

# Debt and Growth: Friends or Foes? A Political Economy Approach

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## **The paper considers:**

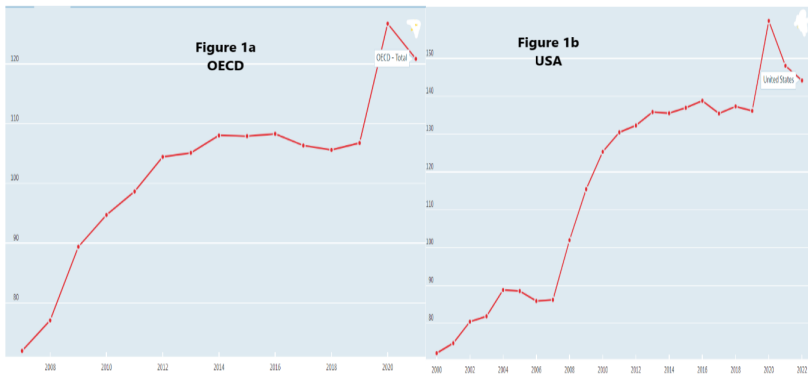
Debt and Growth in Developed Democracies

## **Political economy approach:**

Debt and growth are determined endogenously

## **Why is this issue important?**

Great increase in debt to output in these countries following the great recession and the covid crisis



**Figure 1:** Figures 1a and 1b respectively portray the evolution of debt to output after 2000 in OECD and the USA countries.

## **Why is this issue important?**

Fears that excessive debt will harm growth and welfare

## Standard Economic Theory

Excessive debt causes a decrease in growth because:

- The increase in interest payments crowds out government investment
- It increases taxes and thus crowds out private investments as well
- The increased taxation reduces labour supply (distortive taxation)
- It increases the country risk premium and therefore interest rates

## Empirical Findings

Mixed. Examples:

- Chudik et al. (2017) find a negative relationship between debt to output and growth
- Ash et al. (2017) find that there is no such relationship
- Reinhart and Rogoff (2010) find that this negative relationship emerges only for high levels of debt.

## Point

Maybe there is a more complex theoretical relationship between the two.



## Political Economy Models on Debt

Recent examples:

- Song et al. (2012)
- Arai et al. (2018)

### Characteristics

- Overlapping generations models, with probabilistic voting and time consistent government policy
- Focus on:
  - Population age
  - Preferences for public goods

## Results in the Literature: Median Age

An increase in the median age of the population:

- Song et al. (2012): Increases debt to output
- Arai et al. (2018): Decreases growth

## Results in the Literature: Preference for Public Goods

An increase in the preference for public goods in both models:

- Decreases debt to output
- Increases growth

### Explanation:

Public goods exert a disciplining effect on debt. If the young individuals like public goods, they also dislike excessive debt, because they fear that it might impede public good provision in the future.

## Interpretations of Preferences for Public Goods:

- Müller et al. (2016): Political affiliation
  - People who vote for the left like public goods more
  - Implication: A government shift from left to right increases debt.
- Song et al. (2012): Government corruption
  - Implication: Corruption increases debt
- Others: Trust to the government etc.

I use the last two interpretations

# Motivation

## Correlations (developed democracies)

	Growth	Debt to GDP
Median age	-0.63 (0.003)	0.067 (0.01)
CPI	0.42 (0.07)	-0.65 (0.002)

**CPI** : Corruption Perception Index

**Growth** : Over the period 2008-2022

## Variation of Song et al. (2012)

General characteristics:

- Overlapping Generations Model (Young-Old)
- Probabilistic voting
- Time consistent government policy
- Small open economy

## Government Policy

- Government spending: Transfers to the old (e.g. pensions) vs Investment (e.g. education)
- Government revenue : Taxes vs. New debt

## Key results

- Analytical solution
- Transfer to the old controls debt (disciplining effect)
- Simultaneous determination of debt to output and growth in equilibrium. They depend on the parameters and crucially on the size of the two generations and the preferences for the two public goods.
- Transitional dynamics

The two last points are the most important points of departure from the existing literature.



## Key results

### Two equilibria:

- Stable vs. Unstable
- Interior vs. Immiseration

Any of the 4 combinations is possible depending on the parameters.

## Key results

Transitional Dynamics:

- Explicit relationship between debt to output and growth in the short run
- Ambiguous sign (depending on the parameters)

**why?**

An increase in debt:

- Crowds out private and government investment therefore reduces growth (increased interest payments)
- Can be used for government investment therefore increasing growth

## 3 types of agents:

- Households
- Firms
- Government

## Households

**Utility function of the young:**

$$u_y = \ln c_y + \lambda \ln X + \beta(\ln c_o' + \theta \ln \Phi')$$

$\beta \in (0, 1)$ : time discount factor and  $\theta, \lambda > 0$ : preference weights.

**Utility function of the old:**

$$u_o = \ln c_o + \theta \ln \Phi$$

$X$ : Government investment (e.g. education)

$\Phi$ : Transfer (e.g. pensions)

# Model Assumptions

## Households

**Budget constraint of the young:**

$$w - t = c_y + S'$$

$w$ : wage,  $t$ : tax,  $S'$ : Private investment (savings)

**Budget constraint of the old:**

$$c_o = RS$$

$R$ : Return to investment (constant)

# Model Assumptions

## Firms

**Production function:**

$$y = A \left( \frac{G}{L} \right)^{(1-a)} k^a l^{(1-a)}$$

$y$ : output,  $\kappa$ : capital (perfectly mobile and depreciates fully in one period)  $l$ : labour (hired locally),  $\frac{G}{L}$ : public capital to total labour

**Public capital:**

$$G' = (1 - \delta)G + X$$

$\delta \in [0, 1]$ : discount rate.

# Model Assumptions

**Public capital:** The intangible characteristics of an economy that increase productivity and are the result of government effort in the past.

**Example:**

- $X$ : Government spending for education
- $G$ : Literacy or share of subway passengers reading a book

## Government

**Government budget:**

$$b' = Rb + X + \Phi - tN_y$$

$b$ : government debt (internationally negotiated),  $N_y$  number of young



# Model Assumptions

## Government

**Government objective:**

$$V = \mu u_y + u_o, \text{ where } \mu = \frac{\varphi}{(1-\varphi)}$$

$$\varphi = \frac{\omega_y \lambda_y}{\omega_y \lambda_y + \omega_o \lambda_o}$$

$\omega_y, \omega_o$ : shares in the electorate,  $\lambda_y, \lambda_o$ : number of swing voters.

**Probabilistic voting:** Microfoundations of gov. objective based on a model of 2 candidates maximizing probability of election.

## Sequence of Events

- Government chooses policy  $(t, X, \Phi)$
- Firms choose capital and labour  $(k, l)$
- Young households choose private investment  $(S')$

## Households

Young households maximize their utility with respect to  $S$ . **F.O.C.:**

$$\frac{c_o'}{c_y} = \beta R$$

Optimal investment:

$$S' = \frac{\beta}{(1 + \beta)}(w - t)$$

## Firms

Firms maximize profits with respect to  $k$ ,  $l$ . Capital is perfectly mobile so  $R$  is exogenous (small open economy).

**F.O.C.** wrt  $k$ :

$$R = a \frac{y}{k}$$

Solving for  $k$ , substituting in the production function and aggregating across firms yields:

$$Y = EG$$

where  $E = \left(\frac{a}{R}\right)^{\frac{a}{1-a}} A^{\frac{1}{1-a}}$ .

## Firms

Labour is hired locally so  $w$  is endogenous.

**F.O.C.** wrt  $l$ :

$$w = (1 - a)\frac{y}{l}$$

Aggregating across firms yields:

$$wN_y = (1 - a)EG$$

## Government-Political Equilibrium

After substituting the optimal values from the households' and firms' maximization problem in the government objective we get:

$$V = \mu \left[ (1 + \beta) \ln \left[ \frac{(1 - a)EG}{N_y} - t \right] + \lambda \ln X + \beta \theta \ln \Phi' + Con \right] \\ + \ln RS + \theta \ln \Phi$$

Notice  $\Phi'$

## Government-Political Equilibrium

In order to simplify notation define:

$$V = V(T, T'; W, \pi)$$

$T = (t, X, \Phi)$ : policy ,  $W = (b, G)$ : state variables ,

$\pi = (a, A, \beta, \lambda, \theta, \mu, R)$ : parameters

## Government-Political Equilibrium

In a similar manner in order to simplify notation I define:

$$W' = W(T, T'; W, \pi)$$

in order to aggregate:

$$\begin{aligned} G' &= (1 - \delta)G + X \\ b' &= Rb + X + \Phi - tN_y \end{aligned}$$



## Government-Political Equilibrium

- **Problem:** Objective of current government depends on actions of future government. Game between successive governments.

## Government-Political Equilibrium

### Solution concept: Markov Perfect Equilibrium

- $T$  is a function the current values of pay-off relevant state variables which are  $W = (b, G)$
- The current government forms a guess about what the next government does given next period's state variables ( $T' = T(b', G')$ ).
- In equilibrium this guess is consistent with current choice of policy. Optimal current policy is:  $T = T(b, G)$ .

## Government-Political Equilibrium

**Definition.** *Political equilibrium.*

Let  $W \in \mathbb{R}_+^2$ . A function  $T : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+^3$ , such that  $T = T(W) = (t(b, G), X(b, G), \Phi(b, G))$  is a political equilibrium, if it maximizes the political objective function  $V = V(T, T'; W)$ , with respect to  $T$ , subject to  $T' = T(W')$  and  $W' = W(T, T'; W)$ .

## Government-Political Equilibrium

Maximization problem wrt  $t$ ,  $X$  and  $\Phi$  :

$$V = \mu \left[ (1 + \beta) \ln \left[ \frac{(1 - a)EG}{N_y} - t \right] + \lambda \ln X + \beta \theta \ln \Phi' + Con \right] \\ + \ln RS + \theta \ln \Phi$$

s.t.

$$\Phi' = \Phi(b', G')$$

$$G' = (1 - \delta)G + X$$

$$b' = Rb + X + \Phi - tN_y$$

## Government-Political Equilibrium

F.O.C. :

$$\frac{-(1 + \beta)}{\left[\frac{(1-a)EG}{N_y} - t\right]} + \frac{\beta\theta}{\Phi'} \frac{\partial\Phi(b', G')}{\partial b'} \frac{\partial b'}{\partial t} = 0$$
$$\frac{\lambda}{X} + \frac{\beta\theta}{\Phi'} \left( \frac{\partial\Phi(b', G')}{\partial b'} \frac{\partial b'}{\partial X} + \frac{\partial\Phi(b', G')}{\partial G'} \frac{\partial G'}{\partial X} \right) = 0$$
$$\frac{\theta}{\Phi} + \frac{\beta\theta\mu}{\Phi'} \frac{\partial\Phi(b', G')}{\partial b'} \frac{\partial b'}{\partial\Phi} = 0$$

**Point:** F.O.C. are known as Generalized Euler Equations. Generalized used because they describe a functional problem ( The function  $\Phi(\cdot)$  is unknown).

## Government-Political Equilibrium

Derivation of equilibrium:

- Assume and verify  $\Phi(\cdot)$  is linear.  $\Phi' = \Phi(b', G') = \omega_1 b' + \omega_2 G'$   
where  $\omega_1, \omega_2 \in \mathbb{R}$
- Assume  $\mu$  is constant.

## Government-Political Equilibrium

Under the previous conditions the GEE yield:

$$\Phi = \Lambda R \left[ -b + \frac{(1-a)E}{R - (1-\delta)} G \right]$$

where  $\Lambda = \frac{\theta}{\mu[1+\beta(1+\theta)+\lambda]+\theta}$

**Point:** The optimal  $\Phi$  is a linear function of  $b$  and  $G$ . The conjecture is verified.

## Government-Political Equilibrium

and

$$X = \frac{\lambda\mu}{\theta} \Lambda R \left[ \frac{R - (1 - \delta)}{R - (1 - \delta) - (1 - a)E} \right] \left[ -b + \frac{(1 - a)E}{R - (1 - \delta)} G \right]$$
$$tN_y = \frac{(1 + \beta)\mu}{\theta} \Lambda R b + \left[ 1 - \frac{(1 + \beta)\mu}{\theta} \frac{\Lambda R}{R - (1 - \delta)} \right] (1 - a)EG$$



## Transitional Dynamics

Substituting in the constraints yields:

$$G' = -\frac{\lambda\mu}{\theta}\Lambda R \left[ \frac{R - (1 - \delta)}{R - (1 - \delta) - (1 - a)E} \right] b \\ + \left[ (1 - \delta) + \frac{\frac{\lambda\mu}{\theta}\Lambda R(1 - a)E}{R - (1 - \delta) - (1 - a)E} \right] G$$

and

$$b' = (1 - \Xi\Lambda) Rb + \left( \frac{\Xi\Lambda R}{R - (1 - \delta)} - 1 \right) (1 - a)EG$$

## Transitional Dynamics

- $\Xi = \frac{\lambda\mu}{\theta} \frac{R-(1-\delta)}{R-(1-\delta)-(1-a)E} + 1 + \frac{1+\beta}{\theta} \mu$

## Transitional Dynamics

Dividing the first equation by  $Y$  and the second by  $Y'$  yields:

$$g = -\frac{\lambda\mu}{\theta}\Lambda RE \left[ \frac{R - (1 - \delta)}{R - (1 - \delta) - (1 - a)E} \right] \frac{b}{Y}$$
$$+ \left[ (1 - \delta) + \frac{\frac{\lambda\mu}{\theta}\Lambda R(1 - a)E}{R - (1 - \delta) - (1 - a)E} \right]$$
$$\left( \frac{b}{Y} \right)' = \frac{1}{g} \left[ (1 - \Xi\Lambda) R \frac{b}{Y} + \left( \frac{\Xi\Lambda R}{R - (1 - \delta)} - 1 \right) (1 - a) \right]$$

where  $g = \frac{Y'}{Y}$  is the growth rate of output.

## Transitional Dynamics

Combining parameters to simplify notation yields:

$$g = \Omega_1 \frac{b}{Y} + \Omega_2$$
$$\left(\frac{b}{Y}\right)' = \frac{1}{g} \left( \Omega_3 \frac{b}{Y} + \Omega_4 \right)$$

**Point:** These equations hold in the short run and the first is an explicit relationship between debt to output and growth.

## Transitional Dynamics

Substituting the first equation into the second yields:

$$\left(\frac{b}{Y}\right)' = \frac{\Omega_3 \frac{b}{Y} + \Omega_4}{\Omega_1 \frac{b}{Y} + \Omega_2}$$

This equation describes the dynamics of debt.

## Steady State

If we set  $\left(\frac{b}{Y}\right)' = \frac{b}{Y}$  then we can find the steady states. If they exist there at most two.

$$g = \frac{\Omega_3 + \Omega_2 \pm \sqrt{\Delta}}{2}$$
$$\frac{b}{Y} = \frac{\Omega_3 - \Omega_2 \pm \sqrt{\Delta}}{2\Omega_2}$$

where  $\Delta = (\Omega_2 - \Omega_3)^2 + 4\Omega_1\Omega_4$ .

## Steady State

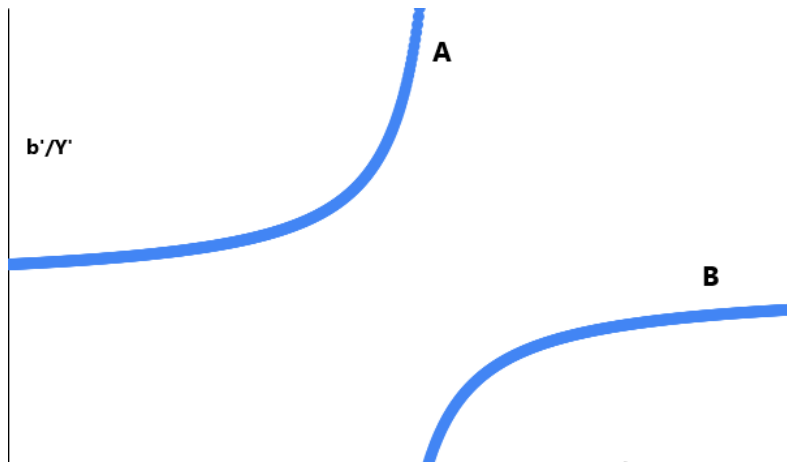
If there are two equilibria :

- One of the two is an **interior** equilibrium: complex function of all parameters
- The other is the **immiseration** equilibrium in which all the tax base is used to repay debt:  $\frac{b}{y} = \frac{1-a}{R-(1-\delta)}$  and  $g = 1 - \delta$ .
- One equilibrium is **stable** and one **unstable**. We don't know which is which (depends on the parameter values)

# Equilibrium

## Stability

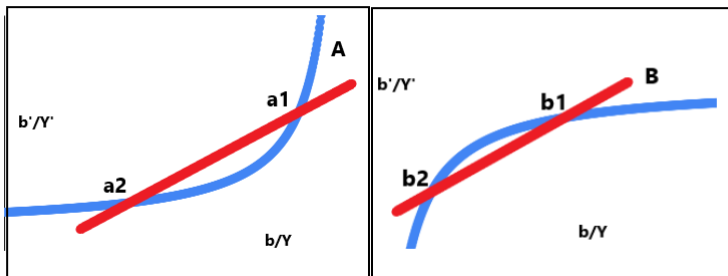
Graphically the equation  $\left(\frac{b}{Y}\right)' = \frac{\Omega_3 \frac{b}{Y} + \Omega_4}{\Omega_1 \frac{b}{Y} + \Omega_2}$  can be portrayed like this:





## Stability

So the possible intersections with the 45 deg line is:



- Stable a2, b1
- Intuitive: Interior in a2.

## Introduction

Next I consider a calibration in order to:

- Show that the model is consistent with data for developed democracies
- Show that the interior equilibrium matches  $a_2$  in the previous graph
- Find whether the relationship between debt and growth in the short run is negative or positive
- Perform comparative statics

# Calibration Description

- Period: 1980-2010
- Synthetic country
- OECD members since 1980 (24 countries)
- Rationale: Homogeneous group of developed democracies
- Exclude:
  - USA, Japan, Germany (small open economy)
  - Turkey (democracy, outlier)

## Use of data:

- Growth between 1980-2010
- Debt to output and government spending to output in 1995

## Calibration overview

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### Preset Parameters

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Time Disc. ( $\beta$ )	$0.995^{30} = 0.8604$	Pol. Power ( $\mu$ )	3
Int. Rate ( $R$ )	$1.05^{30} = 4.3219$	Cap. sh. ( $a$ )	0.3
Pub. Cap. Disc. ( $\delta$ )	0		

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### Targets

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Debt to GDP ( $b/Y$ )	$0.0239 = 0.717/30$	gsp/gdp ( $\frac{X+\Phi}{Y}$ )	0.4927
Growth ( $g$ )	1.8950		

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# Calibration

## Calibrated Parameters

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Pref.-o ( $\theta$ )	3.3475	Pref.-y ( $\lambda$ )	0.72
Effic. par. ( $A$ )	5.0708		

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## Calculated Parameters

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$\Omega_1$	-4.7909	$\Omega_2$	2.3399
$\Omega_3$	0.8578	$\Omega_4$	0.0444
$E$	3.2414	immis. $b/Y$	0.2107 (632%)

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## Important results

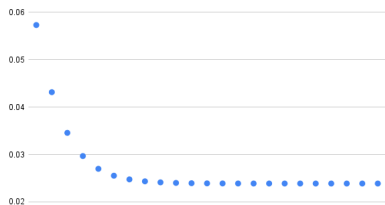
- The Interior equilibrium is stable and smaller than the immiseration equilibrium
- Negative relationship between debt and growth in the short run:  $\Omega_1 < 0$

## Example: Negative relationship between debt and growth

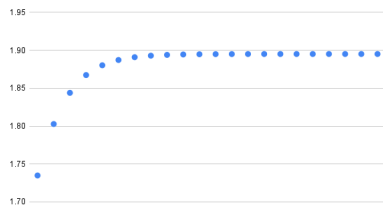
- Exogenous increase in debt to output (e.g. war) from 72 % to 172 %
- Growth drops on an annual base from 2.2 % to 1.9 %
- Both debt to output and growth gradually return to steady state

# Calibration

2a. Debt to output



2b. Growth





# Calibration

**Comparative statics:** Effect of an increase by 1 % in a parameter.

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Elasticities	b/Y SR	b/Y LR	g SR	g LR
$\lambda$	4.2	9	0.4	-0.1
$\theta$	-5.4	-11.3	-0.3	0.4
$\theta + \lambda$	-1.2	-2.3	0.1	0.3
$\mu$	-0.8	-1.5	0.1	0.2
$A$	16.5	34.9	2.2	0

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# Calibration

**Comparative statics:** Effect of an increase by 1 % in a parameter.

Elasticities	$\frac{X+\Phi}{Y}$ SR	$\frac{X+\Phi}{Y}$ LR	$X/\Phi$ SR	$X/\Phi$ LR
$\lambda$	0.7	0.9	1	1
$\theta$	-0.5	-0.8	-1	-1
$\theta + \lambda$	0.2	0.1	0	0
$\mu$	-0.2	-0.2	1	1
$A$	1.8	3.6	3.1	3.2

# Conclusions

- Intuitive effect of population age and government trust
- Negative relationship between debt to output and growth in the short run
- Difficult to identify this relationship in cross country empirical research due to differences in parameters that are difficult to control for
- The same in time series analysis due to the effect of parameter changes on the steady state