Behavioral Sticky Prices

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March 2024

Abstract

We study a model where households make decisions according to a dual-process framework widely used in cognitive psychology. System 1 uses effortless heuristics but is susceptible to biases and errors. System 2 uses mental effort to make more accurate decisions. Through their pricing behavior, monopolistic producers can influence whether households deploy Systems 1 or 2. The strategic use of this influence creates a new source of price inertia and provides a natural explanation for the "rockets and feathers" phenomenon: prices rise quickly when costs increase but fall slowly when costs fall. Our model implies that price stability is not optimal.

J.E.L. Classification: E31, E32, E52, E71
Keywords: Sticky prices, cognitive costs, System 1 and 2, rockets and feathers.

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1 Introduction

We study a model where households make decisions according to a dual-process framework widely used in the cognitive psychology literature to describe human decision making (see, e.g. Stanovich and West (2000)). System 1 uses heuristics to make quick decisions that require little or no effort but are prone to biases and systematic errors. System 2 uses mental effort to make slower, more deliberate decisions that are more accurate. Our paper builds on the elegant formulation of dual process reasoning proposed by Ilut and Valchev (2023).

In our model, households make errors in their purchase decisions because of cognitive costs. Monopolistic producers for whom these errors result in high levels of demand relative to the rational optimum have an incentive to keep their prices constant to discourage households from activating System 2 and reconsider their purchasing decisions. This behavior generates a novel type of price inertia.

This form of inertia is consistent with the "sticky winners" phenomenon documented by Ilut et al. (2020): firms that receive a high demand realization are less likely to change their prices.

Our model offers a natural explanation for an intriguing empirical regularity documented by Karrenbrock (1991), Neumark and Sharpe (1992), and Peltzman (2000) known as "rockets and feathers": prices increase rapidly when costs rise but decrease slowly when costs fall. This phenomenon arises naturally from the strategic interaction between monopolistic producers and households.[1]

When costs rise significantly in our model, all firms increase prices to avoid losses, so costs and prices rise together. When costs fall, the firms that benefit from

[1] Matějka (2015) is another interesting example of strategic interaction between producers and households. In his model, monopolistic producers choose prices as a function of unit input costs. However, households cannot observe prices perfectly due to limited information-procession ability. As a result, it is optimal for monopolists to implement simple pricing policies where prices take only a few values. These policies make prices easier to observe for households, thereby reducing pricing uncertainty and increasing sales.

favorable demand have an incentive to keep their prices constant so that households do not reoptimize their purchase decisions. So, on average, prices decline by less than costs.

Price stability is generally optimal in cashless economies with sticky prices because it eliminates the relative price distortions produced by inflation (see Woodford (2003)). In our model, price stability is not optimal because of the strategic interaction between monopolists and boundedly rational households. When average inflation is zero, firms that receive favorable demand due to behavioral mistakes maintain their prices. The other firms increase or decrease their prices slightly to try to obtain a more favorable demand. As a result, sizeable behavioral mistakes become ingrained, and households end up choosing a significantly inefficient consumption bundle. It is generally optimal to deviate from zero inflation to reduce this inefficiency.

We now discuss three observations consistent with the elements of our model. The first is the "shrinkflation" phenomenon whereby manufacturers reduce product sizes while keeping prices constant. The UK Office for National Statistics (2019) found 206 instances between September 2015 and June 2017 where products were downsized, yet their prices remained largely unchanged. This practice suggests that some manufacturers are prepared to incur considerable expenses to keep prices stable, presumably to avoid triggering a re-optimization of household purchasing decisions.[2]

The second observation is the increasing adoption of subscription-based business models, such as streaming or software-as-a-service, and the tendency for subscription prices to remain stable. This stability can be interpreted as a tactic used by pro-

[2]President Biden deemed shrinkflation important enough to merit discussion in a February 2024 Super Bowl video broadcast. The president noted that "sports drinks bottles are smaller, a bag of chips has fewer chips, but they're still charging us just as much [...] ice cream cartons have shrunk in size but not in price. [...] Some companies are trying to pull a fast one by shrinking the products little by little and hoping you won't notice."

ducers to dissuade households from engaging System 2 and reassessing the value of their subscriptions.[3]

The Amazon Prime subscription prices are remarkably sticky. Initially offered at an annual rate of $79 in 2011, the fee has only been adjusted a few times: to $99 in 2014, $119 in 2018, and $139 in 2022. These adjustments were often accompanied by enhancements in service offerings, including the introduction of Amazon Prime Day, which served to justify the higher fees.

Netflix provides a case study of both price stability and shrinkflation. The standard subscription price remained at $7.99 from November 2010 until May 2014. At that point, the price was increased to $8.99, but only for new subscribers. Existing subscribers were grandfathered in at the $7.99 rate for an additional two years. Concurrently, Netflix rolled out a new basic plan priced at $7.99, which offered only standard-definition video on a single screen, a downgrade from the two high-definition screens available under the regular plan. The price for this basic plan remained unchanged until 2019.

The third observation consistent with the elements of our model is the widespread use of convenient prices that are slightly below a round number (e.g., $9.99 instead of $10) documented by Kashyap (1995), Blinder et al. (1998), and Levy et al. (2011). This practice can be interpreted as a way to exploit System 1 thinking, creating the perception that the price is lower than its actual value.

The advent of Artificial Intelligence is likely to make the strategic exploitation of consumer behavioral biases suggested by these examples and emphasized in our model more prevalent.

Our paper is organized as follows. Section 2 describes our model. Section 3 shows that our model is consistent with the rockets and feathers phenomenon. Section 4 discusses optimal fiscal and monetary policy. Section 5 summarizes our find-

[3]See Della Vigna and Malmendier (2006) for evidence that consumers often fail to assess the value of subscription services rationally.

ings.

2 Model

In this section, we describe the household problem, the monopolistic producers' problem, the government's fiscal and monetary policy, and the economy's equilibrium.

2.1 Household problem

There is a representative household with preferences

$$U = \frac{C^{1-\sigma} - 1}{1-\sigma} - \frac{N^{1+\eta}}{1+\eta} - \int_0^1 \mathcal{I}_i di, \;\; \sigma, \eta > 0,$$

where consumption, C, is a composite of differentiated goods, c_i,

$$C = \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \;\; \theta > 1.$$

The variable N denotes the labor supply and $\mathcal{I}_i$ is the cognitive cost of using System 2 to choose how much of good i to buy. We discuss this cost in more detail below.

Households make their decisions in two steps. First, given a planned level of aggregate consumption, C^p, and prices of differentiated goods $(P_i)_{i\in[0,1]}$, the household chooses $(c_i)_{i\in[0,1]}$ under bounded rationality. Second, given the choices of $(c_i)_{i\in[0,1]}$, the household chooses C and N under full rationality. We assume that the latter choices are fully rational to isolate the effect of bounded rationality on the composition of the consumption basket.

Fully rational solution Consider first the solution to the household problem under full rationality. The optimal consumption basket is the solution to the following problem

$$\min_{(c_i)_{i\in[0,1]}} \int_0^1 P_i c_i \, di,$$
$$\text{s.t.} \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} \, di\right)^{\frac{\theta}{\theta-1}} \geq C^p,$$

where C^p, the planned level of aggregate consumption, is taken as given.

The familiar solution to this problem is

$$c_i^* \left(P_i, P^p, C^p\right) = \left(\frac{P_i}{P^p}\right)^{-\theta} C^p,$$

where

$$P^p \equiv \left(\int_0^1 P_i^{1-\theta} di\right)^{\frac{1}{1-\theta}}.$$

Substituting $\left(c_i^*\right)_{i\in[0,1]}$ in the consumption aggregator yields $C = C^p$ and $\int_0^1 P_i c_i^* di = P^p C$.

Bounded rationality solution Now, consider the household problem with bounded rationality. Throughout, we use the formulation of dual process reasoning proposed by Ilut and Valchev (2023).

There is a pre-period in which households purchased $c_{i,0}$ units of good i at a price $P_{i,0}$. The household perfectly observes the state $\left(P_i, P^p, C^p\right)$ but, due to cognitive costs, it does not know the optimal value of each c_i. For each P_i, P^p, and C^p, the rational log demand for good i is $x^* \left(P_i, P^p, C^p\right)$. The household is uncertain about the optimal log demand for good i, so it treats this demand as a random variable $\hat{x} \left(P_i, P^p, C^p\right)$.

The household relies on a signal to make its best prediction of $\hat{x} \left(P_i, P^p, C^p\right)$. For convenience, we drop the dependence of $\hat{x}$ on P^p and C^p.

The household starts with priors such that for any P_i, P_i',

$$\begin{bmatrix} \hat{x}\left(P_i\right) \\ \hat{x}\left(P_i'\right) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{x}_i\left(P_i\right) \\ \overline{x}_i\left(P_i'\right) \end{bmatrix}, \begin{bmatrix} \overline{\sigma}_i^2\left(P_i\right) & \overline{\sigma}_i\left(P_i, P_i'\right) \\ \overline{\sigma}_i\left(P_i, P_i'\right) & \overline{\sigma}_i^2\left(P_i'\right) \end{bmatrix}\right).$$

The distribution of signal s_i conditional on $\hat{x}(P_i)$ is

$$s_i | \hat{x}(P_i) \sim \mathcal{N}\left(\hat{x}(P_i), \sigma_{\epsilon,i}^2\right).$$

The household chooses the optimal level of $\sigma_{\epsilon,i}$. This value can potentially be infinite, in which case the signal is not informative. Since the prior and the signal are normally distributed, the posterior distribution is also normal with mean function $x_i\left(P_i\right)$ and covariance function $\sigma_i\left(P_i, P_i'\right)$.

The timing is as follows, P_i is observed, $\sigma_{\epsilon,i}$ is chosen, and the cost of the signal is incurred. This cost is proportional to the expected reduction in the entropy of $\hat{x}_i\left(P_i\right)$ brought about by knowing s_i, also known as the Shannon mutual information,

$$\mathcal{I}_i = \kappa \ln\left(\frac{\overline{\sigma}_i^2\left(P_i\right)}{\sigma_i^2\left(P_i\right)}\right), \;\; \kappa > 0.$$

The optimal prediction is given by the conditional normal distribution formulas,

$$x_i\left(P_i\right) = \overline{x}_i\left(P_i\right) + \alpha_i\left(P_i\right)\left[s_i - \overline{x}_i\left(P_i\right)\right],$$

$$\alpha_i\left(P_i\right) = \frac{\overline{\sigma}_i^2\left(P_i\right)}{\overline{\sigma}_i^2\left(P_i\right) + \sigma_{\epsilon,i}^2}.$$

The posterior variance is given by

$$\sigma_i^2\left(P_i\right) = \left[1 - \alpha_i\left(P_i\right)\right]\overline{\sigma}_i^2\left(P_i\right).$$

Log-demand $\widetilde{x}_t\left(P_{i,t}\right)$ is chosen to minimize

$$\mathbb{E}^p\left\{\left[\hat{x}\left(P_i\right) - \widetilde{x}_i\left(P_i\right)\right]^2 \mid s_i\right\},$$

where $\mathbb{E}^p\left[\cdot\right]$ is the subjective expectation.

At stage 2, it is optimal to set

$$\widetilde{x}_i(P_i) = x_i(P_i) = \mathbb{E}^p\left[\hat{x}(P_i) \mid s_i\right].$$

The optimal signal variance is the solution to the following problem,

$$\min_{\sigma_i^2(P_i)} \sigma_i^2(P_i) + \kappa \ln\left[\frac{\overline{\sigma}_i^2(P_i)}{\sigma_i^2(P_i)}\right],$$
$$\text{s.t. } \sigma_i^2(P_i) \leq \overline{\sigma}_i^2(P_i).$$

where the constraint ensures that $0 \leq \sigma_{\epsilon,i}^2 \leq \infty$. The optimal solution is

$$\sigma_i^2(P_i) = \min\left\{\kappa; \overline{\sigma}_i^2(P_i)\right\}.$$

When learning occurs, the posterior variance at the observed state is always set equal to κ. The optimal signal-to-noise ratio in the observed state is

$$\alpha_i(P_i) = \max\left\{0, 1 - \frac{\kappa}{\overline{\sigma}_i^2(P_i)}\right\},$$

The optimal signal variance satisfies

$$\alpha_i(P_i)\,\sigma_{\epsilon.i} = \sqrt{\kappa\alpha_i(P_i)\,.}$$

The prior covariance function is

$$\overline{\sigma}_i^2(P_i) = \begin{cases} \sigma_c^2, & P_i \neq P_{i,0} \\ \kappa, & P_i = P_{i,0} \end{cases},$$

and

$$\overline{\sigma}_i\left(P_i, P_i'\right) = 0 \text{ for all } P_i \neq P_i',$$

where $\sigma_c^2 > \kappa$.

If $P_i \neq P_{i,0}$, uncertainty is high. Therefore, the household draws a signal. If $P_i = P_{i,0}$, the posterior variance at the observed state is κ, and the household does not want to obtain more information by drawing another signal. For any P_i, P_i', knowledge of P_i is uninformative about demand at P_i'.

Using the fact that we can write the signal realization as

$$s_i = x^* \left(P_i\right) + \sigma_\epsilon \epsilon_i',$$

we obtain

$$x_i\left(P_i\right) = \begin{cases} \overline{x}_i\left(P_{i,0}\right), & \text{if } P_i = P_{i,0} \\ \overline{x}_i\left(P_i\right) + \alpha\left[x^*\left(P_i\right) + \sigma_\epsilon \epsilon_i' - \overline{x}_i\left(P_i\right)\right], & \text{if } P_i \neq P_{i,0} \end{cases},$$

where

$$\alpha \equiv 1 - \frac{\kappa}{\sigma_c^2}; \ \ \sigma_\epsilon \equiv \sqrt{\frac{\kappa}{\alpha}}.$$

Assume that learning in the pre-period occurred in a similar fashion so that

$$\overline{x}_i\left(P_i\right) = \begin{cases} \overline{x}_{i,0}\left(P_i\right), & \text{if } P_i \neq P_{i,0} \\ \overline{x}_{i,0}\left(P_{i,0}\right) + \alpha\left[x^*\left(P_i\right) + \sigma_\epsilon \epsilon_i - \overline{x}_{i,0}\left(P_{i,0}\right)\right], & \text{if } P_i = P_{i,0} \end{cases},$$

where ϵ_i is the pre-period's noise. In the pre-period, the household does not use the signal $s_{i,0}$ to predict demand at prices $P_i \neq P_{i,0}$.

As in Ilut and Valchev (2023), we assume that the mean of the prior coincides with the fully rational value of x to ensure that our results are not generated by ex-ante biases, $\overline{x}_{i,0}\left(P_i\right) = x^*\left(P_i\right)$. Under this assumption,

$$\overline{x}_i\left(P_i\right) = \begin{cases} x^*\left(P_i\right), & \text{if } P_i \neq P_{i,0} \\ x^*\left(P_{i,0}\right) + \alpha\sigma_\epsilon\epsilon_i, & \text{if } P_i = P_{i,0} \end{cases}.$$

Replacing $\overline{x}_i\left(P_i\right)$ in $x_i\left(P_i\right)$, we obtain

$$x_i\left(P_i\right) = \begin{cases} x^*\left(P_{i,0}\right) + \alpha\sigma_\epsilon\epsilon_i, & \text{if } P_i = P_{i,0} \\ x^*\left(P_i\right) + \alpha\sigma_\epsilon\epsilon_i', & \text{if } P_i \neq P_{i,0} \end{cases}.$$

Let $\gamma \equiv \alpha\sigma_\epsilon$ and $p_i \equiv P_i / P^p$. Then, the demand for good i is

$$c_i = e^{\gamma\widetilde{\epsilon}_i} p_i^{-\theta} C^p, \ \ \widetilde{\epsilon}_i \equiv \begin{cases} \epsilon_i, & \text{if } P_i = P_{i,0} \\ \epsilon_i', & \text{if } P_i \neq P_{i,0} \end{cases}.$$

Given these demands, aggregate consumption is

$$C = \Delta_u C^p,$$

and total consumption expenditure can be written as

$$\int_0^1 P_i c_i \, di = \Delta_b P^p C^p,$$

where Δ_u and Δ_b are distortions to utility and expenditure caused by deviations from rationality. These distortions are given by,

$$\Delta_u \equiv \left(\int_0^1 e^{\left(\frac{\theta-1}{\theta}\right)\gamma\tilde{\epsilon}_i} p_i^{1-\theta} \, di \right)^{\frac{\theta}{\theta-1}},$$

$$\Delta_b \equiv \int_0^1 e^{\gamma\tilde{\epsilon}_i} p_i^{1-\theta} \, di.$$

If producers did not behave strategically, the average value of these distortions produced by bounded rationality would be,

$$\Delta_u = e^{\frac{1}{2}\left(\gamma^2 \frac{\theta-1}{\theta}\right)}, \quad \Delta_b = e^{\frac{1}{2}\gamma^2}.$$

As we discuss below, these distortions are much larger in equilibrium because of the strategic behavior of producers.

The household budget constraint is

$$PC \leq WN + \int_0^1 \Pi_i \, di - \mathcal{T},$$

where $P \equiv (\Delta_b/\Delta_u)P^p$, W is the nominal wage rate, and $\mathcal{T}$ are lump-sum taxes.

The first-order condition for N is

$$C^\sigma N^\eta = \frac{W}{P}.$$

We define the real wage in terms of the actual price level (w) and the price level that obtains under full rationality (w^p) as,

$$w \equiv \frac{W}{P}; \;\; w^p \equiv \frac{W}{P^p}.$$

2.2 Firms' problem

The producers of the differentiated goods are monopolistically competitive and are not subject to behavioral biases. Firm i produces y_i units of good i using labor (n_i) according to the production function

$$y_i = An_i.$$

The realized nominal profit of firm i is

$$\Pi_i = \left(P_i - (1-\tau)\frac{W}{A}\right)e^{\gamma\widetilde{\epsilon}_i}p_i^{-\theta}C^p = \left(p_i - (1-\tau)\frac{w^p}{A}\right)p_i^{-\theta}e^{\gamma\widetilde{\epsilon}_i}P\frac{C}{\Delta_b}.$$

where τ denotes a government-provided labor subsidy that we discuss below. The real profit is

$$V_i = e^{\gamma\widetilde{\epsilon}_i}v\left(p_i, (1-\tau)\frac{w^p}{A}\right)C^b,$$

where $C^b \equiv C/\Delta_b$ and

$$v\left(p, (1-\tau)\frac{w^p}{A}\right) \equiv \left[p - (1-\tau)\frac{w^p}{A}\right]p^{-\theta}.$$

If the firm decides to change its price, its relative price is p_i. If the firm does not change the price, its relative price is $p_{i,0}/\pi^p$, where $\pi^p \equiv P^p/P_0^p$.

The firm observes ϵ_i, the demand shock in the pre-period, but not ϵ_i', the current demand shock. Let z be a standard normal random variable. The firm chooses between changing the price and obtaining the expected profit

$$\max_p \mathbb{E}\left[e^{\gamma z}\right]v\left(p, (1-\tau)\frac{w^p}{A}\right)C^b,$$

or keeping the price and obtaining profit

$$e^{\gamma\epsilon_i}v\left(\frac{p_{i,0}}{\pi^p}, (1-\tau)\frac{w^p}{A}\right)C^b.$$

The optimal reset price is

$$p^* = \frac{\theta}{\theta-1}(1-\tau)\frac{w^p}{A},$$

so,

$$v\left(p^{*},(1-\tau)\frac{w^{p}}{A}\right)\equiv v^{*}\left((1-\tau)\frac{w^{p}}{A}\right)=\frac{1}{\theta}\left[\left(\frac{\theta}{\theta-1}\right)(1-\tau)\frac{w^{p}}{A}\right]^{1-\theta}.$$

The firm's pricing policy is,

$$p_i=\begin{cases}p^{*}, & \text{if } e^{\gamma\epsilon_i}v\left(\frac{p_{i,0}}{\pi^p},(1-\tau)\frac{w^p}{A}\right)<\mathbb{E}\left[e^{\gamma z}\right]v^{*}\left((1-\tau)\frac{w^p}{A}\right)\\ \frac{p_{i,0}}{\pi^p}, & \text{if } e^{\gamma\epsilon_i}v\left(\frac{p_{i,0}}{\pi^p},(1-\tau)\frac{w^p}{A}\right)\geq\mathbb{E}\left[e^{\gamma z}\right]v^{*}\left((1-\tau)\frac{w^p}{A}\right)\end{cases}.$$

There is a minimum demand shock, ℓ, such that whenever $\epsilon_i\geq\ell$ the firm chooses to keep its price constant. The value of ℓ is given by

$$\ell=\begin{cases}\frac{1}{\gamma}\ln\frac{\mathbb{E}\left[e^{\gamma z}\right]v^{*}\left((1-\tau)\frac{w^p}{A}\right)}{v\left(\frac{p_{i,0}}{\pi^p},(1-\tau)\frac{w^p}{A}\right)}, & \text{if } \frac{p_{i,0}}{\pi^p}>(1-\tau)\frac{w^p}{A}\\ \infty, & \text{if } \frac{p_{i,0}}{\pi^p}\leq(1-\tau)\frac{w^p}{A}\end{cases}.$$

When the pre-period price is equal to the optimal price but $\epsilon_i<\ell$, the firm changes the price by an infinitesimal amount to get a new demand draw.

Suppose $p_{i,0}=1$ for all i, and let $\chi\equiv1-\Phi(\ell)$. The cross-sectional distribution of prices is

$$p_i=\begin{cases}\frac{1}{\pi^p}, & \text{with probability }\chi\\ p^{*}, & \text{with probability }1-\chi\end{cases}.$$

Using $P^p=\left(\int_0^1 P_i^{1-\theta}di\right)^{\frac{1}{1-\theta}}$ and $p^{*}=\frac{\theta}{\theta-1}(1-\tau)\frac{w^p}{A}$, we obtain

$$1=\chi\left(\pi^p\right)^{\theta-1}+(1-\chi)\left(\frac{\theta}{\theta-1}(1-\tau)\frac{w^p}{A}\right)^{1-\theta}.$$

Since χ is a function of ℓ and ℓ is a function of π^p and $(1-\tau)\frac{w^p}{A}$ only, this equation implicitly defines the reset relative price, which we denote as $f(\pi^p)$,

$$f(\pi^p)\equiv\left(\frac{\theta}{\theta-1}\right)(1-\tau)\frac{w^p}{A}.$$

With some abuse of notation, we can then write $\ell=\ell(\pi^p)$, $\chi=\chi(\pi^p)$.

2.3 Government

The government uses monetary policy to control nominal expenditure. It also implements a uniform *ad valorem* subsidy on labor costs at a rate τ, which is financed with lump-sum taxes.

The growth rate of nominal expenditure, μ, is given by

$$\mu = \pi \frac{C}{C_0}.$$

2.4 Equilibrium conditions

Equilibrium in the labor market requires

$$\int_0^1 n_i di = N$$

Equilibrium in the market for good i requires

$$c_i = An_i.$$

Combining these two equations, we obtain

$$C = \frac{\Delta_u}{\Delta_c} AN,$$

where

$$\Delta_c = \int_0^1 e^{\sigma \widetilde{\epsilon}_i} p_i^{-\theta} di.$$

By concavity $\Delta_u \leq \Delta_c$. This result implies that consumption and leisure choices are in the interior of the production possibilities frontier because of cognitive costs.

Given the relative reset price, $f(\pi^p)$, the distortions are

$$\Delta_u(\pi^p) = \Big\{ \chi(\pi^p) \mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)z} \mid z \geq \ell(\pi^p) \right] (\pi^p)^{\theta-1} + \\ [1-\chi(\pi^p)] \mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)z} \right] [f(\pi^p)]^{1-\theta} \Big\}^{\frac{\theta}{\theta-1}},$$

$$\Delta_b(\pi^p) = \chi(\pi^p)\,\mathbb{E}\left[e^{\gamma z} \mid z \geq \ell(\pi^p)\right](\pi^p)^{\theta-1} + \left[1-\chi(\pi^p)\right]\mathbb{E}\left[e^{\gamma z}\right]\left[f(\pi^p)\right]^{1-\theta}$$

$$\Delta_c(\pi^p) = \chi(\pi^p)\,\mathbb{E}\left[e^{\gamma z} \mid z \geq \ell(\pi^p)\right](\pi^p)^{\theta} + \left[1-\chi(\pi^p)\right]\mathbb{E}\left[e^{\gamma z}\right]\left[f(\pi^p)\right]^{-\theta}$$

Using the property,

$$\mathbb{E}\left[e^{az} \mid z \geq \underline{z}\right] = \mathbb{E}\left[e^{az}\right]\frac{1-\Phi(\underline{z}-a)}{1-\Phi(\underline{z})},$$

and defining δ_u and $\delta(\pi^p)$ as

$$\delta_u(\pi^p) \equiv 1-\Phi\left(\ell(\pi^p)-\gamma\left(\frac{\theta-1}{\theta}\right)\right),$$

$$\delta(\pi^p) \equiv 1-\Phi\left(\ell(\pi^p)-\gamma\right),$$

we obtain,

$$\mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)z} \mid z \geq \ell(\pi^p)\right] = \mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)z}\right]\frac{\delta_u(\pi^p)}{\chi(\pi^p)}.$$

$$\mathbb{E}\left[e^{\gamma z} \mid z \geq \ell(\pi^p)\right] = \mathbb{E}\left[e^{\gamma z}\right]\frac{\delta_u(\pi^p)}{\chi(\pi^p)}.$$

From the intratemporal condition, we obtain

$$C^\sigma N^\eta = w,$$

We can rewrite this equation as,

$$C^\sigma N^\eta = \frac{\Delta_u(\pi^p)}{\Delta_b(\pi^p)}w^p = \left[\frac{\Delta_u(\pi^p)}{\Delta_b(\pi^p)}\right]\left(\frac{\theta-1}{\theta}\right)\frac{f(\pi^p)}{1-\tau}A.$$

Using market clearing,

$$C = \left\{\left[\frac{\Delta_u(\pi^p)}{\Delta_c(\pi^p)}\right]^\eta\left[\frac{\Delta_u(\pi^p)}{\Delta_b(\pi^p)}\right]\left(\frac{\theta-1}{\theta}\right)\frac{f(\pi^p)}{1-\tau}\right\}^{\frac{1}{\sigma+\eta}} A^{\frac{1+\eta}{\eta+\sigma}},$$

where $A^{\frac{1+\eta}{\eta+\sigma}}$ is aggregate consumption in the frictionless economy. From the definition of the price level,

$$\pi = \frac{\Delta_b\left(\pi^p\right)}{\Delta_u\left(\pi^p\right)}\frac{\Delta_{u,0}}{\Delta_{b,0}}\pi^p.$$

The equilibrium conditions only depend on ratios of distortions. Let

$$\omega_c\left(\pi^p\right) \equiv \frac{\Delta_u\left(\pi^p\right)}{\Delta_c\left(\pi^p\right)},$$

$$\omega_b\left(\pi^p\right) \equiv \frac{\Delta_u\left(\pi^p\right)}{\Delta_b\left(\pi^p\right)},$$

$$\omega_{b,0} \equiv \frac{\Delta_{u,0}}{\Delta_{b,0}},$$

and

$$\widetilde{A}\left(\pi^p\right) \equiv \omega_c\left(\pi^p\right) A.$$

The equilibrium conditions are as follows.

$$C = \left\{\left(\frac{\theta-1}{\theta}\right)\frac{1}{1-\tau}\frac{\omega_c\left(\pi^p\right)}{\omega_b\left(\pi^p\right)}f\left(\pi^p\right)\right\}^{\frac{1}{\sigma+\eta}}\left[\widetilde{A}\left(\pi^p\right)\right]^{\frac{1+\eta}{\sigma+\eta}},$$

$$\widetilde{A}\left(\pi^p\right) N = C,$$

$$1 = \chi\left(\pi^p\right)\left(\pi^p\right)^{\theta-1} + \left[1-\chi\left(\pi^p\right)\right]\left[f\left(\pi^p\right)\right]^{1-\theta},$$

$$\pi = \frac{\omega_{b,0}}{\omega_b\left(\pi^p\right)}\pi^p,$$

$$\mu = \pi\frac{C}{C_0}.$$

3 Rockets and Feathers

We now study the impact of cost shocks and show that our model is consistent with the rockets and feathers phenomenon: prices rise quickly when costs increase but fall slowly when costs fall.

We establish our results by doing comparative statics using two levels of productivity: $A = 1 + v$ and $A = 1/(1 + v)$, where $v > 0$. Log inflation responds symmetrically to cost shocks in the economy with fully rational households since $\pi^f = 1/C_f$.

To study the response of our economy, we set $1 - \tau = (\theta - 1)/\theta$ and the growth rate of nominal expenditure, μ, to one. We choose C_0 and $\omega_{b,0}$ so that at $A = 1$, $\pi^p = \pi = 1$.

Given these initial conditions, the equilibrium conditions are

$$1 = \pi \frac{C}{\left[\omega_c\left(1\right)\right]^{\frac{1+\eta}{\sigma+\eta}}},$$

$$\pi = \frac{\omega_b\left(1\right)}{\omega_b\left(\pi^p\right)} \pi^p.$$

A cost increase (a productivity fall from $A = 1$ to $A = 1/(1 + v)$) generates inflation, while a cost decrease (a productivity rise from $A = 1$ to $A = 1 + v$) creates deflation. To compare the response of prices to these two types of shocks, we plot in Figure 1 the absolute value of the logarithm of gross inflation as a function of the magnitude of the shocks, v. The orange and blue lines correspond to a cost increase and decrease, respectively. In a fully rational model, these two lines coincide. In absolute value, inflation responds in the same way to positive and negative cost shocks.

This symmetry is preserved in our model for infinitesimal cost shocks. However, for larger cost shocks, prices respond more to cost increases than declines. When costs rise significantly in our model, all firms increase prices to avoid losses, so costs and prices rise together. When costs fall, the firms that benefit from favorable demand have an incentive to keep their prices constant so that households do not re-optimize their purchase decisions. So, on average, prices decline by less than costs.

In light of these results, it's noteworthy that the rockets and feathers phenomenon is especially common in gasoline and agricultural commodities (e.g. Karrenbrock

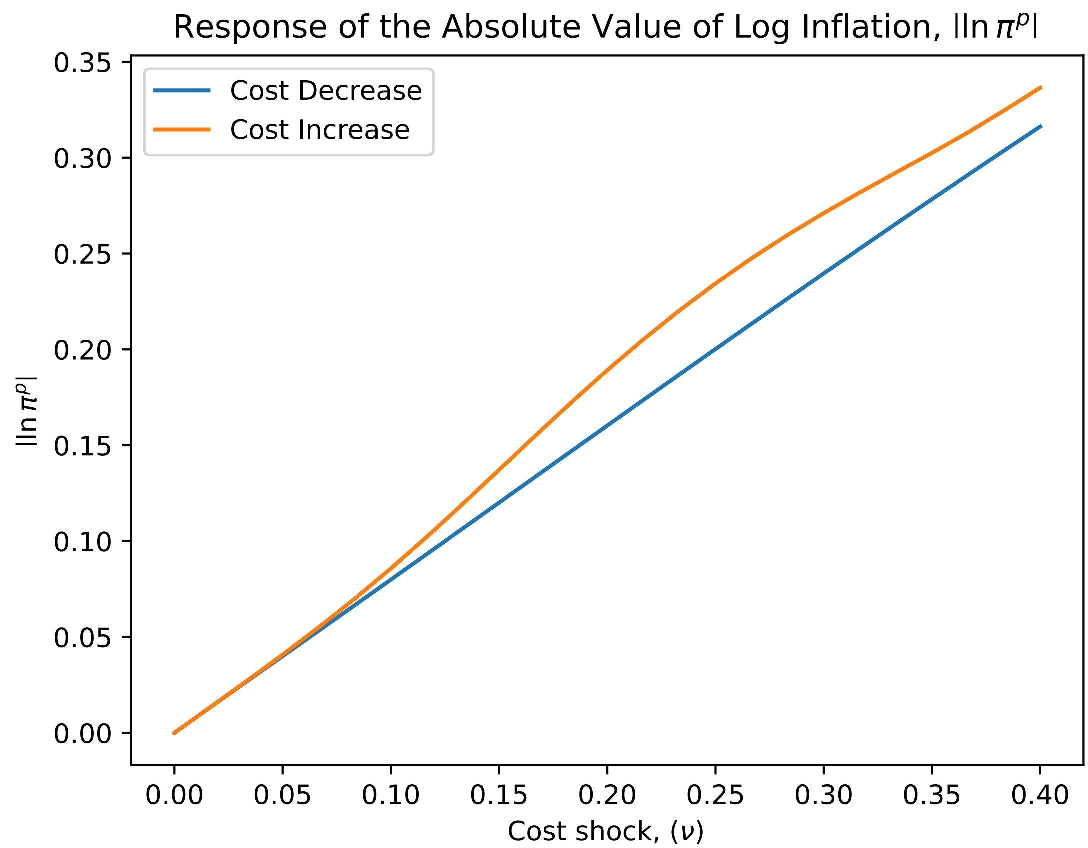

Figure 1: The impact of cost shocks on the absolute value of the logarithm of inflation

(1991)), where there are large cost shocks.

For cost shocks higher than 43 percent, all firms change their prices. But for cost shocks lower than -43 percent, there are firms that do not lower their prices. As the absolute value of the cost shock increases, the orange and blue lines in Figure 1 eventually converge.

The following proposition shows the main theoretic result for a configuration of parameters that makes the equilibrium analytically tractable.

Proposition 1. *Suppose $\sigma = 1$ and $\eta = 0$. There is $\overline{A}$ such that if $A > \overline{A}$,*

$$-\ln\left[\pi^{p*}(A)\right] < \ln\left[\pi^{p*}\left(\frac{1}{A}\right)\right].$$

See the Appendix for proof.

This proposition implies that for large enough shocks, the percentage response of inflation is higher than the percentage response of deflation to cost shocks with the same absolute value.

4 Optimal Monetary Policy

We now characterize the optimal values for the labor subsidy rate, τ, and the growth rate of nominal expenditure, π^p. We assume that the government is able to implement any desired level of inflation.

Social welfare is given by

$$\mathcal{W}(\tau, \pi^p) = \frac{\left[C(\tau, \pi^p)\right]^{1-\sigma} - 1}{1-\sigma} - \frac{\left[N(\tau, \pi^p)\right]^{1+\eta}}{1+\eta} - \kappa\left[1 - \chi(\pi^p)\right]\ln\left(\frac{\sigma_c^2}{\kappa}\right),$$

where given an inflation level π^p, the equilibrium allocations are

$$C(\tau, \pi^p) = \left\{\left(\frac{\theta-1}{\theta}\right)\frac{1}{1-\tau}\frac{\omega_b(\pi^p)}{w_c(\pi^p)}f(\pi^p)\right\}^{\frac{1}{\sigma+\eta}}\left[\widetilde{A}(\pi^p)\right]^{\frac{1+\eta}{\sigma+\eta}},$$

and

$$C\left(\tau,\pi^{p}\right)=\widetilde{A}\left(\pi^{p}\right)N\left(\tau,\pi^{p}\right).$$

We can choose τ to satisfy the first equation. The fraction of sticky firms $\chi\left(\pi^{p}\right)$ does not depend on τ. Therefore, the problem of choosing τ given π^{p} can be rewritten as

$$\max\frac{C^{1-\sigma}}{1-\sigma}-\frac{N^{1+\eta}}{1+\eta}\ \text{ s.t. }\ C\leq\widetilde{A}\left(\pi^{p}\right)N.$$

Lemma 1. *Given π^{p}, the optimal labor subsidy rate $\tau\left(\pi^{p}\right)$ is*

$$1-\tau\left(\pi^{p}\right)=\left(\frac{\theta-1}{\theta}\right)\frac{\omega_{b}\left(\pi^{p}\right)}{w_{c}\left(\pi^{p}\right)}f\left(\pi^{p}\right),$$

and consumption and labor are

$$C\left(\pi^{p}\right)=\left[\widetilde{A}\left(\pi^{p}\right)\right]^{\frac{1+\eta}{\sigma+\eta}},$$

$$N\left(\pi^{p}\right)=\left[\widetilde{A}\left(\pi^{p}\right)\right]^{\frac{1-\sigma}{\sigma+\eta}}.$$

We now discuss some properties of the optimal inflation rate, π^{p}.

Proposition 2 (Price stability is better than high inflation)**.** *Let $\mathcal{W}_{s}$ be the welfare level that is attained when gross inflation, $\pi^{p}\geq\frac{\theta}{\theta-1}$. There is a value $\overline{\sigma}_{c}^{2}$ such that when the household's prior uncertainty about the optimal consumption is higher than $\overline{\sigma}_{c}^{2}$ ($\sigma_{c}^{2}\geq\overline{\sigma}_{c}^{2}$), price stability is better than high inflation, $\mathcal{W}\left(1\right)>\mathcal{W}_{s}$.*
See the Appendix for proof.

The intuition for this proposition is as follows. Recall that when households opt to gather information regarding the optimal consumption policy, they reduce their uncertainty to κ. When the prior uncertainty is high, this reduction involves significant cognitive effort. But households deem these efforts justified at times of high inflation. When inflation is zero, only a few firms adjust their pricing, so households incur low cognitive costs.

Proposition 3 (Price stability is not optimal)**.** *There is a value of* $\pi^p < 1$ *such that* $\mathcal{W}(\pi^p) > \mathcal{W}(1)$*.*
See the Appendix for proof.

The intuition for this result is as follows. When average inflation is zero, firms experiencing high demand due to household decision errors do not adjust their prices. Other firms slightly increase or decrease their prices to draw a new demand shock. As a result, sizeable behavioral mistakes become ingrained, leading households to select a highly suboptimal consumption basket. Moving away from zero inflation mitigates this inefficiency by improving consumption choices.

Why is deflation locally better than inflation? The intuition for this result is as follows. Because of cognitive costs, households do not choose the optimal value of c_i. Instead, they consume an amount of good i that is proportional to the optimal value. The planner would like to reduce the consumption of the firms that have sticky prices, since these are the firms that received large demand shocks that drive consumption far away from the optimum. When inflation is positive, the relative price of the goods produced by firms with sticky prices falls, inducing households to consume more of these goods and exacerbating the impact of behavioral biases. In contrast, when inflation is negative, the relative price of the goods produced by firms with sticky prices rises. As a result, the consumption of these goods falls, mitigating the impact of behavioral biases.

To sharpen our intuition, it is useful to compare the price distortions that emerge in our economy with those in a model with Calvo (1983) sticky prices. Consider a version of our model in which the threshold value of the shock above which firms keep their prices constant, ℓ, is fixed. We adopt this setup because we are interested in studying the behavior of inflation around zero, and locally, ℓ is constant. In this economy, the firms that change prices are the same for rates of inflation and deflation with the same absolute value.

Figure 2 compares the production distortion, Δ_c, in this version of our model with that in an analogous model with Calvo pricing. In the Calvo economy, there is no selection–the probability that a firm changes prices is the same for all firms. Consequently, as depicted in Figure 2, the production distortion reaches its lowest point when prices are stable ($\pi^p = 1$). In contrast, in the version of our economy with constant ℓ, the production distortions are minimized when the rate of inflation is negative ($\pi^p < 1$).

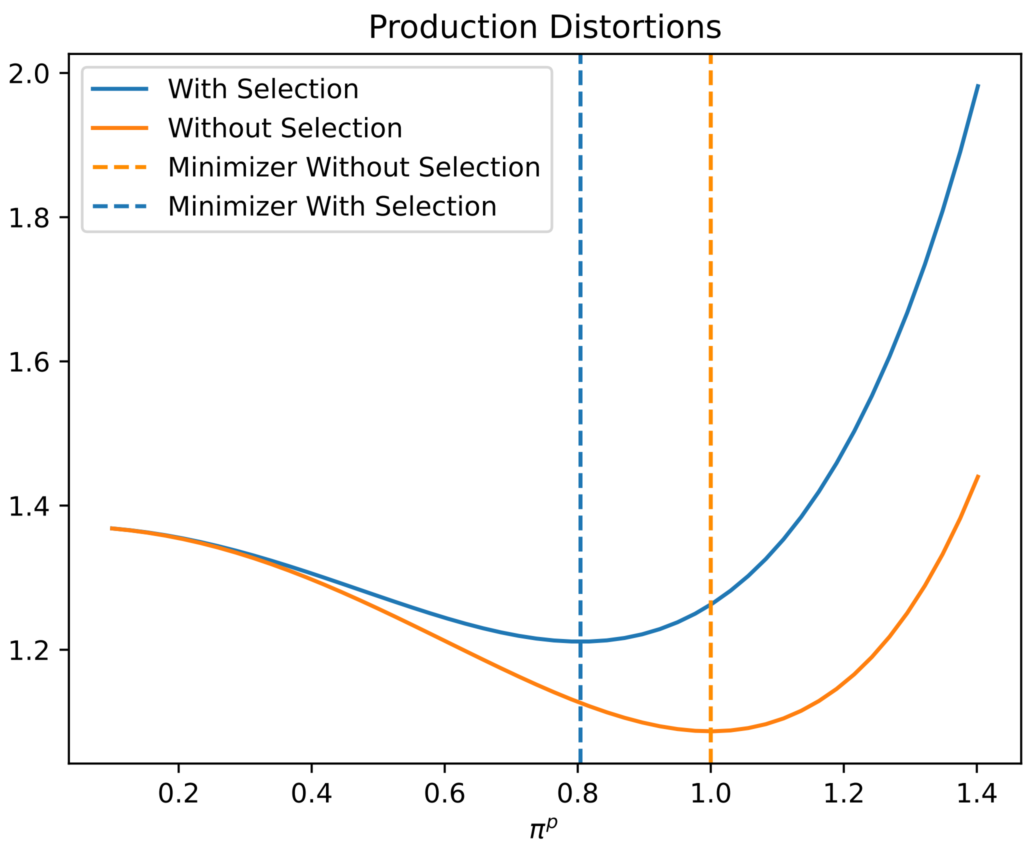

Figure 2: Production distortions as a function of the rate of inflation in a model with Calvo pricing and in a version of our economy with ℓ constant

In our economy, there is a selection effect. Firms with large demand shocks are the ones that keep prices constant. As a result, production distortions are high under price stability.

5 Conclusion

In this paper, we explore a framework where a dual process mechanism drives household choices. This framework gives rise to a new kind of price rigidity that emerges from the interaction between consumers and monopolistic suppliers. There is a range of cost shocks for which some producers refrain from adjusting prices so that households do not reassess their purchasing decisions.

Our model explains the intriguing ”rockets and feathers” phenomenon: prices rise quickly when costs increase but fall slowly when costs fall. In addition, we show that, unlike in other cashless economies with sticky prices, price stability is not optimal in our model.

6 Appendix

This appendix contains the proofs of our three propositions and a characterization of the properties of the price distribution generated by the model.

6.1 Proof of Proposition 1

Proof. Set $\sigma = 1$ and $\eta = 0$. Using the fact that $\omega_b(1) = \omega_c(1) \equiv \omega(1)$, the equilibrium conditions become

$$C = A\omega_b(\pi^p) f(\pi^p),$$

$$\pi = \frac{\omega(1)}{\omega_b(\pi^p)}\pi^p,$$

and

$$1 = \pi \frac{C}{\omega(1)}.$$

Substituting C and π in the policy rule yields

$$C = A\omega_b(\pi^p) f(\pi^p),$$

$$\pi = \frac{\omega(1)}{\omega_b(\pi^p)}\pi^p,$$

and

$$\frac{1}{A} = \pi^p f(\pi^p).$$

Let

$$g(\pi^p) \equiv \pi^p f(\pi^p).$$

This equation implicitly defines the equilibrium value of π^p, as a function of productivity, A. That is,

$$\frac{1}{A} = g[\pi^{p*}(A)],$$

where $\pi^{p*}(A)$ is the equilibrium value of $\pi^p(A)$.

Proposition 4. *Suppose $\sigma = 1$ and $\eta = 0$. $\pi^{p*}(A)$ exists and is unique. Moreover, $\pi^{p*}(A)$ is decreasing in productivity.*

Proof. Since $f(\pi^p) \to 1$ when $\pi^p \to 0$ and $f(\pi^p) = 1$ for $\pi^p \geq \frac{\theta}{\theta-1}$, $g(\pi^p) \to 0$ as $\pi^p \to 0$ and $f(\pi^p) \to \infty$ as $\pi^p \to \infty$. Therefore an equilibrium exists.

To show that the equilibrium is unique, it suffices to show that $g'(\pi^p) > 0$. Taking the derivative,

$$g'(\pi^p) = f(\pi^p)\left[\frac{f'(\pi^p)}{f(\pi^p)}\pi^p + 1\right].$$

Therefore

$$g'(\pi^p) > 0 \iff \frac{f'(\pi^p)}{f(\pi^p)}\pi^p > -1.$$

To continue, we first need the following proposition.

Proposition 5. *For* $\pi^p \in \left(0, \frac{\theta}{\theta-1}\right)$, $f(\pi^p)$ *satisfies the following properties:*

1. $\pi > 1 \iff f(\pi) > 1$, $\pi < 1 \iff f(\pi) < 1$, *and* $f(1) = 1$;

2. $f'(\pi^p)$ *is well-defined, with*

$$\frac{f'(\pi^p)}{f(\pi^p)}\pi^p = \frac{\chi(\pi^p)\left(\frac{1}{\pi^p}\right)^{1-\theta} - \Omega(\pi^p)}{[1-\chi(\pi^p)]\,[f(\pi^p)]^{1-\theta} + \Omega(\pi^p)},$$

   where

$$\Omega(\pi^p) \equiv \frac{\phi[\ell(\pi^p)]}{\gamma}\left[\frac{\left[\frac{1}{\pi^p} - f(\pi^p)\right]\left\{[f(\pi^p)]^{1-\theta} - \left(\frac{1}{\pi^p}\right)^{1-\theta}\right\}}{\frac{1}{\pi^p} - \left(\frac{\theta-1}{\theta}\right)f(\pi^p)}\right] \geq 0,$$

   with $\Omega(\pi^p) = 0$ *only at* $\pi^p = 1$.

Proof. Part 1 follows immediately from the equation

$$1 = \chi(\pi^p)\left(\frac{1}{\pi^p}\right)^{1-\theta} + [1-\chi(\pi^p)]\,[f(\pi^p)]^{1-\theta}.$$

. For part 2, we have from the definition of $\ell(\pi^p)$ that

$$\ell'(\pi^p) = \frac{1}{\gamma}(\theta-1)\left[\frac{f(\pi^p) - \frac{1}{\pi^p}}{\frac{1}{\pi^p} - \left(\frac{\theta-1}{\theta}\right)f(\pi^p)}\right]\left[\left(\frac{1}{\pi^p}\right) + \frac{f'(\pi^p)}{f(\pi^p)}\right].$$

Since

$$\chi\left(\pi^{p}\right)=1-\Phi\left[\ell\left(\pi^{p}\right)\right],$$

we have

$$\chi'\left(\pi^{p}\right)=-\phi\left[\ell\left(\pi^{p}\right)\right]\ell'\left(\pi^{p}\right).$$

Implicitly differentiating

$$1=\chi\left(\pi^{p}\right)\left(\frac{1}{\pi^{p}}\right)^{1-\theta}+\left[1-\chi\left(\pi^{p}\right)\right]\left[f\left(\pi^{p}\right)\right]^{1-\theta},$$

substituting $\ell'\left(\pi^{p}\right)$ and $\chi'\left(\pi^{p}\right)$ yields the result. □

This proposition implies that,

$$\frac{\chi\left(\pi^{p}\right)\left(\frac{1}{\pi^{p}}\right)^{1-\theta}-\Omega\left(\pi^{p}\right)}{\left[1-\chi\left(\pi^{p}\right)\right]\left[f\left(\pi^{p}\right)\right]^{1-\theta}+\Omega\left(\pi^{p}\right)}>-1$$

which, in turn, implies

$$\chi\left(\pi^{p}\right)\left(\frac{1}{\pi^{p}}\right)^{1-\theta}+\left[1-\chi\left(\pi^{p}\right)\right]\left[f\left(\pi^{p}\right)\right]^{1-\theta}>0,$$

This equation always holds given the definition of the price level in an economy with full rationality.

The fact that $\pi^{p*}\left(A\right)$ is decreasing in productivity follows immediately from the fact that $g'\left(\pi^{p}\right)>0$ and the equilibrium condition $1/A=g\left(\pi^{p}\right)$. □

Consider productivity levels $A\leq\frac{\theta-1}{\theta}$. Then clearly $\pi^{p}=1/A$ satisfies the equilibrium condition

$$\frac{1}{A}=g\left(\pi^{p}\right)=f\left(\pi^{p}\right)\pi^{p},$$

as $f\left(\pi^{p}\right)=1$ for any $\pi^{p}\geq\frac{\theta}{\theta-1}$. Moreover, by the previous proposition, this equilibrium is unique.

Now consider productivity levels $A \geq \frac{\theta}{\theta-1}$. We want to show that $\pi^{p*}(A) > \frac{1}{A}$. Since $g(\pi^p)$ is strictly increasing, we simply need to show that $g\left(\frac{1}{A}\right) < \frac{1}{A}$. Now

$$g\left(\frac{1}{A}\right) < \frac{1}{A} \iff f\left(\frac{1}{A}\right) < 1.$$

But since $A \geq \frac{\theta}{\theta-1}$, $\frac{1}{A} \leq 1$. We show in another proposition that $\pi^p < 1 \iff f(\pi^p) < 1$. These facts prove the result.

There are clearly rockets and feathers when the increase in cost is such that all firms raise prices ($\overline{A} < \frac{\theta-1}{\theta}$). The reason is that for a symmetric fall in costs, there are still some firms with favorable demand that keep their prices constant. Since $g(\cdot)$ is continuous, $\pi^{p*}(A)$ is also continuous. This property implies that even when the cost rises does not induce all firms to increase prices there are values of $\overline{A}$ that produce rockets and feathers. □

6.2 Proof of Proposition 2

Proof. At any $\pi^p \geq \frac{\theta}{\theta-1}$, $\omega_c(\pi^p) = \omega$, a constant that is independent from π^p. Therefore $C(\pi^p) = C_s$ and $N(\pi^p) = N_s$ are also independent from inflation. We can write

$$C(1) = \left[\frac{\omega_c(1)}{\omega}\right]^{\frac{1+\eta}{\sigma+\eta}} C_s,$$

and

$$N(1) = \left[\frac{\omega_c(1)}{\omega}\right]^{\frac{1-\sigma}{\sigma+\eta}} N_s.$$

Substituting $C(1)$ and $N(1)$ in $\mathcal{W}(1)$ we get

$$\mathcal{W}(1) - \mathcal{W}_s = \left\{\left[\frac{\omega(1)}{\omega}\right]^{\frac{(1+\eta)(1-\sigma)}{\eta+\sigma}} - 1\right\}\left[\frac{C_s^{1-\sigma}-1}{1-\sigma} - \frac{N_s^{1+v}}{1+v}\right] + \frac{\left[\frac{\omega(1)}{\omega}\right]^{1-\sigma} - 1}{1-\sigma} + \kappa\chi(1)\ln\left(\frac{\sigma_c^2}{\kappa}\right).$$

As $\sigma_c^2 \to \infty$, the first two terms go to a finite number. The third term goes to infinity. Therefore there must be $\overline{\sigma}_c^2$ such that $\sigma_c^2 \geq \overline{\sigma}_c^2$ implies that $\mathcal{W}(1) - \mathcal{W}_s > 0$. □

6.3 Proof of Proposition 3

Proof. For any π^p,

$$\mathcal{W}'(\pi^p) = \widetilde{A}(\pi^p)^{\frac{(1+\eta)(1-\sigma)}{\sigma+\eta}-1}\widetilde{A}'(\pi^p) + \kappa \ln\left(\frac{\sigma_c^2}{\kappa}\right)\chi'(\pi^p).$$

At $\pi^p = 1$, $\ell'(\pi^p) = 0$, so $\chi'(1) = \delta_u'(1) = \delta'(1) = 0$. Since $\widetilde{A}(\pi^p) = A\omega_c(\pi^p)$,

$$\mathcal{W}'(1) \propto \hat{\omega}_c(1),$$

where $\hat{\omega}_c(\pi^p) \equiv \frac{d \ln \omega_c(\pi^p)}{d \ln(\pi^p)}$.

At $\pi^p = 1$,

$$\hat{\omega}_c(1) = \theta\left[\frac{\delta_u(1) - \chi(1)}{\delta_u(1) + 1 - \chi(1)} - \frac{\delta(1) - \chi(1)}{\delta(1) + 1 - \chi(1)}\right].$$

Since $\delta_u = 1 - \Phi\left[\ell - \left(\frac{\theta-1}{\theta}\right)\gamma\right]$ and $\delta = 1 - \Phi(\ell - \gamma)$, $\delta_u(1) < \delta(1)$. Therefore $\hat{\omega}_c(1) < 0$ and $\mathcal{W}'(1) < 0$. □

7 Price Distribution

We now describe the equilibrium relation between the optimal relative reset price, $f(\pi^p)$, and the inflation rate. In this section, we measure inflation with the price index in an economy with fully rational households, which we denote by π^p for analytical convenience.

The following proposition characterizes some key properties of the reset price that are illustrated in Figure 3.

Proposition 6. *For* $\pi^p \geq \frac{\theta}{\theta-1}$, $f(\pi^p) = 1$. *For* $\pi^p \in \left(0, \frac{\theta}{\theta-1}\right)$,

1. $\lim_{\pi^p \to 0} f(\pi^p) = \lim_{\pi^p \to \frac{\theta}{\theta-1}} (\pi^p) = f(1) = 1$,

2. $\ell(\pi^p)$ *is minimized at* $\pi^p = 1$ *with* $\ell(\pi^p) = \frac{1}{2}\gamma$,

3. *$f(\pi^p)$ has exactly one global minimum in $\pi^p \in (0,1)$ and one global maximum in $\pi^p \in \left(1, \frac{\theta}{\theta-1}\right)$.*

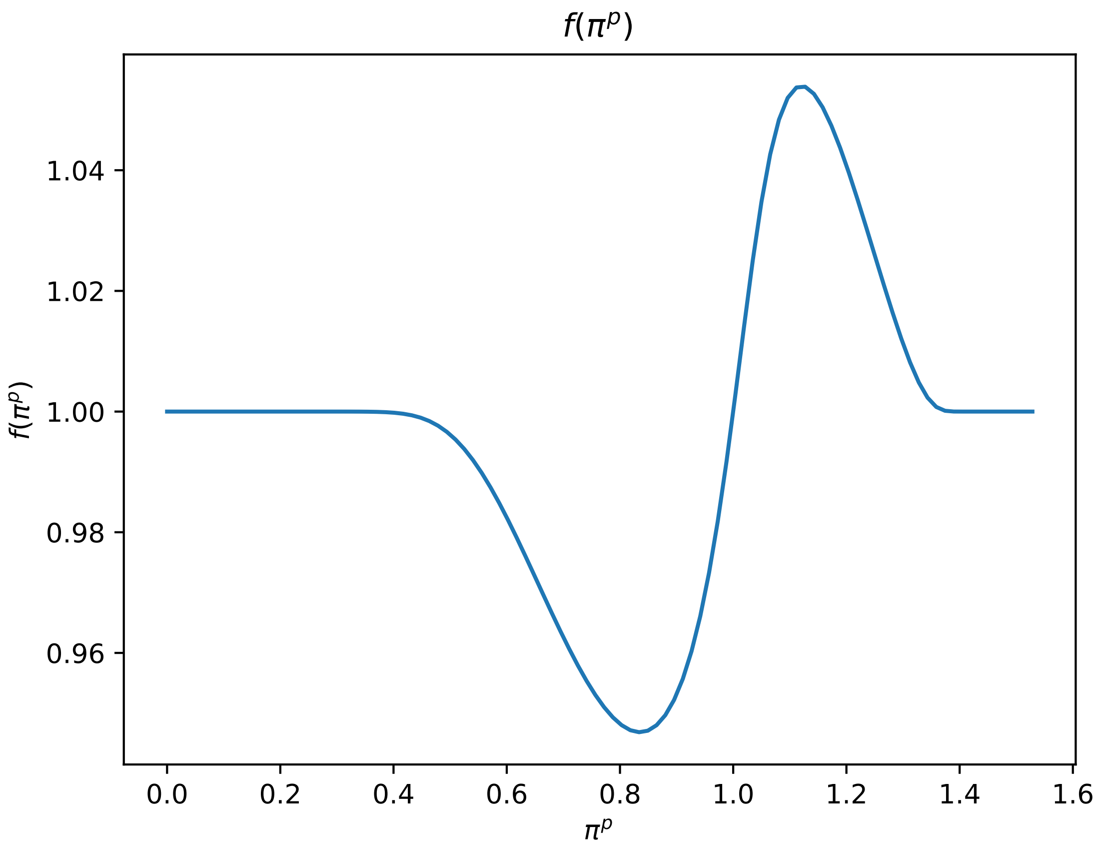

Figure 3: Reset relative price as a function of the inflation rate

The intuition for the behavior of the reset price is as follows. When inflation is sufficiently high, nominal marginal costs are such that the profit margin at the old price is negative. As a result, all producers reset their prices, and therefore the relative reset price is equal to one.

As gross inflation goes to zero (and inflation goes to -100 percent), the real price charged by sticky firms, $\frac{P_0^p}{P^p} = \frac{1}{\pi^p}$ goes to infinity. In this case, almost no firm has a demand shock ϵ_i that makes it worthwhile to keep a real revenue near zero. In the limit, all producers reset their prices, again implying that the relative reset price must equal one.

When gross inflation is equal to one, the definition of P^p implies that the reset price is also equal to one regardless of the fraction of sticky firms:

$$1 = \chi(\pi^p)(\pi^p)^{\theta-1} + [1 - \chi(\pi^p)][f(\pi^p)]^{1-\theta},$$

which implies that

$$f(1) = 1.$$

In this case, the old price is equal to the nominal reset price, P^*, since $f(\pi^p) = 1$ implies that $P^* = P^p = P_0^p$. Therefore firms with $\epsilon_i \geq \ell(1)$ keep their price, and firms with $\epsilon_i < \ell(1)$ change their price by an infinitesimal amount to induce the household to draw a new signal.

Figure 4 shows that the minimum demand shock that makes it worthwhile for firms to keep their price is minimized at $\pi^p = 1$. The old price maximizes the rational component of demand, so it takes a relatively small demand shock to induce firms to keep their price. This fact implies that the fraction of firms with sticky prices and high demand is large around $\pi^p = 1$.

We now explore the non-monotonicity of the reset price with respect to the rate of inflation implicit in part 3 of proposition 6. This non-monotonicity reflects the interplay between the intensive and extensive margins of price adjustment. Using the definition of P^p,

$$1 = \chi(\pi^p)(\pi^p)^{\theta-1} + [1 - \chi(\pi^p)][f(\pi^p)]^{1-\theta},$$

we obtain the elasticity $\hat{f}(\pi^p) \equiv \frac{f'(\pi^p)}{f(\pi^p)}\pi^p$:

$$\hat{f}(\pi^p) = \overline{f}(\pi^p) + \vartheta(\pi^p)\hat{\chi}(\pi^p),$$

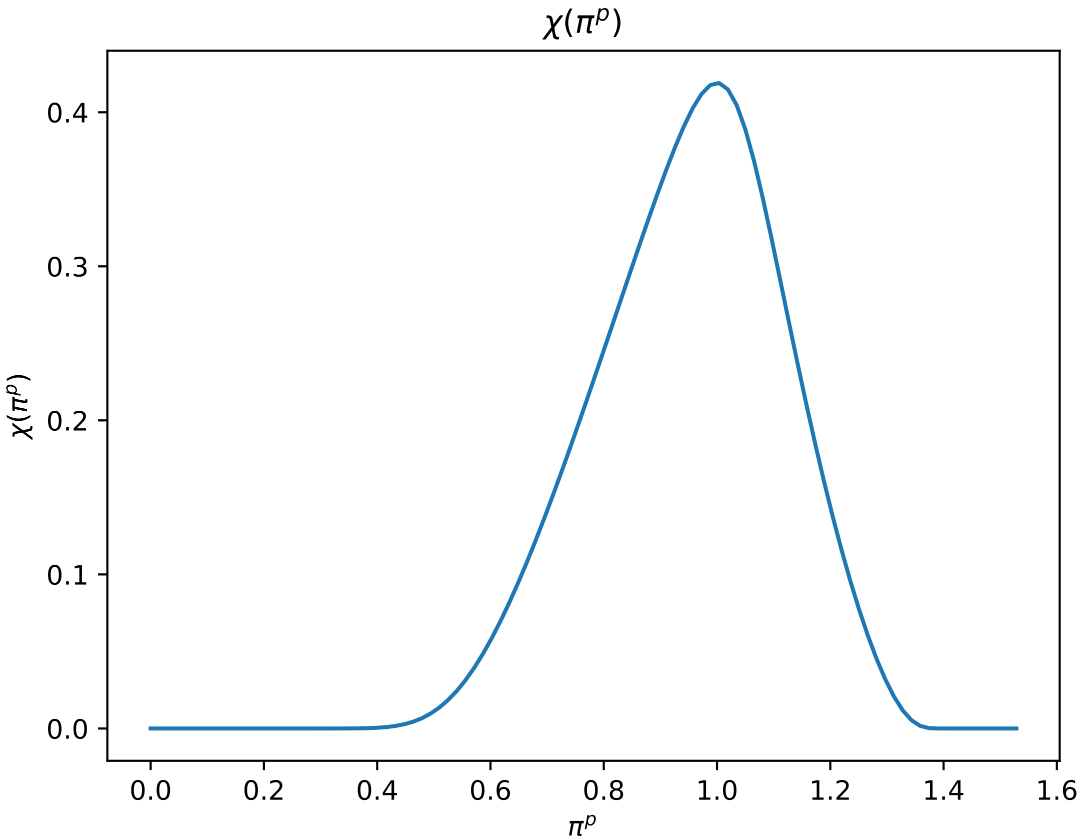

Figure 4: Fraction of firms with sticky prices as a function of the inflation rate

where

$$\overline{f}\left(\pi^p\right) \equiv \left[\frac{\chi\left(\pi^p\right)}{1-\chi\left(\pi^p\right)}\right]\left[\frac{\frac{1}{\pi^p}}{f\left(\pi^p\right)}\right]^{1-\theta} > 0,$$

$$\vartheta\left(\pi^p\right) \equiv \left(\frac{1}{\theta-1}\right)\left[\frac{\chi\left(\pi^p\right)}{1-\chi\left(\pi^p\right)}\right]\left\{\left(\frac{1}{\pi^p}\right)^{1-\theta} - \left[f\left(\pi^p\right)\right]^{1-\theta}\right\},$$

and

$$\hat{\chi}\left(\pi^p\right) \equiv \frac{\chi'\left(\pi^p\right)}{\chi\left(\pi^p\right)}\pi^p.$$

It is easy to show that $\vartheta\left(\pi^p\right) > 0$ when $\pi^p > 1$ and $\vartheta\left(\pi^p\right) < 0$ when $\pi^p < 1$.

The first term of $\hat{f}\left(\pi^p\right)$, $\overline{f}\left(\pi^p\right)$, relates to the intensive margin of price adjustment, and the second term, $\vartheta\left(\pi^p\right)\hat{\chi}\left(\pi^p\right)$, to the extensive margin.

On the intensive margin, there is a positive relation between the relative reset price and inflation ($\overline{f}\left(\pi^p\right) > 0$). If inflation is high, sticky firms charge a low relative price. In equilibrium flexible firms must charge a high relative price so that $\mathbb{E}_i\left[p_i^{1-\theta}\right] = 1$.

On the extensive margin, for π^p there is a negative relation between the relative reset price and inflation ($\vartheta\left(\pi^p\right)\hat{\chi}\left(\pi^p\right) < 0$). if inflation is high, the fraction of sticky firms is low ($\hat{\chi}\left(\pi^p\right) < 0$) because fewer demand shocks make keeping a low nominal price with high nominal marginal costs worthwhile. Flexible firms must charge a smaller relative price so that in equilibrium $\mathbb{E}_i\left[p_i^{1-\theta}\right] = 1$.

It turns out that there is a gross inflation level $\overline{\pi}^p > 1$ such that if $\pi^p > \overline{\pi}^p$, the effect of the extensive margin dominates and $\hat{f}\left(\pi^p\right) < 0$.

The dynamics of deflation are analogous to those of inflation. As inflation becomes more negative, the firms that change prices reduce these prices by more (the intensive margin). But since more firms change prices (the extensive margin), prices do not have to fall by much to ensure that the harmonic mean of the relative prices is one. Again, there is an inflation level $\underline{\pi}^p$ such that if $\pi^p < \underline{\pi}^p$, $f\left(\pi^p\right) > f\left(\underline{\pi}^p\right)$.

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