Information Aggregation with Costly Information Acquisition

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Motivation

- Do financial markets aggregate information through prices?
  - Central question at least since [Hayek, 1945]

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- Do financial markets aggregate information through prices?
    - Central question at least since [Hayek, 1945]
- [Ostrovsky, 2012]: 'Separable' securities (e.g. Arrow-Debreu) aggregate information in ALL equilibria, as long as agents trade for many periods
    - Does not depend on the market power of individual traders
    - Powerful result which shows that markets are efficient

Separability is not robust

- BUT separability is not robust to changes in the information structure
- If we add/subtract more traders or the initial private information changes, a security may no longer be separable
- In practice, we don't know the information structure and therefore we cannot know whether markets are efficient and prices are good predictors of an asset's value

Cheap information

- Can this problem be fixed by the availability of cheap information?
- Information is now cheaper than ever to acquire, analyze, and act upon
- We enhance the model of [Ostrovsky, 2012] by allowing traders to acquire signals in every period

Preview of results

- Without information acquisition, very few securities are separable for all information structures
- κ separability securities characterise information aggregation when cost of signals is κ
- As cost goes to zero, almost all securities become κ separable **irrespective** of the information structure
- Information aggregation can happen both faster and slower unless the security is Arrow-Debreu

Market Scoring Rule

- We model trading using Market Scoring Rules (MSR) [Hanson, 2003]
- There are no noise traders and no strategic market makers, as in [Kyle, 1985], where information *aggregation* is intertwined with information *revelation*
- Prediction markets use MSR
- Can be re-interpreted as a model of trading with an automated market maker (AMM)

Related literature

- No trade theorems: [Aumann, 1976], [Milgrom and Stokey, 1982], [Geanakoplos and Polemarchakis, 1982], [Sebenius and Geanakoplos, 1983]
- Market Scoring Rules: [McKelvey and Page, 1990] and [Hanson, 2003, Hanson, 2007]
- Separability and information aggregation: [DeMarzo and Skiadas, 1998], [Ostrovsky, 2012], [Chen et al., 2012], [Dimitrov and Sami, 2008]
- Rational inattention: [Sims, 2003, Matějka and McKay, 2015, Caplin et al., 2019].

Model

Preliminaries

- Finite state space Ω and $i = 1, ..., n$ traders
- Security $X : \Omega \rightarrow \mathbb{R}$
- Asymmetric information, where $\Pi = \{\Pi_1, ..., \Pi_n\}$ is the information structure and their join is the finest partition

Sequence of play

- Market maker's initial announcement is y_0
- In period t_1, player 1 announces y_1, in t_2 player 2 announces y_2 ...
- Before announcing, each player can buy a signal
- There are infinitely many periods
- Information gets aggregated if sequence $\{y_k\}_{k=1}^{\infty}$ converges in probability to random variable X

Payoffs

- Trader's payoff from announcing y_t at time t when x^* is the true value of X:

$$s(y_t, x^*) - s(y_{t-1}, x^*)$$

- $s(y_t, x^*)$ is a *strictly proper scoring rule*
- For example, quadratic scoring rule: $s(y, x^*) = -(y - x^*)^2$
- For every prior μ and security X, the expectation of $s(y, x)$ is maximised, uniquely, for $y = E_\mu[X]$

Separability [Ostrovsky, 2012]

A security X is non-separable under partition structure Π if there exists probability μ and value $v \in \mathbb{R}$ such that:

(i) $X(\omega) \neq v$ for some $\omega \in Supp(\mu)$

(ii) $E_\mu[X|\Pi_i(\omega)] = v$ for all $i = 1, ..., n$ and $\omega \in Supp(\mu)$

We then say that X is non-separable at μ. Otherwise, it is separable.

Separability

- Separable securities aggregate information in **ALL** equilibria ([Ostrovsky, 2012])
- A security X is separable if, whenever there is no trade, there is no uncertainty about its value
- **BUT** separability is not robust to changes in the information structure
- If the market designer does not know it, she cannot be sure that the market will aggregate information

Most securities are not always separable

Proposition

The only securities that are separable for all information structures in $\mathcal{P}$ *are*

1. *Constant, e.g.* $X = (0, 0, 0, 0, 0)$,
2. *Arrow-Debreu, e.g.* $X = (1, 0, 0, 0, 0)$,
3. $X(\omega_{min}) < X(\omega) < X(\omega_{max})$ *for all* $\omega \neq \omega_{min}, \omega_{min}$.

Information acquisition

Statistical experiments

- Before making an announcement, each trader can buy a statistical experiment $\mathcal{R} : \Omega \to \Delta(Z)$ that costs $h(\mathcal{R}, \mu)$
- For some results we require that it is impossible to buy an experiment that excludes a state with certainty
- $cK(\mu, Q)$: cost of random posterior Q
- $c > 0$: unit cost of information

Myopic problem

A myopic trader with belief μ solves the problem

$$\sup_{Q \in \mathcal{Q}(\mu)} \left(\sum_{\gamma \in Supp(Q)} Q(\gamma) \sum_{\omega' \in \Omega} \gamma(\omega') \left[s(E_\gamma[X], X(\omega')) - s(y_{-1}, X(\omega')) \right] - cK(\mu, Q) \right)$$

where y_{-1} is the previous announcement

Generalisation of separability

- Security X is non-separable at some prior μ if everyone makes the same myopic announcement at all states, yet there is uncertainty about the value of X
- X is κ non-separable if, additionally, no trader is willing (myopically) to acquire any new information

Characterization of information aggregation

Theorem

Fix information structure Π and cost structure κ. Then:

- *If security X is κ separable under Π, then in any Nash equilibrium of the game Γ^S, information gets aggregated.*
- *If security X is κ non-separable under Π, then there is game Γ^S and a Perfect Bayesian equilibrium where information does not get aggregated.*

Theorem

Theorem

Suppose that Ω has at least four states. Under Assumption 1, the following are equivalent:

- *X is κ non-separable given some Π and for all κ,*
- *X pays (a, b, d, d) in four states, where either $a, b < d$ or $a, b > d$.*

Robustness

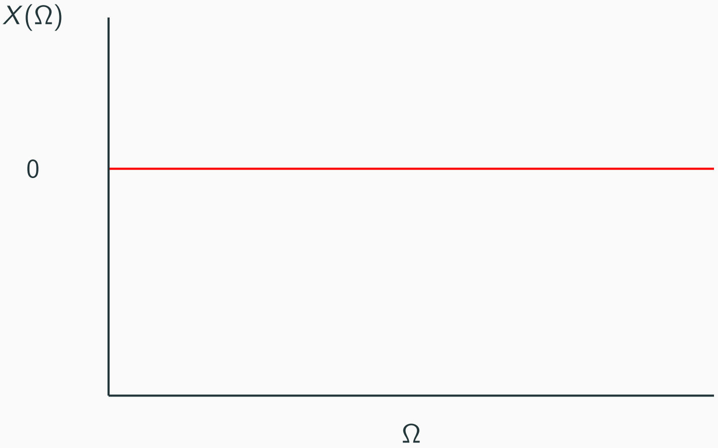
Corollary

If $X(\omega) \neq X(\omega')$ for all $\omega, \omega' \in \Omega$, then, given any Π, X is κ separable for some κ

- If information acquisition is cheap, information aggregation is robust to changes in the information structure
- [Grossman, 1976], [Radner, 1979], [Allen, 1981] show that if trade can be made on sufficiently many events, then generically there is information aggregation in the context of perfect competition and rational expectations

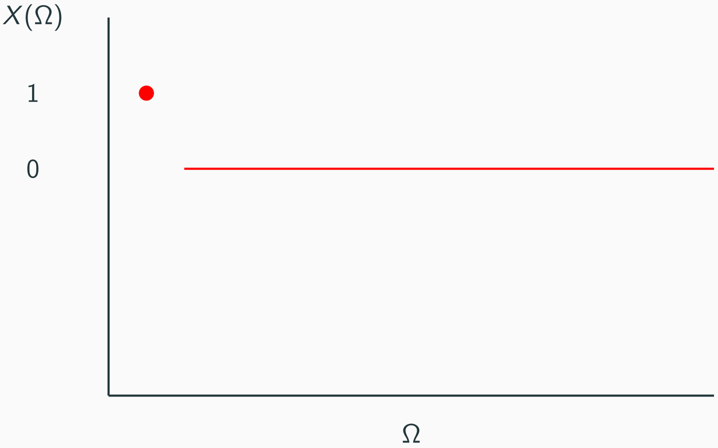
Case 1: X is always separable (for all Π)

Constant



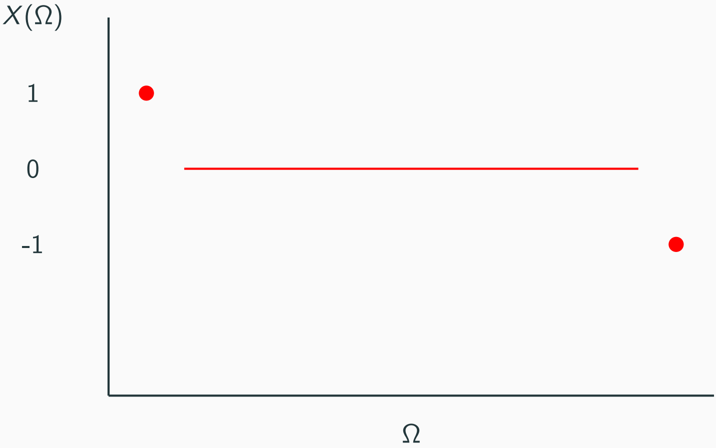
Case 1: X is always separable (for all Π)

Arrow-Debreu



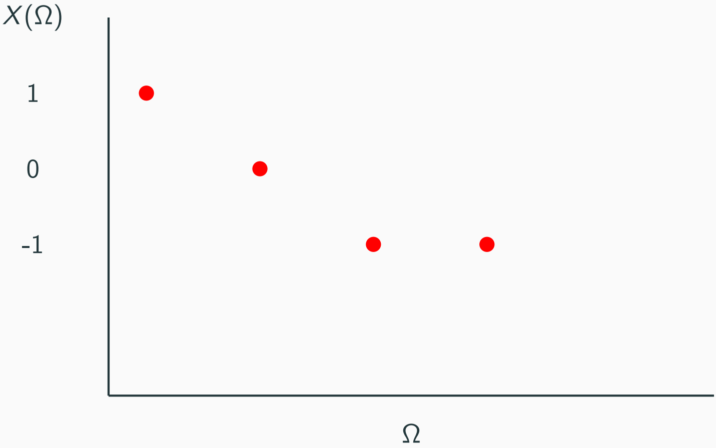
Case 1: X is always separable (for all Π)

Three values



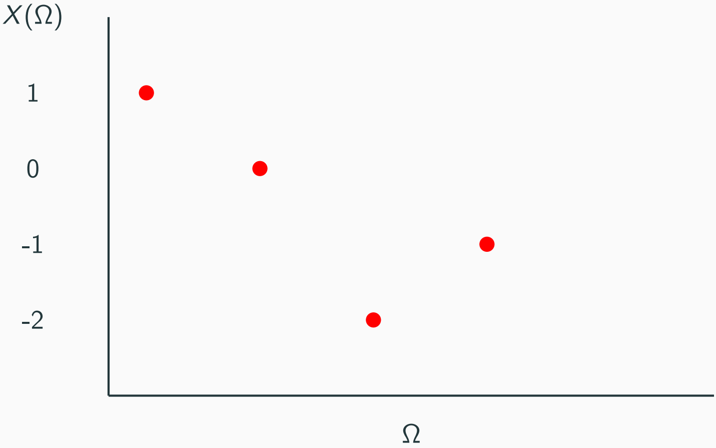
Case 2: X is κ non-separable for all κ, for some Π

There exist four states such that

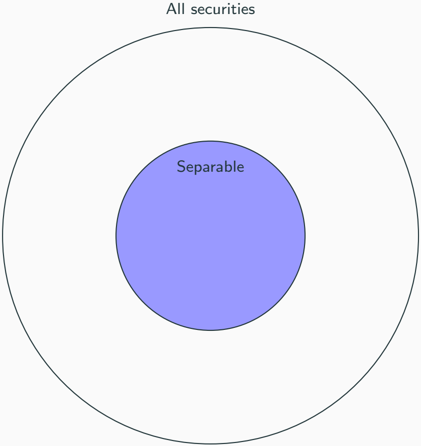


Case 3: X is κ separable for some κ, for all Π

No two states pay the same



Classes of securities



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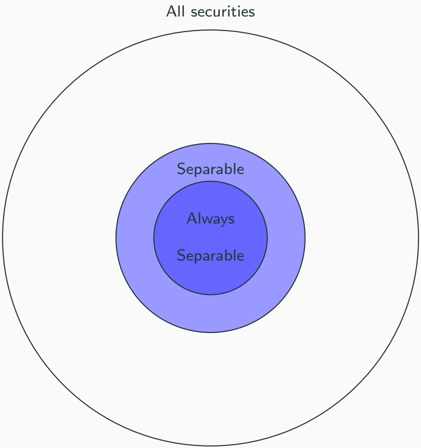

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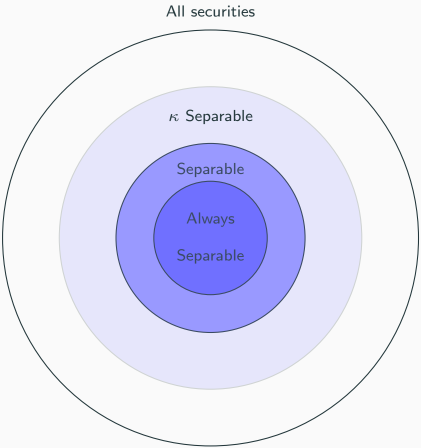

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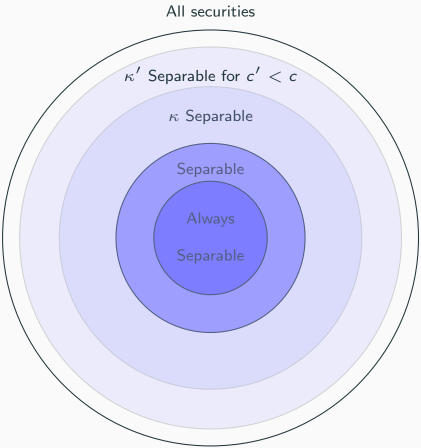
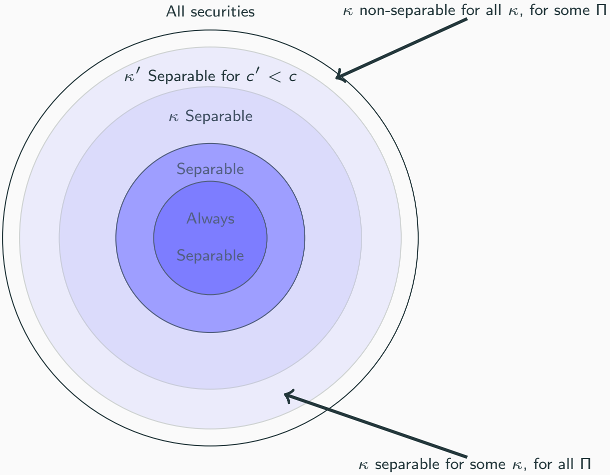


Classes of securities



Speed

Speed of information aggregation

- Does the availability of cheap information make markets unambiguously more efficient, by aggregating information faster?

Speed of information aggregation

- Does the availability of cheap information make markets unambiguously more efficient, by aggregating information faster? **Not necessarily**
- Even with myopic traders, aggregation can happen both FASTER and SLOWER with information acquisition
- The only exception is Arrow-Debreu, for which the speed is unaffected

Discussion

- [Ostrovsky, 2012] showed that separable securities aggregate information in all equilibria but this result is not robust
- Robustness is achieved if we allow costly information acquisition and the cost is sufficiently low
- We characterize the classes of always separable and κ separable securities
- Are markets relevant if $c \to 0$? Yes, because full aggregation kicks in long before that, hence information acquisition makes them even more valuable

Thank you!

Separability [Ostrovsky, 2012]

A security X is non-separable under partition structure Π if there exists probability μ and value $v \in \mathbb{R}$ such that:

(*i*) $X(\omega) \neq v$ for some $\omega \in Supp(p)$

(*ii*) $E_p[X|\Pi_i(\omega)] = v$ for all $i = 1, ..., n$ and $\omega \in Supp(p)$

We then say that X is non-separable at μ. Otherwise, it is separable.

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Posterior Separability [Ostrovsky, 2012]

Definition
A cost of information function K is posterior separable if, given $\mu \in \Delta(\Omega)$ and any Bayes-consistent posteriors $Q \in \mathcal{Q}(\mu)$,

$$K(\mu, Q) = \sum_{\gamma \in Supp(Q)} Q(\gamma) T_\mu(\gamma)$$

for some function $T_\mu : \Delta(Supp(\mu)) \to \overline{\mathbb{R}}$ which is strictly convex and continuous in γ, $T_\mu(\gamma) < \infty$ on $int\Delta(Supp(\mu))$ and $T_\mu(\gamma) \geq 0$.

BACK

No Information Acquisition

Definition
There is no information acquisition for security X, given prior μ and previous announcement y_{-1}, cost structure $\kappa = (K, c)$ and information structure $\Pi \in \mathcal{P}$, if for all states $\omega \in Supp(\mu)$, all traders i, and all $Q \in \mathcal{Q}(\mu_{\Pi_i(\omega)})$,

$$\sum_{\gamma \in Supp(Q)} Q(\gamma) \sum_{\omega' \in \Omega} \gamma(\omega') \Big[s(E_\gamma[X], X(\omega')) - s(y_{-1}, X(\omega')) \Big] - cK(\mu_{\Pi_i(\omega)}, Q) \leq 0.$$

Otherwise, there is information acquisition.

BACK

Faster information aggregation

- Security $X = \{0, 2, 1, 1\}$
- Common prior $\mu = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\}$
- **A**'s partition: $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$
- **B**'s partition: $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$

No information acquisition

- Suppose ω_1 is the true state
- At all states, **A** announces 1, so no information transmission
- **B** announces 0.5, but at ω_4 he would have announced 1.5
- **A** learns that ω_1 is the true state and announces 0
- There is information aggregation in period 3

Information acquisition (faster)

- Before announcing, traders can acquire experiment $\mathcal{R}$ with signal z: $\mathcal{R}_{\omega_1}(z) = 1$ and $\mathcal{R}_{\omega_2}(z) = \mathcal{R}_{\omega_3}(z) = \mathcal{R}_{\omega_4}(z) = 0.5$
- At ω_1, **A**'s updated beliefs are $\{\frac{2}{3}, \frac{1}{3}, 0, 0\}$ and she announces $\frac{2}{3}$
- At ω_3, her posterior would be $\{0, 0, \frac{1}{4}, \frac{1}{4}\}$ and she would have announced 1
- **B** learns that ω_1 is the true state and announces 0
- There is information aggregation in period 2 (vs. 3)
- **B** benefits from the signal, even though he did not buy it

BACK

Slower information aggregation

- Security $X = \{0, 2, 1, 1\}$
- Common prior $\mu = \{\frac{3}{16}, \frac{6}{16}, \frac{7}{32}, \frac{7}{32}\}$
- **A**'s partition: $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$
- **B**'s partition: $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_4\}\}$

No information acquisition

- Suppose ω_1 is the true state
- At ω_1, **A** announces $\frac{4}{3}$
- **B** learns that ω_1 is the true state and announces 0
- There is information aggregation in period 2

Information acquisition (slower)

- Before announcing, traders can acquire experiment $\mathcal{R}$ with signal z: $\mathcal{R}_{\omega_1}(z) = 1$ and $\mathcal{R}_{\omega_2}(z) = \mathcal{R}_{\omega_3}(z) = \mathcal{R}_{\omega_4}(z) = 0.5$
- At ω_1, **A**'s updated beliefs are $\{\frac{1}{2}, \frac{1}{2}, 0, 0\}$
- She announces 1, so no information transmission
- Hence, **B** announces 0.5
- Then, **A** learns that ω_1 is the true state and announces 0
- Thus, there is information aggregation in period 3 (vs. 2)

BACK

Related literature

- No trade theorems: [Aumann, 1976], [Milgrom and Stokey, 1982], [Geanakoplos and Polemarchakis, 1982], [Sebenius and Geanakoplos, 1983]
- Market Scoring Rules: [McKelvey and Page, 1990] and [Hanson, 2003, Hanson, 2007]
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BACK

Example

- Two traders, **A**lice and **B**ob, with asymmetric information but common prior
- State space $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$
- Security $X : \Omega \to \mathbb{R}$
- Trade takes place by sequentially exchanging announcements about the price of X
- Traders announce information truthfully (MSR)

Effect of information acquisition on aggregation

- Security $X = (0, \frac{10}{7}, 0, 1)$
- Common prior $\mu = \{\frac{1}{4}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}\}$
- **A**'s partition: $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$
- **B**'s partition: $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$

No information acquisition

- Suppose ω_1 is the true state, so the true value of X is 0
- Then, **A** knows that the state is in $\{\omega_1, \omega_4\}$
- **A**'s beliefs are $(\frac{3}{5}, \frac{2}{5}, 0, 0)$ and she announces $\frac{4}{7}$
- The same announcement would be made if the state was ω_3
- **B** knows that the state is in $\{\omega_1, \omega_3\}$, hence he learns nothing from **A**'s announcement

No information acquisition

- **B**'s posterior beliefs at ω_1 are $(\frac{3}{7}, 0, 0, \frac{4}{7})$ and he announces $\frac{4}{7}$
- The same announcement would be made at ω_2, hence **A** learns nothing from **B**'s announcement
- Hence, there is no information aggregation because traders agree on the $\frac{4}{7}$ price, yet the true value of X is 0

Information acquisition

- Before announcing, traders can acquire experiment $\mathcal{R}$ with signal z: $\mathcal{R}_{\omega_1}(z) = 1$ and $\mathcal{R}_{\omega_2}(z) = \mathcal{R}_{\omega_3}(z) = \mathcal{R}_{\omega_4}(z) = 0.5$
- At ω_1, **A**'s beliefs are $(\frac{3}{5}, \frac{2}{5}, 0, 0)$ and after the signal they are $(\frac{3}{4}, \frac{1}{4}, 0, 0)$
- She announces $\frac{5}{14}$
- At ω_3, her posterior would be $(0, 0, \frac{4}{7}, \frac{3}{7})$
- She would have announced $\frac{4}{7}$
- Hence, **B** learns that ω_1 is the true state and announces 0
- **B** benefits from the signal, even though he did not buy it

BACK

Why markets are more efficient than polls?

- Security is $X = (0, 1, 2, 3)$
- **A**'s partition: $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$
- **B**'s partition: $\{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}$
- At prior $\mu = (\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8})$, X is non-separable with announcement $v = \frac{3}{2}$
- Assume Shannon cost of information
- Prediction Polls are the average of the individual estimates of traders' myopic predictions:

$$p^{polls} = \frac{\sum_i p_i}{n}$$

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References i

Allen, B. (1981).
Generic existence of completely revealing equilibria for economies with uncertainty when prices convey information.
Econometrica, pages 1173–1199.

Aumann, R. (1976).
Agreeing to disagree.
Annals of Statistics, 4:1236–9.

Caplin, A., Dean, M., and Leahy, J. (2019).
Rational inattention, optimal consideration sets, and stochastic choice.
The Review of Economic Studies, 86(3):1061–1094.

Chen, Y., Ruberry, M., and Vaughan, J. W. (2012).
Designing informative securities.
Mimeo.

DeMarzo, P. and Skiadas, C. (1998).
Aggregation, determinacy, and informational efficiency for a class of economies with asymmetric information.
Journal of Economic Theory, 80(1):123–52.

Dimitrov, S. and Sami, R. (2008).
Non-myopic strategies in prediction markets.
In *Proceedings of the 9th ACM Conference on Electronic Commerce*, pages 200–9.

Geanakoplos, J. and Polemarchakis, H. (1982).
We can't disagree forever.
Journal of Economic Theory, 28:192–200.

Grossman, S. (1976).
On the efficiency of competitive stock markets where trades have diverse information.
The Journal of Finance, 31(2):573–85.

Hanson, R. (2003).
Combinatorial information market design.
Information Systems Frontiers, 5(1):107–19.

Hanson, R. (2007).
Logarithmic market scoring rules for modular combinatorial information aggregation.
Journal of Prediction Markets, 1(1):3–15.

Hayek, F. A. (1945).
The use of knowledge in society.
The American Economic Review, pages 519–30.

Kyle, A. S. (1985).
Continuous auctions and insider trading.
Econometrica, 53(6):1315–35.

Matějka, F. and McKay, A. (2015).
Rational inattention to discrete choices: A new foundation for the multinomial logit model.
American Economic Review, 105(1):272–298.

McKelvey, R. D. and Page, T. (1990).
Public and private information: An experimental study of information pooling.
Econometrica, 58(6):1321–39.

Milgrom, P. and Stokey, N. (1982).
Information, trade, and common knowledge.
Journal of Economic Theory, 26:17–27.

Ostrovsky, M. (2012).
Information aggregation in dynamic markets with strategic traders.
Econometrica, 80(6):2595–647.

References v

Radner, R. (1979).
Rational expectations equilibrium: Generic existence and the information revealed by prices.
Econometrica, 47(3):655–78.

Sebenius, J. and Geanakoplos, J. (1983).
Don't bet on it: Contingent agreements with asymmetric information.
Journal of the American Statistical Association, 78(382):424–6.

Sims, C. A. (2003).
Implications of rational inattention.
Journal of Monetary Economics, 50(3):665–690.