) + \left(1-\beta\right) v_h > 0$.

At the first stage of the game, the FOCs $d\pi_D/ds_\ell = 0, : d\pi_U/ds_h = 0$ give:

$$s_{\ell 1}^{**} = (c')^{-1}\left(\frac{n_\ell v_\ell + n_h v_\ell - \beta n_h v_h}{n_\ell + (1-\beta)\, n_h}\right) > 0, s_{h1}^{**} = (c')^{-1}\left(\frac{v_h}{1-e}\right), \tag{19}$$

with $s_{\ell 1}^{**} > 0$ since $v_h \leq \overline{v}_h$.

Only the high-type consumers served if negotiations fail Now $\pi_D^0 = \pi_{D2}^0$. From Nash product maximization, we get: $w_{h2}(s_\ell, s_h) =$

$$\beta(1-e)\, c(s_h) + (1-\beta)\left[(n_h - n_\ell)\, c(s_\ell) + s_\ell v_\ell (n_\ell + n_h) + n_h v_h (s_h - 2 s_\ell)\right]/n_h \tag{20}$$

with $\frac{dw_{h2}}{ds_\ell} = \frac{(1-\beta)(n_h - n_\ell)c'(s_\ell) + (1-\beta)(v_\ell(n_\ell + n_h) - 2n_h v_h)}{n_h}$, $\frac{dw_{h2}}{ds_h} = \beta(1-e)\, c'(s_h) + (1-\beta)\, v_h > 0$.

At the first stage of the game, the FOCs give:

$$s_{\ell 2}^{**} = (c')^{-1}\left(\frac{v_\ell \beta (n_\ell + n_h) + v_h n_h (1-2\beta)}{(1-\beta)\, n_h + \beta n_\ell}\right) > 0, s_{h2}^{**} = (c')^{-1}\left(\frac{v_h}{1-e}\right). \tag{21}$$

Subgame perfect equilibrium As in the full information benchmark, the equilibrium where only the high-type consumers are served in the event of a negotiation failure arises if such consumers are present in sufficient proportion and otherwise the other equilibrium arises, where all consumers are served when negotiations fail.

Lemma 2. *There exist a threshold $\widehat{r} \in [r(s_{\ell 1}^{**}), r(s_{\ell 2}^{**}))$ such that the equilibrium low-end quality is $s_{\ell 2}^{**}$ if $n_\ell/n_h < \widehat{r}$ and $s_{\ell 1}^{**}$ otherwise.*

Proof. Similar to the proof of Lemma 1. □

By comparison of the equilibrium levels of quality, we obtain the following result.

Proposition 3. *Under asymmetric information, outsourcing yields a strictly higher quality s_ℓ and a (strictly) higher quality s_h whenever the independent contractor is (strictly) more cost-efficient.*

Proof. After direct comparison of (19) and (21) to (17) , we find i) $s_{\ell 2}^{**} > s_{\ell 1}^{**} > s_{\ell}^{**}$ ii) $s_{h1}^{**} = s_{h2}^{**} > s_{h}^{**}$ if and only if $e > 0$. □

Under asymmetric information, the quality of the low-end version strictly increases with outsourcing, regardless of the strategy adopted by the firm in the event of a negotiation failure. The difference with the full-information benchmark, where s_ℓ increases if only the high-type consumers are served in the event of a negotiation breakdown but remains unchanged otherwise, is explained by the negative effect of the low-end quality on the high-end profits. This effect implies that higher low-end quality always improves the firm's disagreement payoff more than it raises its profits when negotiations succeed, thereby improving the firm's bargaining position and reducing the wholesale price of the high-end version. Additionally, an increased s_ℓ increases the information rent obtained by the high-end consumers but, under outsourcing, this negative effect of s_ℓ is not fuly internallized compared to the in-house production leading to a less severe reduction in s_ℓ.

The equilibrium qualities under full and asymmetric information are summarized in Table 3.2. The welfare implications of these results can be summarized as follows.

Equilibrium qualities

	Full information		Asymmetric inf
	s_ℓ	s_h	s_ℓ
In-house production	$s_\ell^* = (c')^{-1}(v_\ell)$	$s_h^* = (c')^{-1}(v_h)$	$s_\ell^{**} = (c')^{-1}\left(\frac{n_\ell v_\ell - n_h(v_h - v_\ell)}{n_\ell}\right)$
High n_ℓ/n_h	$s_{\ell 1}^* = (c')^{-1}(v_\ell)$	$s_{h1}^* = (c')^{-1}\left(\frac{v_h}{1-e}\right)$	$s_{\ell 1}^{**} = (c')^{-1}\left(\frac{n_\ell v_\ell + n_h v_\ell - \beta n_h v_h}{n_\ell + (1-\beta) n_h}\right.$
Low n_ℓ/n_h	$s_{\ell 2}^* = (c')^{-1}\left(\frac{(1-\beta)n_h v_h + \beta n_\ell v_\ell}{(1-\beta)n_h + \beta n_\ell}\right)$	$s_{h2}^* = (c')^{-1}\left(\frac{v_h}{1-e}\right)$	$s_{\ell 2}^{**} = (c')^{-1}\left(\frac{v_\ell \beta(n_\ell + n_h) + v_h n_h (1-2}{(1-\beta)n_h + \beta n_\ell}\right.$

Proposition 4. *i) For all $i \in \{1, 2\}$, there exists a threshold $\widehat{e}_i(\beta)$, where $\widehat{e}_i(\beta) > 0$ and $\widehat{e}'_i(\beta) < 0$, such that outsourcing occurs if and only if $e \geq \widehat{e}_i(\beta)$. ii) When outsourcing enhances cost-efficiency ($e \geq 0$), it improves social welfare if $n_\ell/n_h \geq \widehat{r}$ and if $n_\ell/n_h < \widehat{r}$ and $\beta \geq 0.5$.*

Proof. i) For all $i \in \{1,2\}$, $d\pi_{Di}^{**}/d\beta = (\partial \pi_{Di}/\partial w_{hi})(\partial w_{hi}/\partial \beta) > 0$, $d\pi_{Di}^{**}/de = (c')^{-1}(v_h) > 0$, $\pi^{**} > \pi_{Di}^{**}$ if $\beta = 0$, and $\pi_{Di}^{**} > \pi^{**}$ if $\beta = 1$ and $e > 0$. Therefore, by continuity, there exists a threshold $\widehat{e}_i(\beta) > 0$, where $\widehat{e}_i'(\beta) = -(d\pi_{Di}^{**}/d\beta)/(d\pi_{Di}^{**}/de) < 0$ such that $\pi_{Di}^{**} \geq \pi^{**}$ if $e \geq \widehat{e}_i(\beta)$ and $\pi_{Di}^{**} < \pi^{**}$ otherwise. ii) The high-end quality is set at the socially optimal level $s_{h1}^{**} = s_{h2}^{**} considering cost efficiency as well. Besides,s_{\ell 1}^{**} = (c')^{-1}\left(\frac{n_\ell v_\ell + n_h v_\ell - \beta n_h v_h}{n_\ell + (1-\beta) n_h}\right) \leq (c')^{-1}(v_\ell)$ and $s_{\ell 2}^{**} = (c')^{-1}\left(\frac{v_\ell \beta (n_\ell + n_h) + v_h n_h (1-2\beta)}{(1-\beta) n_h + \beta n_\ell}\right) \leq (c')^{-1}(v_\ell)$ if $\beta \geq 0.5$. It follows that $W(s_{\ell 1}^{**}, s_{h1}^{**}, e) \geq W(s_\ell^{**}, s_h^{**}, 0)$ if $e \geq 0$ and $W(s_{\ell 2}^{**}, s_{h2}^{**}, e) \geq W(s_\ell^{**}, s_h^{**}, 0)$ if $e \geq 0$ and $\beta \geq 0.5$. □

As in the scenario where full information is available, the quality of the high-end version is set at the socially optimal level regardless of whether it is outsourced or produced in-house (considering the cost efficiency parameter under outsourcing). However, when there is asymmetric information, the quality of the low-end version is distorted downward due to the information rent obtained by high-type consumers. As a result, the low-end quality falls below the socially optimal level. The bargaining effect implies that a higher quality of the low-end version leads to a decrease in the wholesale price of the high-end version and can help mitigate this downward distortion. Additionally, if outsourcing enhances cost-efficiency, it leads to an increase in the quality of the high-end version, resulting in welfare gains. In such cases, outsourcing is Pareto-improving if the firm has sufficient bargaining power in its outsourcing agreements. However, it is important to note that outsourcing may also lead to a level of low-end quality that exceeds the socially optimal level ($s_{\ell 2}^{**} > s_\ell^{*} > s_{\ell 1}^{**} > s_\ell^{**}$). In such situations, the overall impact of outsourcing on social welfare becomes ambiguous.

4 Extensions and discussion

In this section, we first extend our baseline model to irrecoverable costs (Section 4.1). We then consider the double-marginalization problem (Section 4.2) and, afterwards, suppose that the low-end version, instead of the high-end version, is outsourced (Section 4.3). Finally, we discuss whether versioning or pooling is optimal, when either version of the product is outsourced (Section 4.4).

4.1 Irrecoverable costs

Thus far, we have assumed that the firm and the contractor could commit to a level of quality without facing any costs. However, in reality, production often involve upfront investments that are partly irrecoverable. In this section, we suppose that the production costs are incurred in the first stage of the game and focus on the case where a share $\alpha \in [0,1]$ of the contractor's costs are irrecoverable, whereas that of the firm are fully recoverable.[5] For brevity of exposition, we omit the full information benchmark and focus on asymmetric information.

We denote as n_c the level of output chosen by the contractor in the first stage of the game. This level never exceeds the number of high-type consumers because it would aggravate the loss of the contractor in the event of a negotiation breakdown without increasing its profits if negotiations with the firm succeed. Supposing that $n_c \leq n_h$, the profits of the contractor are $\pi_U = n_c[w_h - c(s_h)]$ and its (negative) disagreement payoff $-\alpha n_c\,(1-e)\,c(s_h)$; the profits of the firm are $\pi_D = n_\ell[s_\ell v_\ell - c(s_\ell)] + n_c[s_h v_h - s_\ell(v_h - v_\ell) - w_h]$ and its disagreement payoff is either $(n_\ell + n_h)[s_\ell v_\ell - c(s_\ell)]$, when both groups of consumers are served, or $n_h[s_\ell v_h - c(s_\ell)]$, when only the high-type consumers are served. As in the baseline model, the firm chooses the former option if $n_\ell/n_h \geq r\,(s_\ell)$ and the latter option otherwise.

The equilibrium retail prices are the same as in the baseline model. Solving for the Nash bargaining stage, we find

$$w_{h1}\,(s_\ell, s_h) = \beta(1-\alpha)(1-e)c(s_h) + (1-\beta)[n_h c(s_\ell) + v_h(s_h - s_\ell)n_c + (n_c - n_h)s_\ell v_\ell]/n_c$$

[5] The asymmetry between the contractor and the firm can be motivated by the fact that, as a retailer, the latter has access to other final consumers and can dispose unsold inventory.

if $n_\ell/n_h \geq r\left(s_\ell\right)$ and $w_{h2}\left(s_\ell, s_h\right) =$

$$\beta(1-\alpha)(1-e)c(s_h)+(1-\beta)\left[(n_h-n_\ell)c(s_\ell)+s_\ell v_\ell(n_\ell+n_c)+n_c s_h v_h-(n_h+n_c)s_\ell v_h\right]/n_c$$

otherwise.

Turning to the first stage of the game, we obtain the following result.

Lemma 3. *The equilibrium output of the contractor is such that all high-type consumers are served, that is,* $n_c = n_h$.

Proof. $d\pi_U/dn_c = \partial\pi_U/\partial n_c + (\partial\pi_U/\partial w_h)(\partial w_h/\partial n_c)$ and $d\pi_U/dn_c$ is positive for all $n_c \leq n_h$, because every term on the RHS is positive. □

The contractor may set a level of output n_c such that only a fraction of the high-type consumers is served, in order to reduce its loss in the event of a negotiation breakdown and, thereby, improve its bargaining position. However, such strategy also reduces the profits of both the firm and the contractor when negotiations succeed. All in all, a lower output reduces the wholesale price, which explains why the contractor chooses to serve all high-type consumers. The equilibrium qualities ensue as follows.

Proposition 5. *If a share* $\alpha \in [0,1]$ *of the costs incurred by the contractor is irrecoverable, the equilibrium qualities are* $s_h = c'^{-1}\left[\frac{v_h(1-\beta)}{(1-(1-\alpha)\beta)(1-e)}\right] \leq s_h^{**}$, $s_\ell = s_{\ell 1}^{**}$ *if* $n_\ell/n_h \geq \widehat{r}$, *and* $s_\ell = s_{\ell 2}^{**}$ *otherwise.*

Proof. The equilibrium qualities are obtained by solving for the first order conditions $d\pi_D/ds_\ell = 0$ and $d\pi_U/ds_h = 0$, given the equilibrium wholesale price and for $n_c = n_h$. □

Irrecoverable costs worsen the bargaining position of the contractor and result in a lower wholesale price (using the fact that $n_c = n_h$, we find $w_{hi} = w_{hi}^{**} - \alpha\beta(1-e)c(s_h),$:

$\forall i \in \{1, 2\}$). In order to mitigate this effect, the contractor reduces its quality level as the share of such costs increases.[6]

Using the same approach as in Section 3, it can be shown that this reduces the welfare gains from outsourcing, and more so as the share of irrecoverable costs and the bargaining power of the firm increase. The irrecoverable costs of the contractor also make outsourcing less profitable for the firm. In this regard, the bargaining power parameter plays a more ambiguous role: on the one hand, for all given level of high-end quality, stronger bargaining power allows the firm to obtain a lower wholesale price and, thereby, increase its profits; on the other hand, stronger bargaining power for the firm also reduces the quality level chosen by the contractor, which in turn makes outsourcing less profitable.

4.2 Double marginalization

The presence of two groups of identical consumers yields either full or zero participation, thereby silencing the double marginalization effect that usually characterizes models of vertical relations with linear wholesale prices. In order to incorporate such effect to our model, we consider a unit mass of consumers whose willingness to pay for quality is uniformly distributed over the interval $[0, 1]$. The resulting demands are $n_\ell = (p_h - p_\ell)/(s_h - s_\ell) - p_\ell/s_\ell$ and $n_h = 1 - (p_h - p_\ell)/(s_h - s_\ell)$ when both the high-end and the low-end versions are available, and $n_\ell^o = 1 - p_\ell/s_\ell$ when only the latter is available because negotiations failed. The definitions of the profits under in-house production and outsourcing are the same as in Section 2, but with the new, price elastic demand functions defined above. For the sake of tractability, we assume the contractor has no cost advantage, that is, $e = 0$.

The equilibrium of the retail pricing subgame is $\{p_\ell^{in} = [s_\ell + c(s_\ell)]/2, p_h^{in} = [s_h + c(s_h)]/2\}$ when the two versions are produced in-house and $\{p_\ell^{out} = [s_\ell + c(s_\ell)]/2, p_h^{out} = (s_h + w_h)/2\}$ when the production of the high-end version is outsourced. In the event of a negotiation failure, the firm maximizes $n_\ell^o(p_\ell - c(s_\ell))$, which yields the equilibrium price

[6] Note that the baseline model corresponds to the case where $\alpha = 0$.

$p_\ell^o = p_\ell^{in} = p_\ell^{out}$.

Under outsourcing, the wholesale price is determined by maximization of

$$M = [n_\ell(p_\ell^{out} - c(s_\ell)) + n_h(p_h^{out} - w_h) - n_\ell^o(p_\ell^o - c(s_\ell))]^\beta [n_h(w_h - c(s_h))]^{1-\beta},$$

where the demands are evaluated at the equilibrium retail prices. In this version of the model, the wholesale price does not only determine the allocation of the total industry profit between the firm and the contractor, but also the level of that profit (from the equilibrium retail prices, $\partial(\pi_D + \pi_U)/\partial w_h < 0$ iff $c(s_h) < w_h$). We find

$$w_h^{out} = [(1-\beta)s_h + (1+\beta)c(s_h) - (1-\beta)(s_\ell - c(s_\ell))]/2.$$

The corresponding retail quantities and markup for all given levels of quality are

$$n_\ell^{out}(w_h^{out}) = \frac{s_\ell[(1-\beta)(s_h - s_\ell) + (1+\beta)c(s_h)] - [2s_h - (1-\beta)s_\ell]c(s_\ell)}{4s_\ell(s_h - s_\ell)},$$

$$n_h^{out}(w_h^{out}) = 4(1+\beta)[s_h - s_\ell - c(s_h) + c(s_\ell)]/(s_h - s_\ell),$$

$$p_\ell^{out} - c(s_\ell) = p_\ell - c(s_\ell) = [s_\ell - c(s_\ell)]/2,$$

$$p_h^{out}(w_h^{out}) - w_h^{out} = [(1-\beta)[s_\ell - c(s_\ell)] + (1+\beta)[s_h - c(s_h)]]/4,$$

$$\text{and } w_h^{out} - c(s_h) = [(1-\beta)(s_h - c(s_h) - s_\ell + c(s_\ell)]/2.$$

Note that the equilibrium demands under full in-house production are $n_\ell^{in} = n_\ell^{out}(c(s_h))$ and $n_h^{in} = n_h^{out}(c(s_h))$.

In the baseline model, a higher quality s_ℓ raises the markup on the low-end version and reduces the high-end profits, while a higher quality s_h raises the wholesale markup. In order to obtain similar effects in the present framework and, thereby, ensure its consistency with the baseline model, we suppose that $(s_h - s_\ell)c'(s_\ell) < c(s_h) - c(s_\ell) < (s_h - s_\ell)c'(s_h) < s_h - s_\ell$.[7] Provided that this condition is satisfied, we obtain the following result.

Proposition 6. *Outsourcing yields strictly higher equilibrium qualities s_ℓ and s_h.*

Proof. For all given high-end quality s_h, the equilibrium low-end quality is $s_\ell^{in}(s_h)$ such that

$$\frac{d\pi^{in}}{\partial s_\ell} = \frac{\partial n_\ell}{\partial s_\ell}[p_\ell^{in} - c(s_\ell)] + n_\ell^{in}\left(\frac{\partial p_\ell^{in}}{\partial s_\ell} - c'(s_\ell)\right) + \frac{\partial n_h}{\partial s_\ell}(p_h^{in} - c(s_h)) = 0 \tag{22}$$

under in-house production and $s_\ell^{out}(s_h)$ such that

$$\frac{d\pi_D^{out}}{ds_\ell} = \frac{\partial n_\ell}{\partial s_\ell}[p_\ell^{out} - c(s_\ell)] + n_\ell^{out}\left(\frac{\partial p_\ell^{out}}{\partial s_\ell} - c'(s_\ell)\right) + \frac{\partial n_h}{\partial s_\ell}(p_h^{out}(w_h^{out}) - w_h^{out}) - n_h^{out}\frac{\partial w_h^{out}}{\partial s_\ell} +$$

$$\frac{\partial n_\ell}{\partial w_h}\frac{\partial w_h^{out}}{\partial s_\ell}[p_\ell^{out} - c(s_\ell) - (p_h^{out}(w_h^{out}) - w_h^{out})] = 0 \tag{23}$$

under outsourcing.

For all given pair of quality levels (s_ℓ, s_h), the first terms of (22) and (23) are the same; the second term of (23) is higher than that of (22), because $\partial p_\ell^{out}/\partial s_\ell - c'(s_\ell) = \partial p_\ell^{in}/\partial s_\ell - c'(s_\ell) > 0$ and $n_\ell^{out} > n_\ell^{in} \geq 0$; $(\partial n_h/\partial s_\ell)(p_h^{in} - c(s_h)) < (\partial n_h/\partial s_\ell)(p_h^{out}(w_h^{out}) - w_h^{out}) < 0$, $-n_h^{out}(\partial w_h^{out}/\partial s_\ell) > 0$, and $(\partial n_\ell/\partial w_h)(\partial w_h^{out}/\partial s_\ell)[p_\ell^{out} - c(s_\ell) - (p_h^{out}(w_h^{out}) - w_h^{out})] > 0$. Therefore, $d\pi_D^{out}/ds_\ell > d\pi^{in}/\partial s_\ell \forall (s_\ell, s_h)$ and $s_\ell^{out}(s_h) > s_\ell^{in}(s_h) \forall s_h$.

For all given low-end quality s_ℓ, the equilibrium high-end quality under full in-house production, $s_h^{in}(s_\ell)$ such that $d\pi^{in}/ds_h = 0$, equals the corresponding quality under out-

[7] The above-mentioned condition is satisfied for quadratic costs $c(s) = a + bs + cs^2$, which are commonly found in the literature.

sourcing, $s_h^{out}(s_\ell)$ such that $d\pi_U^{out}/ds_h = 0$, because $d\pi_U^{out}/ds_h = (1-\beta^2)(d\pi^{in}/ds_h)/2 =$

$$(1-\beta^2)[s_h - c(s_h) - s_\ell + c(s_\ell)][c(s_h) - c(s_\ell) + (s_h - s_\ell)(1 - 2c'(s_h))]/[8(s_h - s_\ell)^2].$$

Finally,

$$\frac{d^2\pi_U^{out}}{ds_\ell ds_h} = \frac{d^2\pi^{in}}{ds_\ell ds_h} = -\frac{2(1+\beta)(1-\beta)}{8(s_h - s_\ell)^3}\times$$

$$[c(s_\ell) - c(s_h) + (s_h - s_\ell)c'(s_\ell)][c(s_\ell) - c(s_h) + (s_h - s_\ell)c'(s_h)] > 0.$$

Therefore, as long as the standard regularity condition $(s_\ell^i)'(s_h^i)' < 1 \forall i \in \{in, out\}$ satisfied, the equilibrium qualities (s_ℓ, s_h) such that $s_\ell = s_\ell^{out}(s_h)$ and $s_h = s_h^{out}(s_\ell)$ are higher than the qualities such that $s_\ell = s_\ell^{in}(s_h)$ and $s_h = s_h^{in}(s_\ell)$. □

Quality choice follows the same logic as in the baseline model. For all given level of high-end quality, the low-end quality that maximizes the profits of the firm is higher under outsourcing because of a positive bargaining effect. Additionally, in the present framework, the decrease in wholesale price ensuing from a rise in low-end quality reallocates consumption from the low-end version to the more profitable high-end version. On the other end, as in Section 3, the contractor sets the high-end quality as if it were extracting the whole marginal surplus. The explanation for why outsourcing improves both qualities, even if it doesn't decrease costs, is that the equilibrium quality of the high-end version increases with that of the low-end version.

4.3 Outsourcing the low-end version

In this section, assume the low-end version of the product may be outsourced and focus on the case where consumers have private information about their type.[8] Denoting the wholesale price of the outsourced low-end version as w_ℓ, we obtain the following profit

[8] Note that in practice both the low-end and high-end versions may be outsourced. Our baseline model is not well-suited to study such market configurations, because the duality of the disagreement payoffs results in too many cases. Considering a continuum of consumers, as in Section 4.2, solves that problem, but additional assumption regarding the timing of the bargaining subgame are necessary.

functions: $\pi_D = n_\ell[p_\ell - w_\ell] + n_h[p_h - c(s_h)]$ and $\pi_u = n_\ell w_\ell - (1-e)\,c\,(s_\ell)$. The equilibrium retail prices are the same as in the baseline model and the equilibrium wholesale price is obtained by maximization of

$$M = [n_\ell(s_\ell v_\ell - w_\ell) + n_h(s_h v_h - s_\ell(v_h - v_\ell) - c(s_h)) - \pi_D^0]^\beta [n_\ell w_\ell - (1-e)\,c\,(s_\ell)]^{1-\beta}.$$

The disagreement payoff of the contractor is zero and that of the firm is

$$\pi_D^0 = \begin{cases} n_h[s_h v_h - c(s_h)] \; if \; n_h/n_\ell \geq r, \\ (n_\ell + n_h)[s_h v_\ell - c(s_h)] \; otherwise, \end{cases}$$

where $r \equiv [s_h v_\ell - c(s_h)]/[s_h(v_h - v_\ell)]$. We find

$$w_\ell = \begin{cases} (1-\beta)s_\ell[n_\ell v_\ell - n_h(v_h - v_\ell)]/n_\ell + \beta(1-e)c(s_\ell) \; if \; n_h/n_\ell \geq r, \\ (1-\beta)(s_h - s_\ell)[n_h(v_h - v_\ell) - n_\ell v_\ell]/n_\ell + \beta(1-e)c(s_\ell) + (1-\beta)c(s_h) \; otherwise. \end{cases}$$

The following result ensues.

Proposition 7. *When the low-end version of the product is outsourced,*

$$s_\ell = c'^{-1}\left[\frac{n_\ell v_\ell - n_h(v_h - v_\ell)}{n_\ell(1-e)}\right] > s_\ell^{**}$$

iff $e > 0$ *and*

$$s_h = \begin{cases} c'^{-1}(v_h) = s_h^{**} \; if \; n_h/n_\ell \geq r \\ c'^{-1}\left[\frac{n_\ell v_\ell(1-\beta) + n_h[v_\ell(1-\beta) + \beta v_h]}{n_\ell(1-\beta) + n_h}\right] < s_h^{**} \; otherwise. \end{cases}$$

.

Proof. From the first-order conditions $d\pi_U/ds_\ell = 0$ and $d\pi_D/ds_h = 0$. □

As in the baseline model, the quality of the outsourced version (presently, the low-end version) increases if and only if the contractor has a cost advantage. This is explained by the fact that the contractor internalizes a share of the negative effect of the low-end quality on the high-end profits and pays the same share of the costs, as shown by the first-order-condition $d\pi_U/ds_\ell = 0$, which can be written as

$$(1-\beta)[n_\ell v_h - n_h(v_h - v_\ell) - n_\ell(1-e)c'(s_\ell)] = 0.$$

However, by contrast with the baseline model, the quality of the in-house version (presently, the high-end version) decreases compared to the in-house production benchmark. This is explained by a negative bargaining effect through which the high-end quality reduces the low-end profits of the firm. Indeed, the first-order condition that defines the equilibrium quality s_h is

$$d\pi_D/ds_h = -\partial w_\ell/\partial s_h + n_h[\partial p_h/\partial s_h - c'(s_h)] = 0,$$

where $\partial w_\ell/\partial s_h$ is negative if both groups of consumers are served in the event of a negotiation breakdown and null otherwise.

Proposition 7 implies that the existence of a cost advantage for the contractor is a necessary condition for both profitable and welfare-improving outsourcing.

4.4 When is versioning profitable?

The profitability of versioning compared to pooling is a long-standing question in the literature. Following the approach introduced by **?**, pooling occurs if the low-end version is not offered ($s_\ell \leq 0$ in equilibrium), if both the low-end and the high-end versions reach a maximum quality threshold, or if the quality-ordering condition $s_\ell < s_h$ is violated.

From Proposition 3, a pooling equilibrium where the low-end version is not offered is less likely to exist when the high-end version is outsourced than under in-house production, while an equilibrium where both qualities exceed the threshold is more likely to exist if the contractor is cost-efficient ($e > 0$). A pooling equilibrium where the quality of the low-end version equals or surpasses that of the high-end version is less likely to exist unless the contractor has a strong cost disadvantage. From Proposition 7, outsourcing the low-end version makes the first type of pooling equilibrium less (resp. more) likely to exist if the contractor is cost-efficient (resp. cost-inefficient). It makes the second type of pooling equilibrium less likely to exist and the third type more likely to exist if the contractor is cost-efficient.

The changes in type of equilibrium outsourcing may lead to are important for assessing the welfare implications of outsourcing and regulations related to versioning such as minimum quality standards.

5 Concluding remarks

Considering a framework in which a retail monopolist may outsource either the high-end or the low-end version of a product, we have shown that outsourcing the high-end version may alleviate the downward distortion in quality for the low-end version compared to when the firm produces both versions in-house. This result can be attributed to a bargaining effect, where a higher quality of the low-end version enhances the firm's bargaining power with its contractor, resulting in a lower wholesale price for the high-end version. Moreover, if outsourcing leads to improvements in cost-efficiency, quality of the high-end version tends to increase, resulting in overall welfare gains. This outcome highlights the potential positive implications of outsourcing on quality and demonstrates that it can be Pareto-improving, particularly if the firm possesses sufficient bargaining power in its outsourcing arrangements. However, outsourcing may also result in a level of low-end quality that surpasses the socially optimal level, in which case its overall impact on social welfare is ambiguous.

When introducing a percentage of irrecoverable costs to commit to a specific level of quality, the incentives of the contractor to set increased high-end quality are reduced since in the event of a negotiation breakdown there is a sunk cost to be paid. By contrast, when demand is elastic with a continuum of uniformly distributed consumers, quality increase is reinforced by strategic complementarity of the decisions of the firm and its contractor. When the low-end version of the product is outsourced, instead of the high-end version, welfare is less likely to increase because outsourcing then reduces the quality of the in-house, high-end version.

Our main results relate to the profitability and welfare implications of outsourcing, but our model could be used to study regulations related to discrimination and quality levels. For example, in our framework, minimum quality regulation could affect welfare through its impact on both outsourcing and versioning decisions. Our research potentially could be extended in several directions. For instance, it would be interesting to study upstream price discrimination where the price of an input would depend on the quality of the version it is used for. More complex extensions would involve asymmetric information within the supply chain.