Returns to Housing with Financial Frictions

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Abstract

This study investigates the effects of monetary policy on returns to housing in the context of a New Keynesian DSGE model with financial frictions. We find that contractionary monetary policy with financial frictions, in the form of loan-to-value (LTV) and payment-to-income (PTI) limits, amplifies the expected excess returns to housing compared to a model without financial frictions. Monetary policy has both direct and indirect effects on excess returns to housing. An increase in the nominal interest rate increases excess returns to housing directly by increasing the shadow value of borrowing and, indirectly, through its influence on the covariance between future housing returns and the marginal utility of consumption. These predictions are supported by empirical evidence from Canadian quarterly macro data over the period 2000Q1 to 2022Q2, a time period associated with more restrictive financial frictions.

1 Introduction

In recent years, there has been a notable surge in global housing prices. The significant rise and subsequent fall in housing prices and residential investment in the present time have drawn the interest of many economists. This has led to a growing need to understand housing's role in macroeconomics and formulate effective real estate sector policies. In this study, we introduce a New Keynesian model with financial frictions: loan-to-value (LTV) and payment-to-income (PTI) limits to analyze the impact of monetary policy to the housing returns and whole economy. Our research has two phases: Firstly, we construct a dynamic stochastic general equilibrium (DSGE) model enriched with LTV and PTI frictions. Subsequently, we employ a Structural Vector Autoregression (SVAR) model to empirically investigate the impact of contractionary monetary policies, LTV , and PTI limits on housing returns and the broader economy. Through this analysis, we aim to demonstrate the model's empirical explanatory power in both the housing and non-housing sectors.

The model we propose in this paper is closely related to Monacelli's (2009) model, which examines how credit market frictions can reconcile a standard New Keynesian (NK) model with empirical evidence of monetary policy shocks on durable and non-durable consumption. Monacelli's model predicts that durable and non-durable consumption co-move downwards, but the decline in durable consumption is more significant than non-durable consumption. In this study, we consider a closed economy dynamic stochastic general equilibrium (DSGE) model with income and collateral constraints. We show that the expected excess return to housing is higher in tighter monetary policy with the LTV and PTI constraints than without the aforementioned constraints. Both constraints generate future consumption uncertainty directly and indirectly, leading to a decrease in housing prices and an increase in the expected excess return. Overall, we find that monetary policy with financial frictions has a significant and systematic effect on price stability and absolute economic stability.

Conducting monetary and macroprudential policies has the benefit of enhancing the clarity and transparency of each policy's goals and instruments, enabling easier evaluation and accountability of decision-makers for achieving their objectives. We consider two of the most prevalent macroprudential policies: LTV limit as a collateral constraint and the PTI limit as an income constraint. LTV limits are conventional and restrict the debt issued to borrowers to be a net downpayment fraction of the anticipated future value of the housing goods. In contrast, PTI limits limit interest payments on debt and periodic debt payments to a specific fraction of borrowers' labor income. When these essential macroprudential policy tools are integrated with tighter monetary policies, borrowing costs rise significantly. As a result, only a limited number of borrowers meet the criteria for loan qualification, exerting both direct and indirect influence on household debt. This, in turn, triggers a notable decrease in housing consumption, leading to a drop in housing prices and an increase in anticipated excess returns.

We provide evidence that the incorporation of financial frictions of this nature plays a crucial role in reconciling the the results of the model. First, we examine the model with a single financial friction, namely collateral requirements, and demonstrate that the presence of this friction results in higher excess returns to housing than in the absence of such a friction. The model with only collateral constraint incentivises borrowers to accumulate more debt, as the acquisition of less expensive homes allows them to alleviate this constraint, subsequently enabling them to purchase more expensive properties. To avoid high debt accumulation we introduce two financial frictions: collateral, and income requirements, and demonstrate that the expected excess return to housing is even higher with both frictions than with only one friction. We can interpret this result in two ways. Firstly, the binding collateral and income constraints discourage households from consuming housing services, reducing the relative demand for housing, negatively affecting prices, and increasing the expected excess return to housing. Secondly, the covariance between two variables, the future marginal consumption of nonhousing goods, and the expected excess return, is more likely to be negative, and the value of covariance is more significant due to consumption volatility. This impairs households' ability to smooth consumption, leading to precautionary savings. The intuition here is that the more negative the covariance between these two variables, the collateral constraint coincides with low ex-post housing returns requiring a high premium for housing and services.

In our framework, credit frictions are modeled as stochastic combinations of LTV and PTI borrowing constraints. Specifically, when a borrower's random labor income surpasses a threshold value, the LTV constraint binds, whereas when income falls below the threshold, the PTI constraint binds. In the contractionary monetary policy environment, the PTI constraint binds more frequently. In the context of two constraints, the actual collateralized debt of the borrower is the minimum of the two debt levels, indicating that the LTV and PTI limits significantly limit the households' debt level. With both constraints in place, the debt level sharply decreases compared to only the LTV constraint. In the presence of these two constraints, a contractionary monetary policy that raises the interest rate further complicates households' efforts to smooth their consumption of housing goods, even when housing prices are fully flexible. The inability to smooth consumption of housing goods results in a decrease in the prices of housing and services and an increase in the expected excess return.

We also extend the model by adding adjustment cost in housing consumption and introduce habit parameters in nonhousing consumption. The habit models imply an inverse relation between expected returns and surplus consumption. Adding habits to the model adds an extra consumption smoothing motive to the household, therefore the households require a higher return in order to consume housing goods.

In this study, we also aim to provide empirical evidence to support the results of our proposed model. Specifically, we focus on the Canadian economy and analyze recent quarterly data on macroeconomic variables using a structural vector autoregression (SVAR) framework. Since there is no indication of macroprudential policy being implemented in Canada before the fourth quarter of 1999, we divide the data into two periods, pre and post-2000 for 86 quarters, and perform structural VAR analysis on each period separately. Our findings suggest that in response to a contractionary monetary policy shock, characterized by a 25 basis point increase in interest rates, the excess return to housing is higher in the post-2000 period compared to the pre-2000 period. This result implies that the combination of contractionary monetary policy and financial frictions, as captured by our model, provides greater rewards to borrowers in the post 2000 periods. Overall, our empirical analysis provides support for the theoretical results of our model, highlighting the importance of incorporating financial frictions in macroeconomic models to understand the impact of policy interventions on the housing sector.

Our study contributes to the macroeconomics literature that explores the impact of monetary policy on the housing sector in the context of DSGE models with sticky prices. One relevant study in this literature is the work of Lacoviello and Neri (2010), who estimate a New Keynesian model and investigate the sources and consequences of fluctuations in the US housing market. Their findings indicate that slow technological progress in the housing sector is responsible for the upward trend in real housing prices over the past four decades. Moreover, they show that the housing market spillovers affect consumption rather than business investment and have become increasingly important over time. Another relevant study in this literature is the analysis by Gabriel et al. (2018), who examine the effectiveness of a countercyclical LTV ratio in responding to credit-to-GDP or house price fluctuations compared to a monetary policy rule augmented with house price inflation. Their study investigates the reduction of household indebtedness and housing price fluctuations. Additionally, Monacelli (2009) incorporates a housing goods sector into a general equilibrium model with collateral constraints and analyzes the monetary policy reaction to sectoral fluctuations.

Mendoza and Smith's (2005) work contributes to the understanding of the Sudden Stop phenomenon by demonstrating that the quantitative predictions of an equilibrium asset-pricing model with financial frictions align with its key features. Specifically, a Sudden Stop is characterized by sudden and significant reversals in capital inflows and the current account, notable declines in absorption and production, and collapses in real asset prices and non-tradable goods relative to tradeable ones. The authors illustrate that the binding margin requirement in the current date leads to an increase in the equity premium due to margin calls' pressure on households to liquidate their equity holdings, depressing the current equity price and driving up the excess return.

Our study contributes to the existing literature on the impact of monetary policy and changes in LTV and PTI ratios in a dynamic stochastic general equilibrium (DSGE) framework. This literature includes studies such as Iacoviello (2005), Iacoviello and Neri (2010), Monacelli(2009) and, Mendoza and Smith (2005). A non-exhaustive list includes Christensen, Corrigan, Mendicino, and Nishiyama (2009), Kannan, Rabanal, and Scott (2012), Justiniano, Primiceri, and Tambalotti (2013), Lambertini, Mendicino, Teresa Punzi (2013*b*), Gelain et al. (2013) and Gelain, Ko- Lasa, and Brzoza-Brzezina (2014).

We organize our work in this paper as follows. Section 2 presents the NK-DSGE model and discusses the optimality conditions for borrowers and lenders. Section 3 calibrates the model with credit frictions, compares the numerical results, and performs the sensitivity analysis of expected excess returns to housings. Section 4 discusses the extension of the model with adjustment cost and habit. Section 5 presents the empirical evidence that the effect of overnight rate and LTV ratio limit shocks in the economy and the excess return to housing, with the most recent data for the Canadian economy. The paper ends with concluding remarks.

2 The Dynamic Stochastic General Equilibrium Asset Pricing Model With Financial Frictions

2.1 The Model

We extend the benchmark New Keynesian setup to incorporate household heterogeneity and credit frictions. The economy is possessed of a continuum of households in $[0, 1]$. We consider an economy with two types of households called borrowers and savers. The credit flows are generated by assuming ex-ante heterogeneity in agents' subjective discount factors.[1] Impatient agents differ from patient agents in that they discount the future at a faster rate. Hence, in equilibrium, patient agents are net lenders, while impatient agents are the net borrowers. To prevent borrowing from growing at the desired limit instantaneously, we assume that borrowers face a credit constraint tied to the current value of their collateral.

A fractions $\omega \in [0, 1]$ measures borrowers and $1 - \omega$ measures savers. Each individual household's time endowment is normalized to 1. The housing and nonhousing goods sectors are each populated by a large number of monopolistic competitive firms. Households derive utility from the consumption of nonhousing final goods and from services of final housing goods. Debt accumulation reflects intertemporal trading between borrowers and savers. Only the borrowers are subject to LTV constraints.

Following the literature, we define an index of consumption:

$$X_t \equiv \left[(1-\mu)^{\frac{1}{\eta}}(C_t)^{\frac{\eta-1}{\eta}} + \mu^{\frac{1}{\eta}}(H_t)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where, C_t denotes the consumption of final nonhousing goods, H_t denotes services from the stock of the final housing goods at the end of period t, [2] $\mu > 0$ is the share of housing goods in the composite consumption index, $\eta \geq 0$ is the elasticity of substitution.

2.2 Borrowers Maximization Problem

This type of household maximizes the following utility:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(X_t, N_t) \right\} \tag{2}$$

subject to the average budget constraint:

$$P_{c,t}C_t + P_{h,t}(H_t - (1-\delta)H_{t-1}) + R_{t-1}B_{t-1} = B_t + W_t N_t + T_t \tag{3}$$

and the collateral constraint:

$$R_t B_t \leq \left(1 - \kappa^{ltv}\right)(1-\delta)\mathbb{E}_t\{H_t P_{h,t+1}\} \tag{4}$$

[1] Earlier models that consider heterogenous discount factor, see Monachelli (2009), Iacoviello (2005), Kiyotaki and Moore (1997), Krusell and Smith (1998).

[2] We define housing investment as the flow variable $H_t - (1-\delta)H_{t-1}$

where B_t, the nominal debt at the end of period t, T_t the government lump-sum transfer and W_t the nominal wage, which is common across the sectors, N_t total labor supply, and κ^{ltv} the fraction of the housing goods value that can not be used as collateral.

For optimization in units of nonhousing consumption, budget constraint is then given as:

$$C_t + q_t(H_t - (1-\delta)H_{t-1}) + R_{t-1}\frac{b_{t-1}}{\pi_{c,t}} = b_t + \frac{W_t}{P_{c,t}}N_t + \frac{T_t}{P_{c,t}} \tag{5}$$

where, $q_t \equiv \frac{P_{h,t}}{P_{c,t}}$, is the relative price of housings, $\pi_{c,t} \equiv \frac{p_{c,t}}{p_{c,t-1}}$, is non-housing goods inflation, and $b_t = \frac{B_t}{P_{c,t}}$ is real debt. Rewriting (4) in real terms (i.e., in terms of nonhousing consumption):

$$b_t = \left(1 - \kappa^{ltv}\right)(1-\delta)\mathbb{E}_t\left\{\frac{H_t q_{t+1}}{R_t/\pi_{c,t+1}}\right\} \tag{6}$$

Given initial values, $\{b_{-1}, H_{-1}\}$, the representative borrowers choose $\{C_t, N_t, H_t, b_t\}$ to maximize expected lifetime utility based on information available at time 0:

$$\sum_{t=0}^{\infty}\beta^t\left(log(X_t) - \frac{\nu N_t^{1+\phi}}{1+\phi}\right)$$

subject to, (5) and (6).

Optimality conditions by defining λ_t and $\lambda_t\psi_t$ as multipliers to subjects:

$$\frac{-U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}} \tag{7}$$

$$U_{c,t} = \lambda_t \tag{8}$$

$$U_{c,t}q_t = U_{h,t} + \beta(1-\delta)\mathbb{E}_t[q_{t+1}U_{c,t+1}] + \left(1-\kappa^{ltv}\right)(1-\delta)U_{c,t}\psi_t q_t\mathbb{E}_t(\pi_{h,t+1}) \tag{9}$$

$$R_t\psi_t = 1 - \beta\mathbb{E}_t\left[\frac{U_{c,t+1}}{U_{c,t}}\frac{R_t}{\pi_{c,t+1}}\right] \tag{10}$$

The equation presented in (7) is a crucial condition that establishes an equivalence between the real wage, as measured in units of non-housing goods, and the marginal rate of substitution of borrowers between consumption and leisure. Meanwhile, in equation (8), the marginal utility of consumption for borrowers is made equivalent to the shadow value of relaxing the budget constraint. Equation (9) demands that the borrower match the marginal utility of non-housing consumption to the shadow value of the housing service. The shadow value of housing hinges on three components: the direct utility gain from an additional unit of housing, the anticipated utility from the potential for expanding future consumption by means of the realized resale value of the housing bought in the prior period, and the marginal utility obtained from relaxing the collateral constraint, which is proportional to ψ_t.

Equation (10) takes the form of the Euler equation of debt, which reduces to a standard Euler condition when the collateral constraint is non-binding, that is, when $\psi = 0$. The shadow value of borrowing, denoted as ψ_t, represents the additional utility that could be gained from borrowing an extra dollar, using it to increase current consumption, and subsequently reducing consumption in the next period by an appropriate amount. When the debt constraint is binding, that is, when $\psi_t > 0$, the constrained optimum occurs where the marginal utility of current consumption exceeds the discounted expected marginal utility of future consumption.

2.3 Expected Excess Return

Given the definition of the gross rate of returns to housings,[3]

$$R^h_{t+1} = \frac{q_{t+1}}{q_t} \tag{11}$$

The first-order conditions can be combined to derive the following expression for the expected premium on housings:

$$\mathbb{E}[R^h_{t+1}] - R_t = \frac{1 - \frac{1}{q_t} Z_t - \beta Cov\left(R^h_{t+1}, \lambda_{t+1}\right)}{\beta \mathbb{E}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)} + \frac{\kappa^{ltv}(1-\delta)\psi_t \mathbb{E}_t(\pi_{h,t+1})}{\beta \mathbb{E}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)} - \frac{1}{\psi_t + \beta \mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{c,t+1}}\right\}} \tag{12}$$

If collateral constraints never bind *i.e.* $\psi = 0$ for all t, this expression yields the standard result for the housings' return of a frictionless model:

$$\mathbb{E}[R^h_{t+1}] - R_t = \frac{1 - \frac{z_t}{q_t} - \beta Cov\left(R^h_{t+1}, \lambda_{t+1}\right)}{\beta \mathbb{E}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)} - \frac{1}{\beta \mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{c,t+1}}\right\}}. \tag{13}$$

Equation (12) reveals that the expected excess return to housing assets is subject to various parameters of the model, including the shadow value of borrowing, ψ_t, the user cost of housing, z_t, and the LTV limit, κ^{ltv}. As z_t increases, the expected premium decreases, whereas an increase in both ψ_t and κ^{ltv} raises the premium. When the collateral requirement at date t is binding, that is, when $\psi_t > 0$, it has two-fold implications, namely, the direct and indirect effects.

Direct effect: a binding collateral constraint, $\psi_t > 0$, implies that the utility derived from consuming non-housing goods in the current period is greater than the expected utility from consuming such goods in the next period, i.e., $U_{c,t} > \mathbb{E}t\{Uc, t+1R_t/\pi_{t+1}\}$. This reflects a tighter collateral constraint, leading to a decline in the demand for housing, a decrease in housing prices, and a greater premium required by borrowers.

Indirect effect: the indirect effect is reflected in equation (12), where the covariance between λ_{t+1} and R^h_{t+1} is likely to be more negative, as the risk of binding borrowing constraints at $t+1$ hampers households' ability to smooth consumption, leading to precautionary savings. A more negative covariance between non-housing good consumption in the next period and the expected return from housing goods indicates that the collateral constraint coincides with low ex-post housing returns, which means that housing assets are a bad hedge, and borrowers require a high premium on housing assets.

From equation (9), the user cost of housing equivalent to the marginal rate of substitution between housing and nonhousing consumption:

$$Z_t \equiv \frac{U_{h,t}}{U_{c,t}} = q_t \left[1 - \left\{\psi_t \frac{(1-\kappa^{ltv})(1-\delta)\psi_t \mathbb{E}_t(\pi_{h,t+1})}{R_t}\right\}\right] - \beta(1-\delta)\mathbb{E}_t\left(\frac{\lambda_{c,t+1} q_{t+1}}{\lambda_{c,t}}\right) \tag{14}$$

[3]Mendoza and Smith (2006)

In a frictionless economy, the user cost of housing depends on the current and expected prices, but an economy with borrowing constraints, the user cost also depends on the shadow value ψ_t. Let's log-linearize equation (14) around the deterministic steady-state to obtain:

$$\hat{Z}_t = \Phi^{-1}(1-\delta)\left[\Gamma\hat{q}_t - \beta\mathbb{E}_t[\hat{q}_{t+1}] + \gamma\hat{R}_{r,t} + (\gamma-\beta)(\kappa^{ltv}\hat{\psi}_t - \hat{\zeta}_t)\right] \tag{15}$$

where, $\Gamma \equiv \left[\frac{1-(1-\kappa^{ltv})(1-\delta)(\gamma-\beta)}{(1-\delta)}\right]$, $\Phi \equiv 1-(1-\delta)\left[\beta + (1-\kappa^{ltv})(\gamma-\beta)\right]$ and $\hat{R}_{r,t} \equiv \hat{R}_t - \mathbb{E}_t(\hat{\pi}_{c,t+1})$ is the ex-ante real interest rate in units of nonhousings and $\hat{\zeta}_t \equiv \mathbb{E}_t\left[(1-\kappa^{ltv})\hat{\pi}_{h,t+1} - \hat{\pi}_{c,t+1}\right]$ is the combined sectoral inflation. The demand for housing rises when the expected future price rises due to the expected asset appreciation. When the heterogeneity among households vanishes, i.e., $\gamma = \beta$, the movements in $\hat{Z}_t$ depend positively on the current relative price of housing and negatively on the expected future price of housing.

2.4 Re-visiting Borrowers Optimization Problem With Income and Collateral Constraint

When borrowers seek to obtain loans for purchasing housing, lenders typically impose collateral requirements along with other requirements such as verifying employment and income to ensure that the borrower's cash flow is sufficient to make periodic debt and interest payments. These requirements are critical components of financial frictions that affect credit markets. In this section, we explore the optimization problems faced by borrowers subject to LTV and PTI constraints.

To obtain a new one-period loan $B_{i,t}$, an individual borrower i must satisfy both the LTV and PTI constraints. The LTV constraint is captured in equation (4), while the PTI constraint can be expressed as follows:

$$(R_t - 1 + \tau)\, {B_{i,t}}^{pti} \leq \left(1-\kappa^{pti}\right) W_t N_{i,t} e_{i,t} \tag{16}$$

where, the parameter κ^{pti} denotes the fraction of income that cannot be utilized to cover periodic debt and interest payments, while τ represents taxes, insurance, and other borrowing costs associated with debt issuance and payment. Furthermore, $e_{i,t}$ signifies a stochastic shock to the borrower's labor income that follows a log-normal distribution with a mean of 1 and cumulative distribution function F_e.

The PTI constraint places an upper limit on the interest payments for the debt, restricting it to a specific fraction of each borrower's random labor income. Similarly, the LTV constraint restricts the amount of debt issued to each borrower to a fraction of the expected future value of the housing. Therefore, to obtain a loan, a borrower must satisfy both the LTV and PTI constraints. The maximum level of debt that a borrower can acquire while satisfying both constraints is determined as follows: $B_{i,t} \leq \bar{B}_{i,t} = min\left(\bar{B}^{ltv}_{i,t}, \bar{B}^{pti}_{i,t}\right)$ where, $\bar{B}^{ltv}_{i,t} = \left(1-\kappa^{ltv}\right)(1-\delta)\mathbb{E}_t\{H_t P_{h,t+1}\}$ and $\bar{B}^{pti}_{i,t} = \left(1-\kappa^{pti}\right) W_t N_{i,t} e_{i,t}$.

The maximum average LTV and PTI debt limits when $\mathbb{E}[e_{i,t}] = 1$ are,

$$\bar{B}^{ltv}_t = \frac{\left(1-\kappa^{ltv}\right)(1-\delta)\mathbb{E}_t\{H_t P_{h,t+1}\}}{R_t} \tag{17}$$

$$\bar{B}^{pti}_t = \frac{\left(1-\kappa^{pti}\right) W_t N_t}{(R_t - 1 + \tau)} \tag{18}$$

Let, $\bar{e}_t = \frac{\bar{B}_t^{ltv}}{\bar{B}_t^{pti}}$ be the threshold value of the income shock $e_{i,t}$ such that when $e_{i,t} < \bar{e}_t$ the PTI constraint binds, and when $e_{i,t} > \bar{e}_t$ the LTV constraint binds. The combined average borrower debt $\bar{B}_t$ becomes;

$$\bar{B}_t = \int min\left(\bar{B}_t^{ltv}, \bar{B}_t^{pti} e_{i,t}\right) dF_e(e_{i,t}) = \bar{B}_t^{ltv}\left(1 - F_e(\bar{e}_t)\right) + \bar{B}_t^{pti} G(\bar{e}_t) \tag{19}$$

where,

$$G(\bar{e}_t) = \int_0^{\bar{e}_t} e_{i,t} dF_e(e_{i,t}$$

is the fractions of the borrowers with a binding PTI constraint and

$$\left(1 - F_e(\bar{e}_t)\right) = \int_{\bar{e}_t}^{\infty} \bar{B}_t^{ltv} dF_e(e_{i,t})$$

The final two components of equation (19) depict the borrowing potential of households that are bound by the LTV and PTI constraints. To facilitate our analysis and enable comparisons with existing research, we presume that individual borrowers have access to Arrow-Debreu securities and that the random shock to their labor income, $e_{i,t}$, is log-normally distributed. This allows us to aggregate the optimization problems of individual borrowers into a representative borrower's problem. This framework aligns with the representative agent framework introduced in Monacelli(2009) and permits us to draw meaningful comparisons with previous literature.

Now, the representative borrower's optimization problem is to maximize expected discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \left(log(X_t) - \frac{\nu N_t^{1+\phi}}{1+\phi} \right)$$

with respect to the budget constraint (4) and the combined collateral and income constraint (19).

The optimality conditions are;

$$\frac{-U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}} \left[1 + \psi_t \frac{(1-\kappa^{pti})}{(R_t - 1 + \tau)} G(\bar{e}_t) \right] \tag{20}$$

$$U_{c,t} = \lambda_t \tag{21}$$

$$U_{c,t} q_t = U_{h,t} + \beta(1-\delta)\mathbb{E}_t[q_{t+1}U_{c,t+1}] + \left(1 - \kappa^{ltv}\right)(1-\delta)U_{c,t}\psi_t q_t \mathbb{E}_t(\pi_{h,t+1}) \frac{\left(1 - F_e(\bar{e}_t)\right)}{R_t} \tag{22}$$

$$R_t \psi_t = 1 - \beta \mathbb{E}_t \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{R_t}{\pi_{c,t+1}} \right] \tag{23}$$

After manipulating optimality conditions, expected excess return turns to be,

$$\mathbb{E}[R_{t+1}^h] - R_t = \frac{1 - \frac{1}{q_t} Z_t - \beta Cov\left(R_{t+1}^h, \lambda_{t+1}\right)}{\beta \mathbb{E}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)} + \frac{\kappa^{ltv}(1-\delta)\psi_t \mathbb{E}_t(\pi_{h,t+1})}{\beta \mathbb{E}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1 - G(\bar{e}_t)\right)} - \frac{1}{\psi_t + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{c,t+1}} \right\}} \tag{24}$$

Equation (20) establishes a link between the marginal substitution rate between consumption and leisure and the opportunity cost of leisure, which depends on the presence of the LTV constraint and the level of κ^{pti}. When the LTV constraint is in place and $\kappa^{pti} = 0$, the equation reduces to the standard labor market condition. However, when both constraints are present, the opportunity cost of leisure is higher, as working more hours generates more income and helps meet the PTI constraint. Moreover, the interest rate R_t plays a critical role in equation (20), as it affects the opportunity cost of leisure, and hence, the labor supply of borrowers, creating important implications for the propagation of monetary shocks and the model's dynamics compared to models with only collateral frictions.

Equation (21) establishes the shadow value of housing as proportional to ψ_t, which measures the tightness of the debt constraint. When the constraint is not binding ($\psi = 0$), the shadow value of housing reduces to the sum of the first two terms. Equation (22) provides the Euler equation for debt, where the shadow value of borrowing represents the extra utility from borrowing an additional dollar to increase current consumption and decrease consumption next period. Given the binding debt constraint ($b_t \leq \bar{b}_t$), the shadow value of housing is positive ($\psi_t > 0$), and $Uc, t > \mathbb{E}t\{Uc, t+1R_t/\pi_{t+1}\}$.

In equation (24), the expected excess return to housing depends on various model parameters, such as the shadow value of borrowing (ψ_t), the user cost of housing (z_t), and the limits on PTI (κ^{pti}) and LTV (κ^{ltv}). Specifically, the expected premium decreases as the user cost of housing increases, whereas it increases as the limits on PTI, LTV, and the shadow value of borrowing increase. If only the LTV limit is considered, then $(1 - \kappa^{pti})$ is equal to zero, and the excess return decreases, suggesting that households bind more tightly in the presence of both constraints. As a result, the demand for housing decreases, leading to a decline in housing prices, with only opportunistic borrowers willing to buy at higher premiums.

When a collateral or income requirement is binding at a certain date t, denoted by $\psi > 0$, it has both direct and indirect effects on the excess return, as discussed earlier. The shadow value of borrowing, represented by ψ, is smaller when both LTV and PTI constraints are present compared to when only the LTV constraint is in place. This is due to the fact that an additional hour of work generates more income, which relaxes the PTI constraint. However, in the context of contractionary monetary policy, where an interest rate hike is expected, the rising interest rate reduces the opportunity cost of leisure, resulting in a decrease in the borrower's labor supply, making them more constrained by the PTI limit. In such an environment, durable goods are less attractive to borrowers, leading to a decrease in demand and a consequent increase in the price of housing, which implies a higher excess return. This indirect effect is in line with the collateral constraint alone.

Rearranging equation (22), we obtain the expression for the user cost of housings as;

$$Z_t = q_t \left[1 - \left\{ \psi_t \frac{(1-\kappa^{ltv})(1-\delta)\psi_t \mathbb{E}_t(\pi_{h,t+1})}{R_t} \left(1 - F_e(\bar{e}_t)\right) \right\} \right] - \beta(1-\delta)\mathbb{E}_t \left(\frac{\lambda_{c,t+1} q_{t+1}}{\lambda_{c,t}} \right) \quad (25)$$

Equation (27) after log-linearizing about the deterministic steady state,

$$\hat{Z}_t = \Phi^{-1}(1-\delta) \left[\Gamma \hat{q}_t - \beta \mathbb{E}_t[\hat{q}_{t+1}] + \gamma \hat{R}_{r,t} + (\gamma - \beta)(\kappa^{ltv} + (1-\kappa^{ltv})F(\bar{e})\hat{\psi}_t - \hat{\zeta}_t) - \gamma(\gamma-\beta)\kappa^{ltv}[1 - F(\bar{e})]\hat{R}_t \right] \quad (26)$$

where,

$$\Phi = 1 - (1-\delta)\left(\beta + \gamma\kappa^{ltv}(\gamma - \beta)[1 - F(\bar{e})]\right)$$

$$\Gamma = \frac{(1 - \gamma\kappa^{ltv}(1-\delta)(\gamma-\beta)[1 - F(\bar{e})])}{(1-\delta)}$$

$$\hat{\zeta}_t = \mathbb{E}_t\left(\kappa^{ltv}(1 - F(\bar{e})) - \pi_{c,t+1}\right)$$

Φ is the steady state value of Z_t and $\hat{R}_{r,t} = \hat{R}_t - \mathbb{E}_t[\hat{\pi}_{c,t+1}]$ is the real interest rate steady state deviation in units of nonhousings.

2.5 Savers' Optimization Problem

The savers are owners of the monopolistic competitive firms that produce the intermediate goods, and in each period, they collect the monopolistic profits. Like borrowers, savers consume, work, earn wages, and receive transfers from the government. Infinitely lived representative savers choose $\{\tilde{N}_t, \tilde{C}_t, \tilde{b}_t, \tilde{H}_t\}$ to maximizes:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \gamma^t \left(log(\tilde{X}_t) - \frac{\nu \tilde{N}_t^{1+\phi}}{1+\phi} \right) \right\} \tag{27}$$

subject to,

$$\tilde{C}_t + q_t \tilde{I}_t + R_{t-1} \frac{\tilde{b}_{t-1}}{\pi_{c,t}} = \tilde{b}_t + \frac{\tilde{W}_t}{p_{c,t}} \tilde{N}_t + \frac{\tilde{T}_t}{p_{c,t}} + \frac{\tilde{\Gamma}_{c,t}}{1-\omega} + \frac{q_t \tilde{\Gamma}_{h,t}}{1-\omega} \tag{28}$$

where, where, $\tilde{C}_t$ denotes the consumption of final non-housing goods, $\tilde{H}_t$ denotes services from the stock of the final housing goods at the end of period t, $\tilde{N}_t$ total labor supply γ the savers discount factor, $\tilde{b}_t$ is end of period t real debt, and $\tilde{\Gamma}_{j,t}$ the aggregate nominal profits from the monopolistic competitive firms. The optimality conditions derived from the savers' optimization problems:

$$\frac{-\tilde{U}_{n,t}}{\tilde{U}_{c,t}} = \frac{\tilde{W}_t}{p_{c,t}} \tag{29}$$

$$\tilde{U}_{c,t} = \gamma \mathbb{E} \left\{ \frac{\tilde{U}_{c,t+1}}{\tilde{\pi}_{c,t+1}} R_t \right\} \tag{30}$$

$$q_t = \frac{\tilde{U}_{h,t}}{\tilde{U}_{c,t}} + \gamma(1-\delta) \mathbb{E} \left\{ \frac{\tilde{U}_{c,t+1}}{\tilde{U}_{c,t}} q_{t+1} \right\} \tag{31}$$

This is the standard two-sector sticky price model with no agent heterogeneity and no credit frictions. In solving first-order conditions for savers optimization problem, the same expression yields the standard result for the housings' return of a frictionless model as in the equation (21).

2.6 Firms Maximization Problem

Each producer in sector j operates the production function:

$$Y_{j,t} \equiv \left(\int_0^1 Y_{j,t}(i)^{\frac{e_j - 1}{e_j}} di \right)^{\frac{e_j}{e_j - 1}}, \; e_j > 1, j = c, h \tag{32}$$

where, $Y_{j,t}(i)$ is the intermediate good i demanded to produce final good j, e_j, the elasticity of substitution between differentiated varieties in sector j.

Optimization problem of the final good producer j;

$$max \; P_{j,t} Y_{j,t} - \int_0^1 P_{j,t}(i) Y_{j,t}(i) di$$

subject to equation (32)

Intermediate good firm i in sector j hires labor to operate linear production function:

$$Y_{j,t}(i) = N_{j,t}(i) - F_j \tag{33}$$

where, $N_{j,t}(i)$ the demand for labor by firm i in sector j, and $F_j \geq 0$ is a fixed cost of production Each firm i in sector j chooses the sequence of employment and prices $\{N_{j,t}(i), P_{j,t}(i)\}$, in order to maximize expected discounted profits.

Optimality conditions:

$$\big((1-e_j)+e_j mc_{j,t}\big) = \nu_j(\pi_{j,t}-1)\pi_{j,t} - \gamma\nu_j \mathbb{E}\left\{\frac{\Lambda_{j,t+1}}{\Lambda_{j,t}}\frac{P_{j,t+1}}{P_{j,t}}\frac{Y_{j,t+1}}{Y_{j,t}}(\pi_{j,t+1}-1)\pi_{j,t+1}\right\} \tag{34}$$

where, $\Lambda_{j,t} = \gamma\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}$ is the savers stochastic discount factor, $\tilde{\lambda}_t$ is the savers marginal utility of nominal income, ν_j measures the sectoral nominal price stickiness, $\pi_{j,t}$ is the gross inflation rate in sector j, and, $mc_{j,t} = \frac{W_t}{P_{j,t}}$ is the real marginal cost of production in sector$j$.

2.7 Overnight Rate Policy

We assume that the overnight rate policy follows a simple Taylor-type rule:

$$\frac{R_t}{R} = \left(\frac{\tilde{\pi}_t}{\tilde{\pi}}\right)^{\phi_\pi}\epsilon_t,\;\; \phi_{pi} > 1 \tag{35}$$

where R is the steady-state gross nominal interest rate, $\tilde{\pi}_t = \left(\pi_{c,t}\right)^{(1-\mu)}\left(\pi_{h,t}\right)^{\mu}$ is a composite inflation index, and ϵ_t is an exogenous policy shock which evolves according to

$$exp(\epsilon_t) = exp(\epsilon_{t-1})^{\rho}u_t$$

with u_t, $i.i.d.$ and $(0 < \rho < 1)$.

2.8 Equilibrium Conditions

A sequence of variables $\{N_t, \tilde{N}_t, C_t, H_t, \tilde{C}_t, \tilde{D}_t, N_{j,t}, \tilde{N}_{j,t}, b_t, \tilde{b}_t; j = c, h\}$ and prices $\{P_{c,t}, P_{h,t}\}$, wages $\frac{W_t}{p_{c,t}}, \frac{\tilde{W}_t}{p_{c,t}}$, rate R_t, relative price of housings q_t, inflation on housing and nonhousing respectively $\{\pi_{h,t}, \pi_{c,t}\}$ comprises a competitive equilibrium for this model such that

- Given prices $\{C_t, H_t, b_t, N_t\}$ solves borrowers' problem.
- Given prices $\{\tilde{C}_t, \tilde{H}_t, \tilde{b}_t, \tilde{N}_t\}$ solves borrowers' problem.
- Given inflation, R_t satisfies the monetary policy rule.
- The goods, labor, and credit market clear.

2.9 Market Clearing Conditions

Goods market clears,

$$Y_{c,t} = \omega C_t + (1-\omega)\tilde{C}_t + \frac{\nu_c}{2}\left(\pi_{c,t}-1\right)^2 Y_{c,t} \tag{36}$$

$$Y_{h,t} = \omega I_t + (1-\omega)\tilde{I}_t + \frac{\nu_h}{2}\left(\pi_{h,t}-1\right)^2 Y_{h,t} \tag{37}$$

The labor market clears,

$$N_{c,t} + N_{h,t} = \omega N_t + (1-\omega)\tilde{N}_t \tag{38}$$

The credit market clears,

$$b_t + \tilde{b}_t = 0 \tag{39}$$

And, there is no redistribution i.e. $T_t = \tilde{T}_t = 0$.

2.10 Overnight Rate and Frictions

Overnight rate policy is a more powerful tool with two credit frictions than only with collateral constraint. The policy follows the Taylor rule,

$$\frac{R_t}{R} = \left(\frac{\tilde{\pi}_t}{\tilde{\pi}}\right)^{\phi_\pi}, \tag{40}$$

where, $\phi_\pi > 0$, $\tilde{\pi}_t = \left(\pi_{c,t}\right)^{1-\mu} \left(\pi_{h,t}\right)^{\mu}$, is a composition inflation index, and $\epsilon_t = \rho\epsilon_{t-1} + u_t$, is an exogenous monetary shock.

An increase in the overnight rate has a direct impact on the labor income shock threshold value, denoted as $\bar{e}_t = \bar{b}_t^{ltv}/\bar{b}_t^{pti}$, resulting in an increase in the fraction of borrowers who are constrained by the PTI requirement, captured by the function $G(\bar{e}_t)$. In contrast, the fraction of borrowers constrained by the collateral requirement decreases as a result of this policy, as given by $(1 - F_e(\bar{e}_t))$. Additionally, the overnight rate policy affects the labor market optimality condition both directly, through the interest rate itself, and indirectly through $G(\bar{e}_t)$ and ψ_t. The latter reduces the opportunity cost of leisure and, as a result, labor supply.

3 Calibration of The Model

In this section, we present the calibration of the model using quarterly frequency. The real steady-state interest rate is determined by the saver's discount factor γ, which is selected such that a real annual interest rate of 4% is achieved. Specifically, we set $(\frac{1}{\gamma})^4 = 1.04$, implying $\gamma = 0.99$. Borrowers' discount factor β is set to 0.98 as in Krusell and Smith (1998), and the depreciation rate for housing goods is set to 4%, which translates to a quarterly depreciation rate of $\delta = 0.01$ as in Monacelli (2009).

The elasticity of substitution between varieties ϵ_j is set to 6 in both sectors $j = c, h$, resulting in a steady-state markup of 20%. The elasticity of substitution between housing services and non-housing goods is $\eta = 1$, and the elasticity of labor supply is $\phi = 1$. The degree of nominal price stickiness in non-housing prices ν_c is determined by the frequency of price adjustment, which is set to approximately four quarters. Specifically, we set $\nu_j = \theta(\epsilon_j - 1)/(1 - \theta)(1 - \gamma\theta)$, where θ is the probability that the firm will not reset prices, following the Calvo-Yun standard model. We assume $1/(1 - \theta) = 4$, which implies $\theta = 0.75$ as the average frequency of price adjustment. A price stickiness of two quarters is commonly used in past literature.

In our results, we focus on measuring the reaction of the endogenous variables to a policy tightening when housing prices are fully flexible. Therefore, we set the quarterly price rigidity parameter ν_h to zero. We set the share of housing in total consumption μ to 0.20, and the preference parameter ν such that households' labor supplies are equal to 1/3 of their total normalized time. The fraction of borrowers ω is set to 0.54 as in Justiniano et al. (2015) based on the Survey of Consumer Finances data. The value of the inverse elasticity of labor supply ϕ is 1, and the coefficient in inflation ϕ_π is set to 1.5. The persistence parameter ρ of the monetary shock is 0.5. We choose the LTV ratio κ^{ltv} to be 0.75, κ^{pti} to be 0.28, and the debt cost parameter τ to be 0.0028. The values of borrowers' labor income shock $e_{i,t}$ are simulated from the log-normal distribution.

$$log(e_{it}) \sim N\left(\frac{-\sigma_e^2}{2}, \sigma_e^2\right) \tag{41}$$

where, $\sigma_e = 0.411$ is the borrowers' income dispersion.

Table 1: Parameter Values

Parameter	Description	Values	
ϕ	Inverse elasticity of labor supply	1.000	Monachelli(2009)
ρ	Persistence of monetary shock	0.500	Monachelli(2009)
δ	Depreciation rate of housings	0.010	Monachelli(2009)
ϵ_j	Elasticity of substitution between varieties	6.000	Monachelli(2009)
η	Elast. of subst. between housing and nonhousing	1.167	Monachelli(2009)
ϕ_{pi}	Coefficient of inflation in Taylor rule	1.500	Monachelli(2009)
ω	Fraction of borrowers	0.540	Monachelli(2009)
μ	Share of housing consumption	0.200	Monachelli(2009)
τ	Debt related expenses parameter	0.0028	Greenwald(2018)
σ_e	Income dispersion	0.411	Greenwald(2018)

Table 2: Steady states

Variable	Description	Values	
		LTV	LTV and PTI
$4(R-1)$	Annual real interest rate	0.04	0.04
C/Y	Nonhousing consumption share to GDP	0.5312	0.5141
D/Y	Housing and services consumption share to GDP	0.3428	0.3136
b/Y	Debt to GDP	0.6123	0.5837
$\bar{H}$	Excess return to housing	0.0365	0.0587

3.1 The role of credit frictions on returns to housing

In this section, we explain how the introduction of financial frictions, LTV, and PTI limit, on borrowing affects the consumption of housing and nonhousing, the relative price of housing, and hence the excess return to housing. In the benchmark model, i.e., $\psi = 0$, the shadow value of one unit of housings $U_{c,t}q_t$ is almost constant.From equation (9) we have

$$U_{c,t}q_t = U_{h,t} + \beta(1-\delta)\mathbb{E}_t[q_{t+1}U_{c,t+1}]. \tag{42}$$

The right-hand side of equation (42) depends only on the current and expected future values of the marginal utility of housings $U_{h,t}$, which is about to constant (iterate forward) and indicates that the consumption of housings fluctuates only with the change in price stickiness. However, when there are credit frictions, the quasi-constancy of the utility of housing breaks. Recall, equation (9) the shadow value of housings

$$U^1 \equiv U_{c,t}q_t = \frac{U_{h,t} + \beta(1-\delta)\mathbb{E}_t[q_{t+1}U_{c,t+1}]}{\left[1-(1-\kappa^{ltv})(1-\delta)\psi_t^1\mathbb{E}_t[\pi_{h,t+1}]\right]} \tag{43}$$

The denominator depends negatively on the multiplier ψ_t and the expected housing inflation rate. We see that the shadow value of housing depends not only on the marginal utility of housing in a perfect market but also on the current and expected future values of the shadow

Figure 1: Impulse response to an overnight rate hike with financial frictions

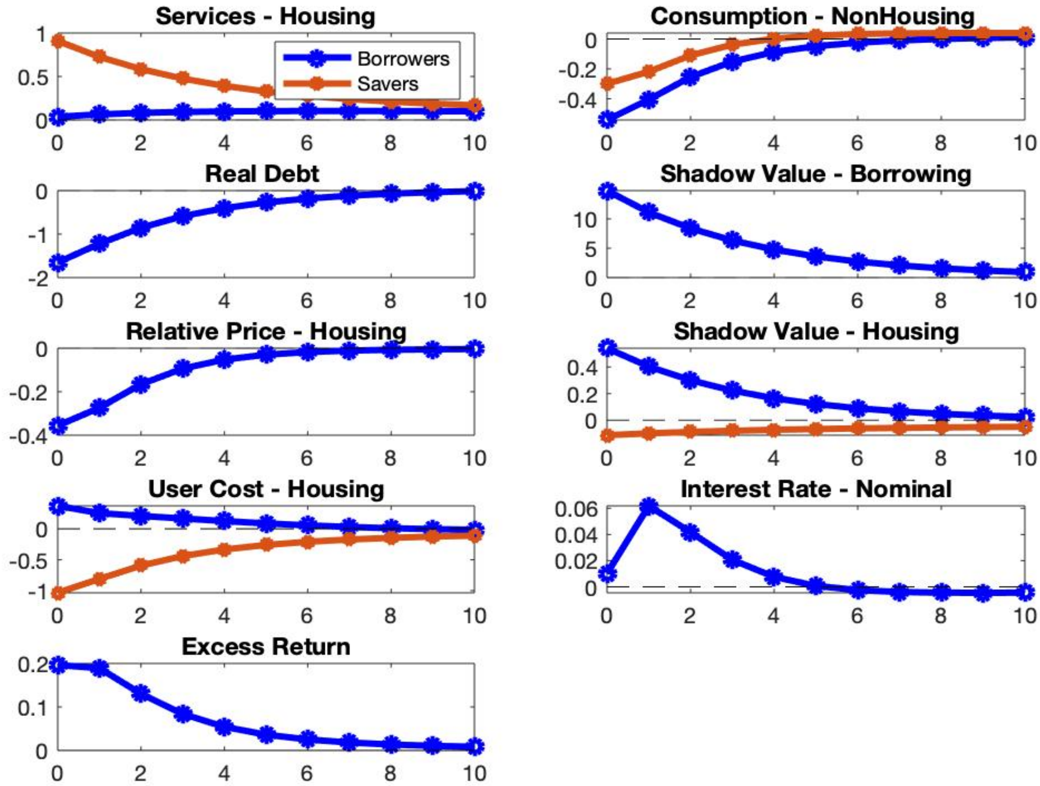
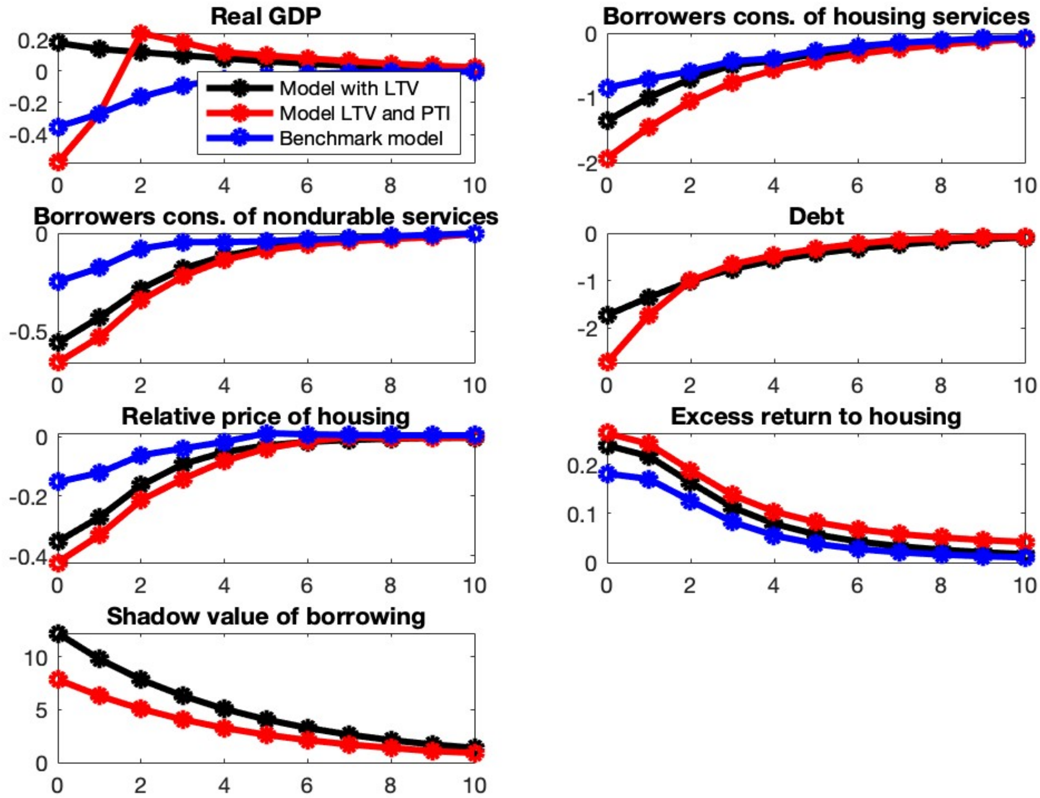


Table 3: Business cycle properties of the model

	Benchmark Model	Model: LTV	Model: LTV & PTI	Data
S.D				
C	0.8132	0.7705	0.4554	0.375
D	0.5811	0.4977	0.4323	0.2418
H	0.1925	0.1767	0.1132	0.0903
Y	0.2709	0.2924	0.4823	0.3979
b	1.3121	1.3978	1.3631	1.1470
q	1.6198	1.7123	1.0198	1.3684
Correlation				
C(Agg.),GDP	0.8769	0.8569	0.7450	0.7196
D(Agg.),GDP	0.8512	0.8412	0.7933	0.6991
H,GDP	0.8882	0.8182	0.6923	0.6396
b,GDP	0.9176	0.9796	0.8101	0.7987
q,GDP	0.8311	0.9111	0.6374	0.7035

Figure 2: Comparison of Impulse responses to an overnight rate hike with and without frictions



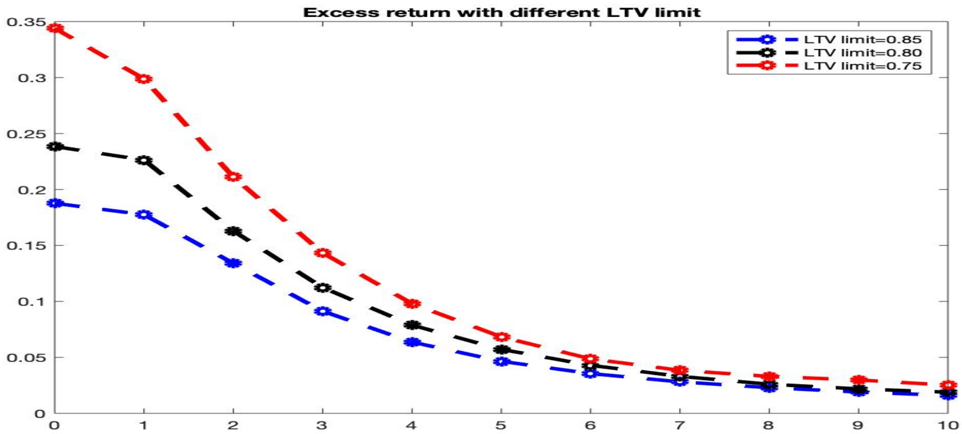
value of borrowing, which fluctuate in response to shocks. In fact, the substitution effect pushes consumers towards nonhousing, and the current housing price decreases with the decrease in consumption. Also, the variation in demand for housing will change with the behavior of the relative price of housing.

The rise in the shadow value of borrowing is less with both LTV and PTI frictions than with only LTV friction. When the government increases the interest rate, the relative housing price falls by the same percentage, which causes similar LTV tightening in both models. However, the rise in the interest rate causes more often binding of the PTI constraint; as a result, the percentage fall in the borrowers' real debt is greater in the model with the two financial frictions than in the model with only one LTV friction. Since the borrowers are constrained by their labor income, they can buy more valuable housing than those allowed by their PTI limit by paying dollar-for-dollar for the difference. This implies that the shadow value of borrowing in the economy with two financial frictions, ψ^2, rises by less. Which can be seen formally from equation (22).

$$U^2 \equiv U_{c,t}q_t = \frac{U_{h,t} + \beta(1-\delta)\mathbb{E}_t[q_{t+1}U_{c,t+1}]}{\left[1-(1-\kappa^{ltv})(1-\delta)\psi_t^2\mathbb{E}_t[\pi_{h,t+1}(1-F_e(\bar{e}_t))]\right]} \tag{44}$$

Equation (43) and (44) implies that $U^2 < U^1$ as $\psi_t^2 < \psi_t^1$ and $(1-F_e(\bar{e}_t)) < 1$. Figure 4 plots the impulse response functions of the model, the consumption services from housings, the

Figure 3: Impulse response to the interest rate increase with financial frictions for different LTV limits.



consumption of nonhousing, real debt, shadow value of borrowings, the relative price of housings, shadow value of housings, the user cost of housings, and the excess return to housings. In the contractionary monetary policy shock by 25 basis points, the consumption of housing falls on the borrowers' side five times more than nonhousing goods, and the lower real price attracts the savers to consume the housing services requiring higher premiums. On the other hand, consumption of nonhousing decreases in both types of households. The real debt decreases, and the shadow value of housing increases. The excess return response is higher in the contractionary policy shock with financial frictions. We will discuss the sensitivity of the excess return later in detail. We apply the positive shock to ϵ_t in the contractionary monetary policy by raising the overnight rate by 25 basis points.

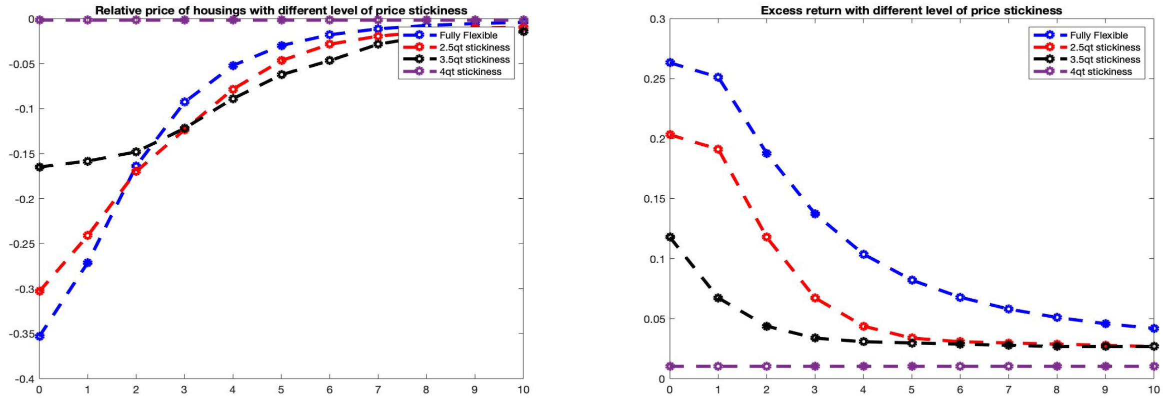
Figure 5 compares the impulse responses from the variables; real GDP, consumption of housing and, nonhousing goods and services, household debt, relative housing prices, shadow value of borrowing and the excess return to housing. All the variables have improved impulse responses in the model with frictions than without frictions to the contractionary monetary policy shocks. The expected excess return to housings with combined friction of LTV and PTI limits is about 5%. The expected excess return to housing with both financial frictions is approximately 2% more than that without frictions. Furthermore, more than that in a perfect financial market. In our sensitivity analysis, we look at the excess return response with different denominations of LTV frictions, such as 75%, 80%, and 85%. The excess return of housings is higher with a high fraction of κ^{ltv} that can not be used as a collateral requirement. See Figure 6.

We examine the sensitivity of the simulation results to successive increases in the degree of durable price stickiness at $0, 2.5, 3.5$ and 4 quarters, while keeping non-durable price stickiness at 4 quarters. Figure 7 shows the impulse response functions of relative price and excess return of housing for different level of price stickiness in housing prices. Fully flexible housing prices can expects high excess return and low current price.

4 Adjustment cost, habit and returns to housing

In this section, we add adjustment cost in housing consumption and introduce habit parameters in nonhousing consumption to smooth the excess return to housing. The habit models imply

Figure 4: Relative price of housings and excess return sensitivity of the different value of price stickiness on housings: $\nu_h = 0, 2.5qt, 3.5qt, 4qt$



an inverse relation between expected returns and surplus consumption. The empirical evidence presented in Yuming Li (2001) implies that the relation between expected returns is inversely related to surplus consumption. According to Christiano et.al. (2005) the final housing goods evolve as:

$$H = (1-\delta)H_{t-1} + \Theta_H(I_t, I_{t-1}) \tag{45}$$

where,

$$\Theta_H(I_t, I_{t-1}) = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{46}$$

is a function representing adjustment cost to purchase of new housing goods I_t. And, $S(.)$ satisfies, $S(1) = 0 = S'(1)$, and $S''(1) > 0$. In other words, we can say that the change in I_t increases the cost, which tends to zero in the steady state. For instance, we define $S(.)$ for simulation as follows:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\Theta}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{47}$$

We introduce habits over nonhousing consumption only, according to Iacoviello and Neri (2010). The composite consumption indices of borrowers and savers are given by

$$X_t = \left[(1-\mu)^{\frac{1}{\eta}}(C_t - \epsilon_{bc}C_{t-1})^{\frac{\eta-1}{\eta}} + \mu^{\frac{1}{\eta}}(H_t)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{48}$$

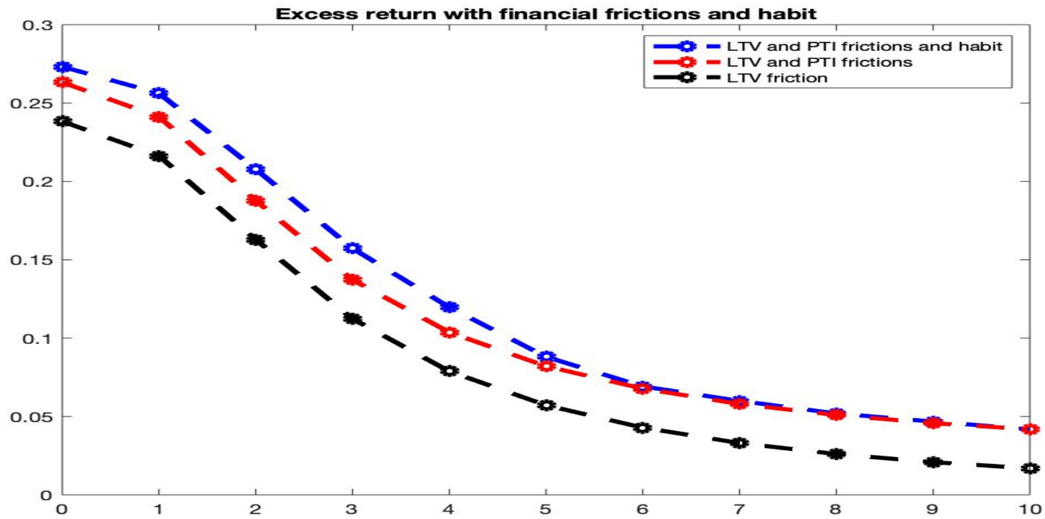
and,

$$\tilde{X}_t = \left[(1-\mu)^{\frac{1}{\eta}}(\tilde{C}_t - \epsilon_{sc}\tilde{C}_{t-1})^{\frac{\eta-1}{\eta}} + \mu^{\frac{1}{\eta}}(\tilde{H}_t)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{49}$$

where, ϵ_{bc} and ϵ_{sc} are the habit parameters of borrowers and savers respectively.

With habits, a unit increase in consumption in period t increases the marginal utility of consumption in period t. However, it decreases the marginal utility of consumption in period $t+1$ because the habit stock is higher in period $t+1$. As a result, nonhousing consumption becomes more expensive, and borrowers might shift to savers seeing the future consumption uncertainty. This leads borrowers to demand a higher risk premium to smooth excess return. The quantitative results for the model with LTV constraints and with or without adjustment

Figure 5: Impulse response to an overnight rate hike with financial frictions and habit



cost and habit parameters are plotted in Figure 5.

For the quantitative results we set $\epsilon_{bc} = 0.75$ for borrowers, $\epsilon_{sc} = 0.45$ for savers and the adjustment cost parameter $\Theta = 0.04$. Since the borrowers are collaterally constrained and need stronger habits to smooth their consumption and the excess return. We choose habit parameters for the borrower higher than that of the savers. In our numerical result, the excess return with LTV constraint, habit parameters of nonhousing, and the adjustment cost of housings are about 1.5% more than that with LTV constraint only.

5 Empirical evidence: returns to housing

Macroprudential policies are financial policies that advanced and emerging economies frequently use to reduce systematic risk in recent years and many economists are interested in examining their effectiveness. Several macroeconomics papers examine the effectiveness of the macroprudential policies in terms of hampering households debt, for example, Kuttner and Shim (2016); Vandenbussche et al. (2015).[4]We can also recall the papers that analyze macroprudential policy together with the monetary policy: Angelini et.al(2014), Bailliu et al. (2015), and so on. The empirical evidence on the macroeconomic effects remains scattered, particularly in the framework that incorporates both monetary and macroprudential policies. We, therefore, want to reconcile the combined effect of monetary and widely used macroprudential policies: LTV and PTI limits to the excess return to housing using the most recent Canadian macro data.

Monetary policy is a set of actions the government takes to control a nation's overall money supply and achieve economic growth. In Canada, monetary policy is conducted by adjusting short-term interest rates, namely the overnight rates, to achieve a rate of monetary expansion consistent with maintaining a low and relatively stable rate of inflation. When the interest rate is increased by the Central Bank, it becomes more expensive for financial institutions to borrow money from one another, increasing their total cost. To make up for this increase in costs, banks increase their prime rates, which makes borrowing money for borrowers or consumers more expensive.[5]

In Canada, commercial banks lend funds to each other for brief periods at the overnight interest rate, a market-determined rate that fluctuates daily. By changing its target for the interest rate, the Bank of Canada can alter the actual overnight rate at which commercial banks transact; such changes in the rate remain at the heart of how monetary policy affects the economy. By changing the target for the overnight rate, the Bank influences the entire range of market interest rates, from the yield on 30-day treasury bills to that on 30-year government bonds and from the rate on 3-month guaranteed investment certificates (GICs) to that on 5-year home mortgages. When the Bank lowers the target for the interest rate, these interest rates fall, firms and households increase their demand for credit, and commercial banks increase their quantity of credit supplied. When the Bank raises the target, interest rates rise, firms and households reduce their demand for credit, and commercial banks decrease the quantity of credit supplied. We will show in this section that the contractionary monetary policy shocks have a positive effect on excess return to housing.[6]

The LTV limit, one of the most widely used macroprudential instruments, determines the maximum amount of a secured loan based on the market value of the asset claimed as collateral. The market value of an asset is the typical price that would be paid for the asset on the open market between two unrelated parties. If the asset offered as collateral is relatively easy to convert into cash, you'll be more likely to be able to obtain a bigger loan. The shocks to the monetary policy has direct effect to the LTV and PTI limits. When interest rate increases borrowers bind more via PTI constraint and when interest rate decreases borrowing cost decreases and the price of houses go up and hence the borrowers bind more via LTV constraint.

In this section, we examine the results of our theoretical model empirically. We show the reaction of the expected excess return to housing and other key macro economic variables that includes household debt in response to the contractionary monetary policy shocks. The results

[4]For details, see also the survey by Galati and Moessner (2018).

[5]Details can be found here: https://www.bankofcanada.ca/publications/books-and-monographs/why-monetary-policy-matters.

[6]See the Bank of Canada publication: Why Monetary Policy Matters: A Canadian Perspective, by Christopher Ragan.

here follow from the most recent Canadian data that replicates the stylized facts such as real GDP, consumption of housing and nonhousing goods, real household debt decline, and expected excess return to housing. The analysis is carried out by using structural vector auto-regression. This methodology has recently gained widespread use in empirical business cycle analysis, as it has proved to be a flexible and tractable way to analyze economic time series. In particular, vector autoregression (VAR) models have been capable of describing the rich, dynamic structure of the relationships between economic variables.

The VAR model was first introduced in 1980 by Christopher Sims.[7] Since then, the methodology has gained widespread use in applied macroeconomic research. Calza et al. (2007) used VAR to analyze the transmission of monetary policy shocks on housing prices in E.U. countries, the U.S., and Canada. The Quarterly VAR model was used to assess the impact of monetary policy shocks on the U.S. economy by Monacelli (2006). Bernanke et al. (2004) conditioned monetary policy simultaneously to a large set of home and foreign variables by using a factor-augmented VAR model to identify monetary policy more realistically.[8]

5.1 Empirical Strategy

We use the more recent quarterly data for Canada available in FRED and Stat Canada over the period (1971/01) to (2022/01). The LTV limit is determined by existing indices by providing quantitative information on both the level and changes of its limits.[9] We use the debt-to-services (DTS) ratio as the PTI limit.

5.2 VAR Model

A VAR model we present here describes the evolution of a set of variables over time. We estimate a quarterly VAR model for the Canadian economy specified as follows:

$$Y_t = \sum_{j=1}^{L} A_j Y_{t-j} + B\epsilon_t \tag{50}$$

where ϵ_t is a vector of contemporaneous disturbances, A_j are $(L \times L)$ coefficient matrices, and L is the number of variables considered in the model. Y_t is a vector consisting of endogenous variables in the framework. As discussed above, we do not have any information about the loan-to-value limit policy before (2000); we concluded that no such policies were applied to maintain business cycles. We estimate the VAR framework considering two cases:

Case I: we use quarterly data from the fourth quarter of (1978) to the first quarter of (2022.) In this case the vector Y_t comprises seven variables, and the error term becomes $\epsilon_t = \left[\epsilon_t^{gdp}, \epsilon_t^{cd}, \epsilon_t^{ndc}, \epsilon_t^{d}, \epsilon_t^{er}, \epsilon_t^{gd}, \epsilon_t^{r}\right]$, consisting of shocks to GDP, housing and nonhousing goods, households debt, expected excess return, GDP deflator and the federal funds rate respectively. We perform VAR twice to compare the impulse responses to the pre and post 2000 periods.

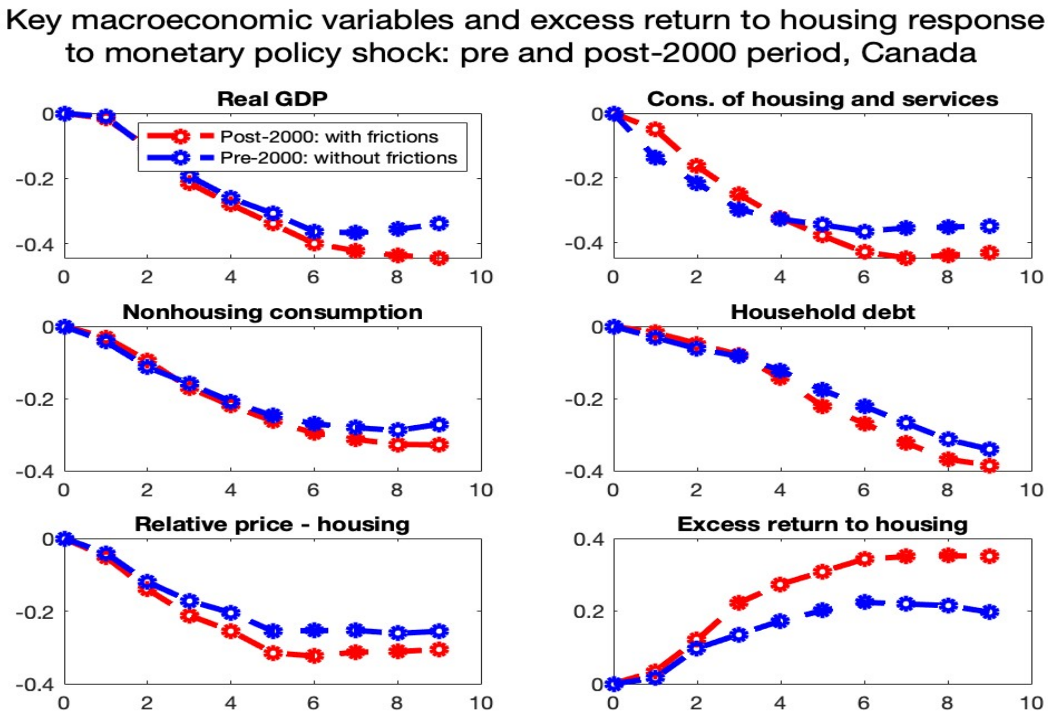
Case II: In this case, we use quarterly data from the first quarter of (2000) to the first quarter of (2022). We add one more variable, the LTV ratio limit, and the vector Y_t comprises eight variables: real GDP, consumption of services from housing, consumption of nonhousing goods, total real household debt, expected excess return to real residential property, GDP deflator, LTV limit and overnight rate for Canada. The error term becomes $\epsilon_t =$

[7] See Canova (1995a, b) for a detailed discussion on the VAR approach.

[8] Applications of VAR models to financial data are given in Hamilton (1994), Campbell, Lo and MacKinlay (1997), Cuthbertson (1996), Mills (1999) and Tsay (2001).

[9] More detailed explanations are available in Alam et al. (2019).

Figure 6: Empirical IRF's to overnight rate increase by 25 basis points: pre and post 2000 periods



$\left[\epsilon_t^{gdp}, \epsilon_t^{cd}, \epsilon_t^{ndc}, \epsilon_t^{d}, \epsilon_t^{er}, \epsilon_t^{gd}, \epsilon_t^{ltv} and or \epsilon_t^{pti}, \epsilon_t^{r}\right]$, consisting of shocks to GDP, housing and nonhousing goods, households debt, expected excess return, GDP deflator, LTV ratio, and the federal funds rate respectively. We do eight-dimensional VAR twice to test the effect of LTV and PTI limits separately on the economy and returns to housing.

All the variables, except the interest rate, LTV, and PTI limit, are in the log and deflated by the GDP deflator. We want to measure the dynamic responses of these variables to one standard deviation increase in ϵ_t^r. We also measure the dynamic responses of these variables to the LTV and PTI limit to one standard deviation innovations in ϵ_t^{ltv}.

We use the standard recursive identification scheme based on the Cholesky decomposition (Christiano et al.,1999) to identify the shocks. The lag order value of 4 is chosen by the minimized values of the Final Prediction Error (FPE), Hannan-Quinn (HQ), Akaike (AIC), and Swartz Bayesian information criterion (BIC). Unit root and co-integration tests suggest that some of the variables in the analysis are non-stationary, co-integrated processes. Since our goal is to analyze short-run effects via impulse responses, non-stationarity is not a concern. Not adding the error correction term will lead to a loss of efficiency, but this will not affect forecasting or impulse responses. However, as a robustness check, a vector error correction model (VECM) was fit to the data, and the results show similar impulse responses.

Figure 6, displays the empirical impulse response functions following the one-standard-deviation increase in the overnight rate. As shown in the figure, the expected excess return is higher in post 2000 period than pre 2000, consumption of housing and services decreases more in comparison to nonhousing goods consumption, and household debt decreases in response to the contractionary policy in the overnight rate, which is as expected. We consider a two-time frame to compare the dynamics of excess return in the absence and presence of the LTV and

PTI limit in the contractionary monetary policy environment. The expected excess return is higher in the presence of LTV and PTI limit frictions.

In this section, we provided a piece of evidence to determine the effects of the contractionary overnight rate policy, the LTV and PTI limit policy in the expected excess return to housing, real GDP, housing and nonhousing goods consumption, household debt in a sample of the Canadian economy over the period (1978Q2) to (1999Q4) and (2000) to (2021Q2). Using vector-auto-regressions, we showed that the overnight rate policy combined with the financial frictions, LTV and PTI limits significantly affect the expected excess return to housing. We also showed that, in response to the contractionary overnight rate policy shocks, the real GDP, housing and nonhousing goods consumption, and household debt decreased. The consumption of housing goods decreases significantly more than nonhousing goods.

From a real-world standpoint, our findings indicate that LTV limits have important macroeconomic consequences. For instance, for the post (2000) period covered in this study, LTV limits achieved their financial stability objectives in Canada by limiting credit and house price appreciation under an inflation-targeting regime. Furthermore, they attained these objectives without posing any threat to its price stability objective.

6 Conclusion

In this paper, we have studied the role of credit frictions in the macroeconomic variables and the excess return to housing goods. The DSGE asset pricing modeling framework we construct here contains two housing and nonhousing goods sectors, heterogenous agents, namely, the savers and borrowers, and two credit constraints, namely, the LTV and PTI ratio limits on borrowers. The two credit constraints are randomly combined by shocks to the borrowers' labor income and jointly determine the number of collateralized loans the borrowers can obtain. We reconcile the expected excess return puzzle following a monetary contraction when the housing goods prices are a fully flexible and different measure of price stickiness. In this scenario, an increase in the nominal overnight rate directly impacts and increases the threshold level of the income shock, and the larger fraction of borrowers binds with the PTI constraints. This results in a fall in the relative price of housing and a sharp decline in the number of collateralized loans to the borrowers and turns back from purchasing housing, along with the relatively more expensive nonhousing. On the other hand, the lenders decrease their purchase of the nonhousing and increase the purchase of the housing by an amount less than the fall in housing purchase by borrowers and can expect a higher return in the future. The model also predicts that an aggregate output, debt level, and consumption of housing and nonhousing goods declines in response to the contractionary monetary policy shocks with financial frictions. We have also presented here in this paper that the predictions of the model are empirically consistent.

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