Modeling Money and Inflation
Radical Critique
Radical Proposal

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(Pradeep Dubey)

Inflation and Money

- Inflation is the change in the commodity price of money.
- Yet the supply of money has virtually disappeared from modern economic theory.
- Curiously this has happened just as we are getting more monies. Instead of modeling the new monies, we exiled all money.

Failure to Predict Inflation

- Failure of economists and the Fed models in predicting the 2021-22 inflation.
- Fed stood idle for almost 10 months while inflation raged from under 2% to close to 10%.
- Disagreement (failure) in predicting how fast inflation would come down. Continues.
- Usual explanation is that Covid and Supply side disturbances are so unprecedented that failure to predict consequences is unsurprising.
- My view is that some things are missing from the Fed monetary models. Need radically new model.

Failure to Understand Inflation

- Disagreement about whether 2008 monetary injections (QE) would create inflation.
- Economists did not foresee the magnitude of the 1970s inflation.
- Even 50 years later, still no explanation for the 1970s inflation.
  - Not Oil shock.
  - Not Anchovies.
  - What?

What's Missing from Fed Models?(1/5)

- Theoretically there are no general theorems. (We will see some soon.)
  - Everything is special production functions, special kind of monopolistic competition.
  - Ad hoc fixed prices, ad hoc rationing rules.
- Empirical Phillips Curves and Beveridge Curves that keep moving around.
  - As if all inflation created in the labor market.
  - As if inflation is about disequilibrium.

What's Missing from Fed Models?(2/5)

- Prices refer to Transactions. Where are they?
  - Quantity of Transactions missing from Fed models.
  - Require money, either at purchase or when credit comes due.
  - The ratio of money to transactions must affect price level.
- Fed injects money every Christmas because of this.
- If it matters at Christmas, why not every other day?

What's Missing from Fed Models? (3/5)

- Agent based accounting of money transactions missing from Fed models.
- Key idea in post Covid inflation was transition from services to goods sector. Need to keep track of who is buying what. (e.g. Straub)
- Key Covid idea is supply chains. Where were they in the Fed models?
  - My little hedge fund Ellington can follow mortgage decisions of almost every household. Why can't Fed follow who has the money and how many transactions they make?
- Impossible to put all those details into a conventional infinite horizon optimizing model.

What's Missing from Fed Models? (4/5)

- Fiat money has disappeared from many macro models. Quantity of money plays no role.
- Expectations about long run quantity of money is missing.

What's Missing from Fed Models? (5/5)

- New money like credit cards, money market charging, crypto. Do they matter?
- What is significance for inflation of Fed paying interest on deposits?

Radical Proposal

- Agent based model of transactions and money in the American economy in GE.
- Finite Horizon makes it tractable. How to give fiat money value in finite horizon. Gains to Trade.
- Different kinds of money like credit cards should be modeled too.
- Some agents must be aware of how long fiscal or monetary intervention will persist. Optimization.

What the Model Also Delivers

- Why money has value: enables gains to trade.
  - Define Gains to Trade.
- Main cause of Inflation: Fiscal deficits financed by printed money = Helicopter Money.
  - Like Biden-Trump Covid relief package.
  - Doesn’t depend on unemployment or vacancies.
  - Worst inflation when not much gains to trade left.
  - In extreme case get Hyperinflation.
- Short Term Money Injections Give Little Inflation
  - Like 2000s Japan, 2010 US Quantitative Easing
  - Eventually get Liquidity Trap
- New moneys can create Inflation.

Credit Card Interlude

- Credit Cards not studied in GE or monetary theory. Never in grad classes.
- In 1950s - 1970s monetarists talked about near monies. But disappeared.
- Apparently nobody thinks quantity of money or credit cards matter.

- 1970s and 1980s $\approx$ 100% Inflation
  - – Not caused by Oil and Opec
  - – Or by Anchovies.
    - Yesterday’s supply side disruptions
  - – Big Mystery still today.

Credit Cards Application

- 1970s and 1980s $\approx$ 100% Inflation
  - – Not caused by Oil and Opec
  - – Or by Anchovies
    - Yesterday's supply side disruptions
  - – Big Mystery still today.
- It coincided with invention and spread of credit cards!
- Theoretically credit cards should produce $\approx$ 100% inflation because money used twice as often per period.

The Model

Central Bank Policy

- M = central bank spending on bonds (pure monetary policy)
- m = fiscal transfers to agents or government spending (fiscal policy) via printed money.
- B = central bank sales of bonds
- r = central bank interest rates for borrowers or now depositors.
- In any government bond market n, bank will set one of M,B,r.
- Set r $\rightarrow \rightarrow$ money supply endogenous.

One Period Warm Up

One Period GE Economy

- Commodities $\ell \in L$ arbitrary number
- Agents $h \in H$ arbitrary number
- Utilities $u^h: R_+^L \to R \;\; \forall h \in H$ heterogeneous
  - continuous, concave, and strictly monotonic utilities
- Endowments $e^h \in R_{++}^L \;\; \forall h \in H$
  - Arbitrary strictly positive endowments

$$E = (u^h, e^h)_{h \in H}$$

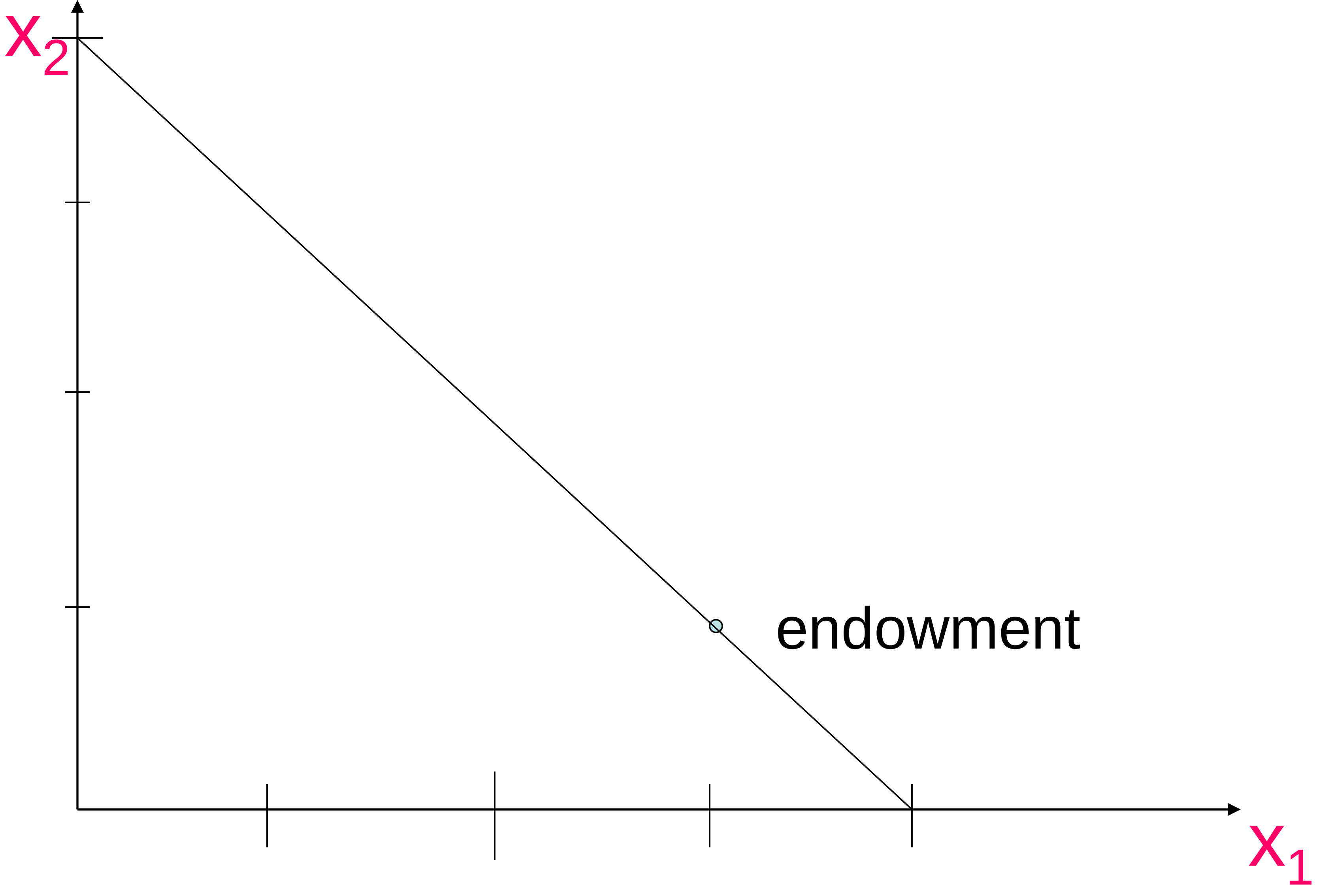
General Equilibrium

$$(p,(x^h)_{h\in H})$$

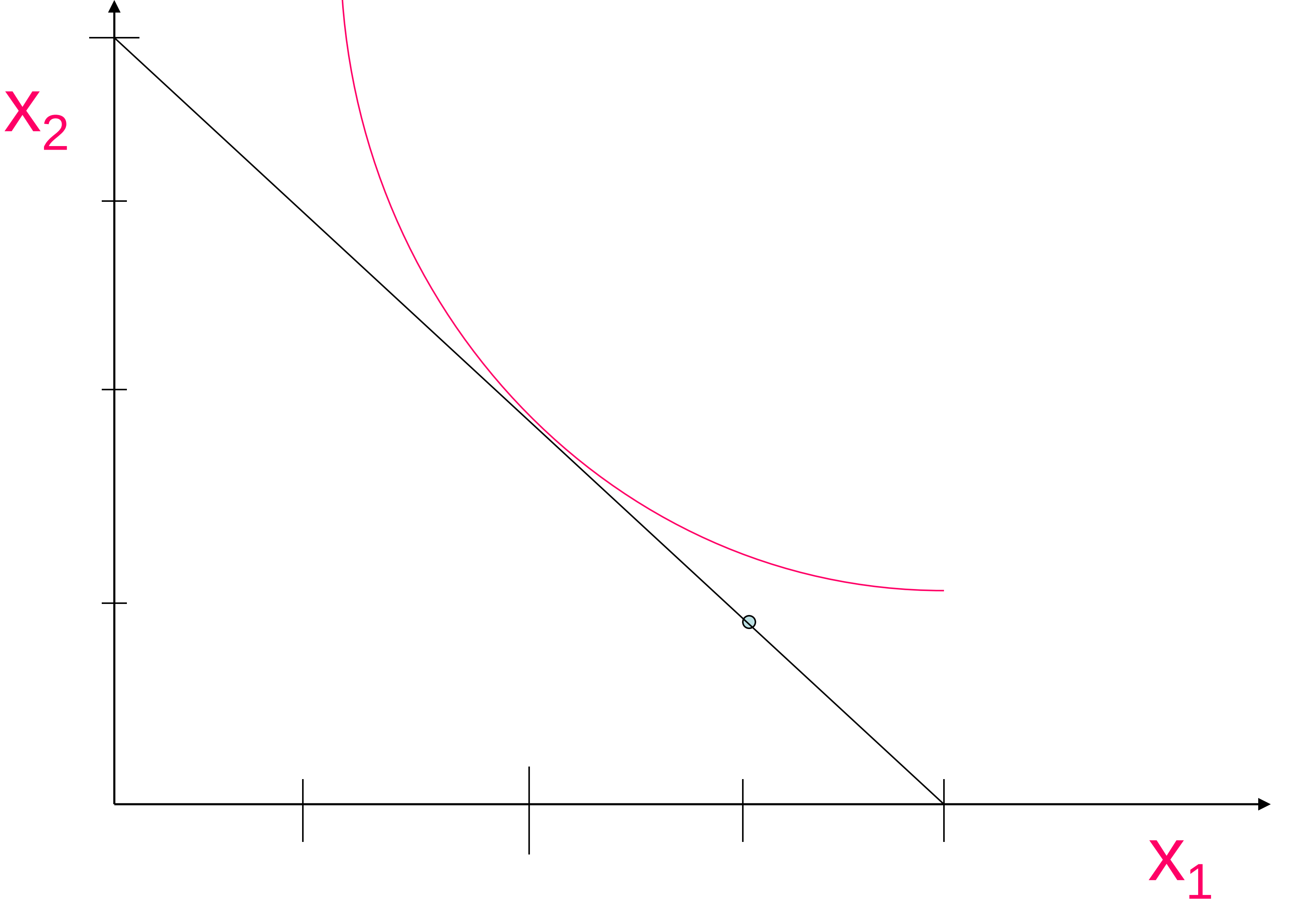
↑ Prices ↑ Final holdings

- $\sum_h(x^h - e^h) = 0$
- $x^h \in B^h(p) = \{x \in R^L_+ : p \cdot (x - e^h) \leq 0\}$
- $x \in B^h(p) \Rightarrow u^h(x) \leq u^h(x^h)\ \forall h$

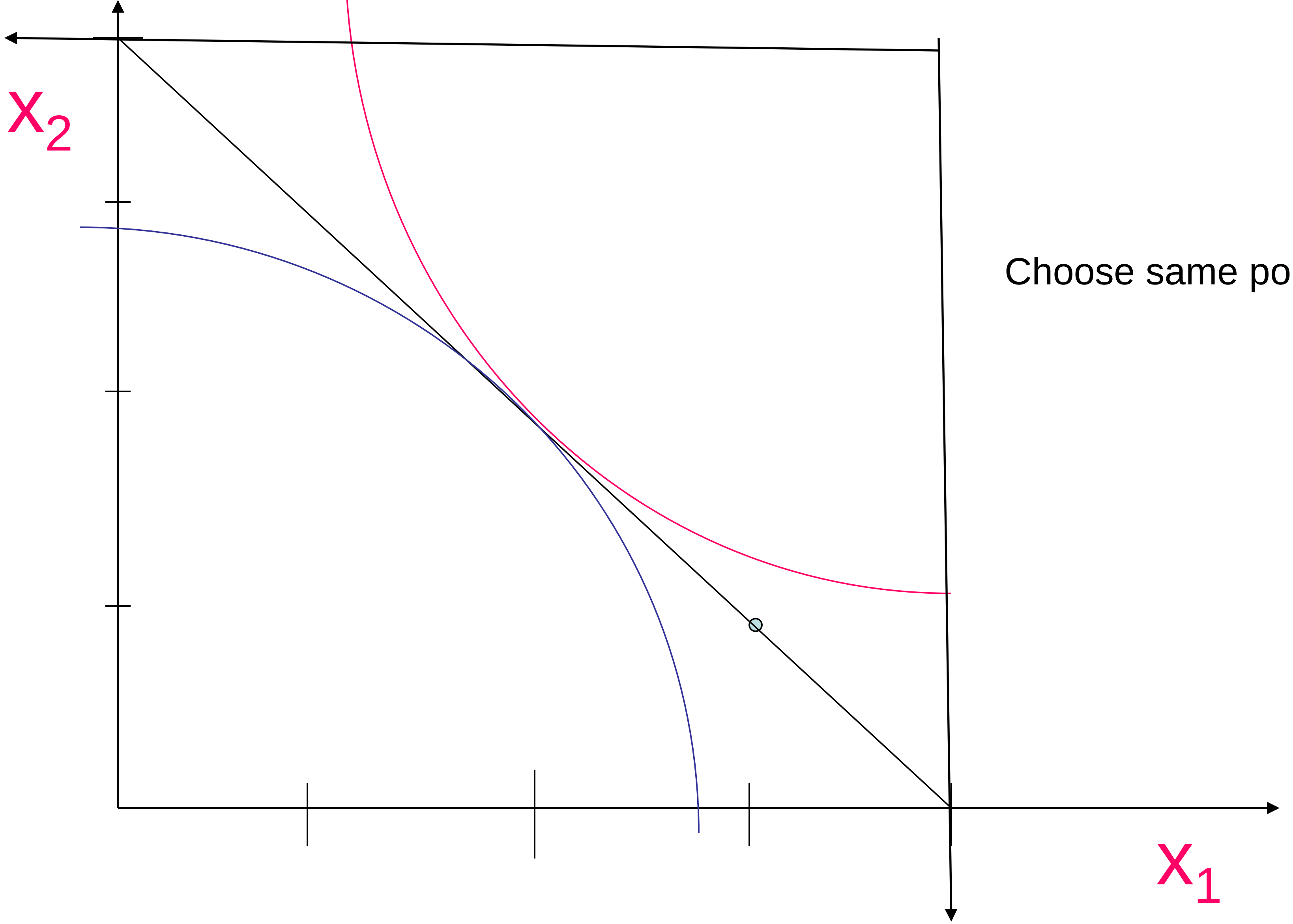
- All markets clear in perfect competition with flexible prices.
- But no theory of price level.



GE budget set if $p_1 = p_2$



Individual Optimization



GE Equilibrium Edgeworth Box for Symmetric Example

GE Symmetric Example

- $u^h(x_1, x_2) = \log x_1 + \log x_2$, $h = 1,2$
- $e^1 = (3,1)$
- $e^2 = (1,3)$
- Equilibrium $(p_1 = p_2, (3-\tau,1+\tau), (1+\tau,3-\tau))$
- $\tau = 1$, $(p_1,p_2) = (p, p)$
- $x^1 = x^2 = (2,2)$

GE Monetary Economy

One Period

No Credit Cards or New Money

One Period Model

- Why would anybody accept money for goods when there is no future?
- Same problem in finite horizon model in last period. By backward induction, money would seem to never have value. Called the Hahn Problem.
- What does Central Bank do?

One Period Model

- How can money have value?
- No money in utility function.
- No taxes paid only in money.
- No infinite horizon.

The monetary economy Exogenous Variables

Outside Money (Fiscal)

$$E = ((u^h, e^h, m^h)_{h \in H}, M)$$ old

Inside Money (Bank)

$$E = ((u^h, e^h, m^h)_{h \in H}, r)$$ new

Central Bank Interest Rate

Dubey-Geanakoplos 1992

GE Model + Fiat Money

- Cash-in-advance required. Model transactions.
- Outside Money: Every agent has an endowment of cash m^h which he owns free and clear, perhaps from fiscal transfers.
- Inside Money: M injected by bank as loan,
- Central Bank: Either lends a fixed amount of money M at endogenous market clearing interest rate r or equivalently it fixes $r > 0$ by lending whatever M the market endogenously demands. (Not borrowing yet.)
  – Collects Debts. Gurley and Shaw, Shubik

Three sub-periods inside one period

- (a) Agents Borrow from Central Bank.
- (b) Agents Trade
- (c) Agents Repay Central bank

  - In one period no reason for any agent to deposit money with central bank.

Three sub-periods inside one period

- (a) Agents Sell Bonds for Money to Central Bank.
- (b) Agents Trade = buy goods with money and simultaneously sell goods for money
  - Revenue from sales of goods too late to spend on buying goods.
  - Result of trade is new holdings of goods, money
- (c) Agents Repay banks

The monetary economy
Exogenous Variables

Outside Money (Fiscal)

$$E = ((u^h, e^h, m^h)_{h \in H}, M)$$ old

Inside Money (Bank)

$$E = ((u^h, e^h, m^h)_{h \in H}, r)$$ new

Central Bank Interest Rate

Endogenous Variables (p,r,q,x)

- Macro: Commodity prices $p \in R_+^L$
- Macro: Interest Rate $r > 0$
- Agent bond sales $q^h_{bm} \in R_+$
  - Bond promises $1 at end of period
- Agent commodity sales $(q^h_{\ell m})_{\ell \epsilon L} \in R_+^L$
- Agent money expenditures on commodities $(q^h_{m\ell})_{\ell \epsilon L}) \in R_+^L$
- Agent commodity consumption $x^h \in R_+^L$
- Who spends how much money on what?

Budget Set $B^h(p,r) = \{(q^h, x^h)$:

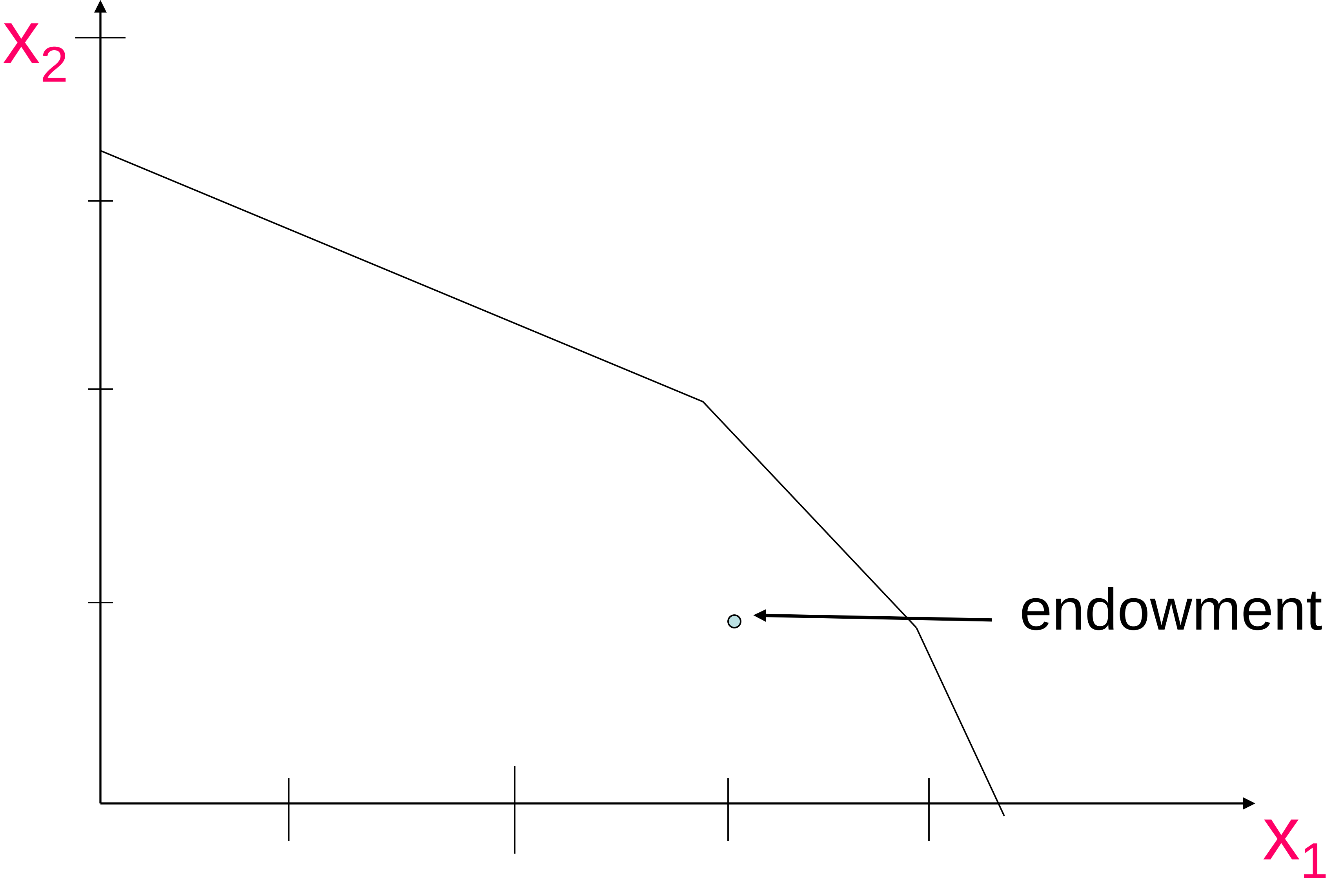
- $\underline{m}^h = q^h_{bm} / (1+r)$ bond trade (1a)
- $\Sigma_\ell q^h_{m\ell} \leq m^h + \underline{m}^h$ goods trade (2b)
- $q^h_{\ell m} \leq e^h_\ell$ goods trade (3ℓb)
- $\underline{x}^h_\ell = q^h_{m\ell} / p_{\ell m}$ (4ℓ)
- $q^h_{bm} \leq \Delta(2) + \Sigma_\ell p_{\ell m} q^h_{\ell m}$ repay (5c)
- $x^h_\ell \leq \Delta(3\ell) + \underline{x}^h_\ell$ (6ℓ)

Interest rates

Equilibrium $(p, r, (q^h, x^h)_{h \in H})$

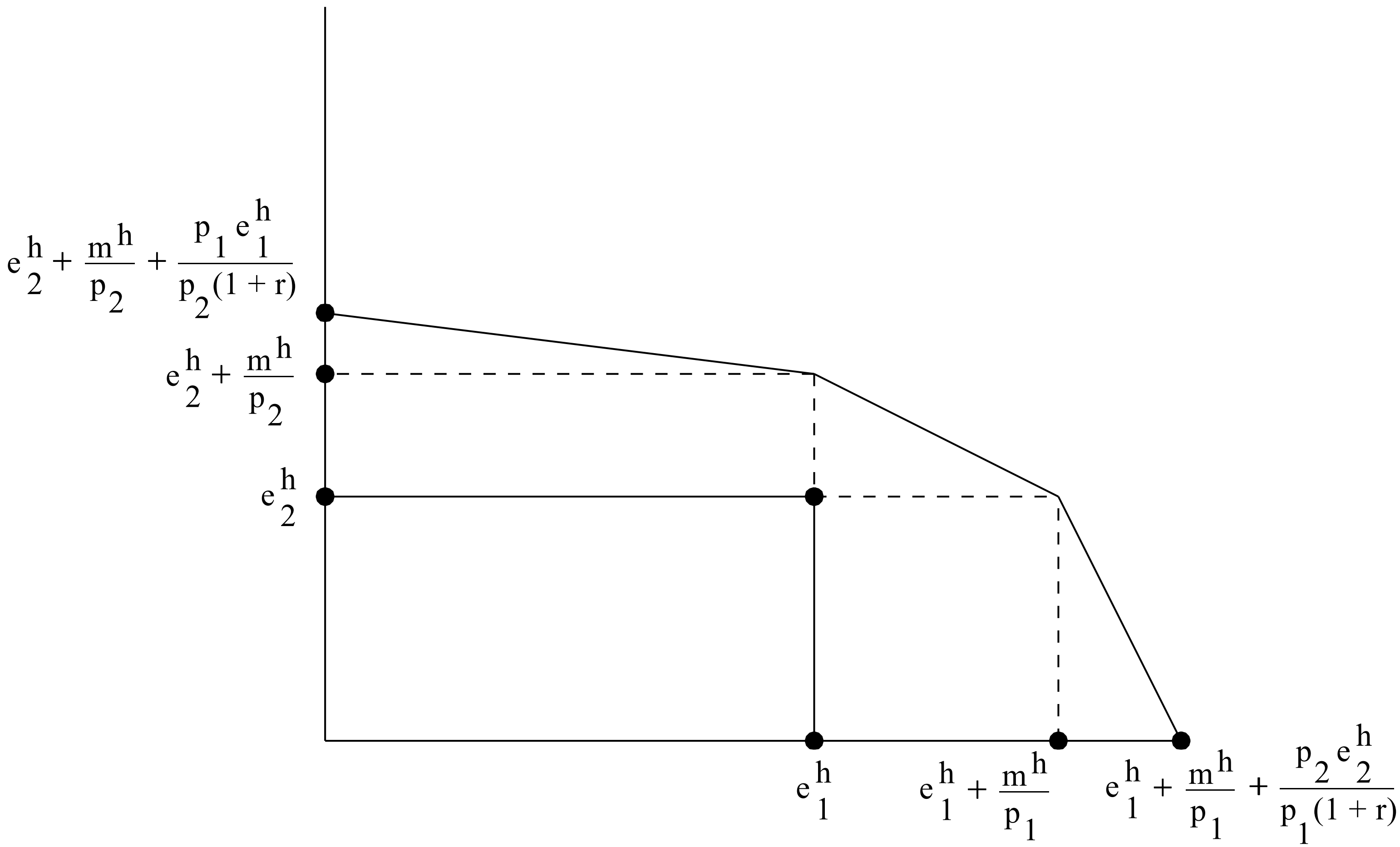
Money spent on goods
Goods sold for money

- $(q^h, x^h) \in B^h(p,r)$
- $u^h(x^h) \geq u^h(x)$ for all $(q, x) \in B^h(p,r)$
- $\Sigma_h \underline{x}^h_\ell = \Sigma_\ell q^h_{\ell m}$ for all $\ell \in L$
- $\Sigma_h \underline{m}^h = M$
- All markets clear in perfect competition with flexible prices. But trade with money.

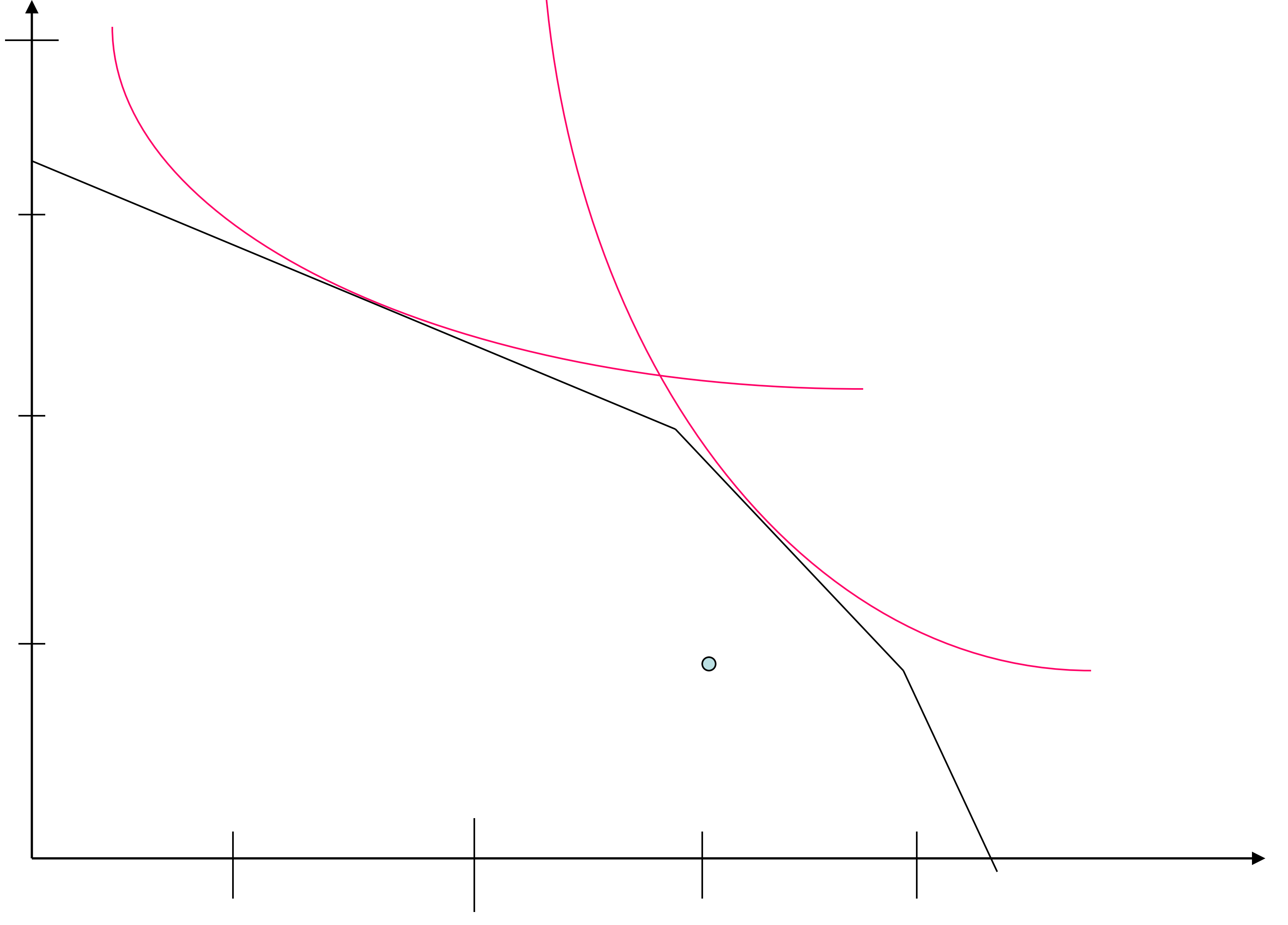


Monetary budget set if $p_1 = p_2$ and $m^h > 0$ and $r>0$

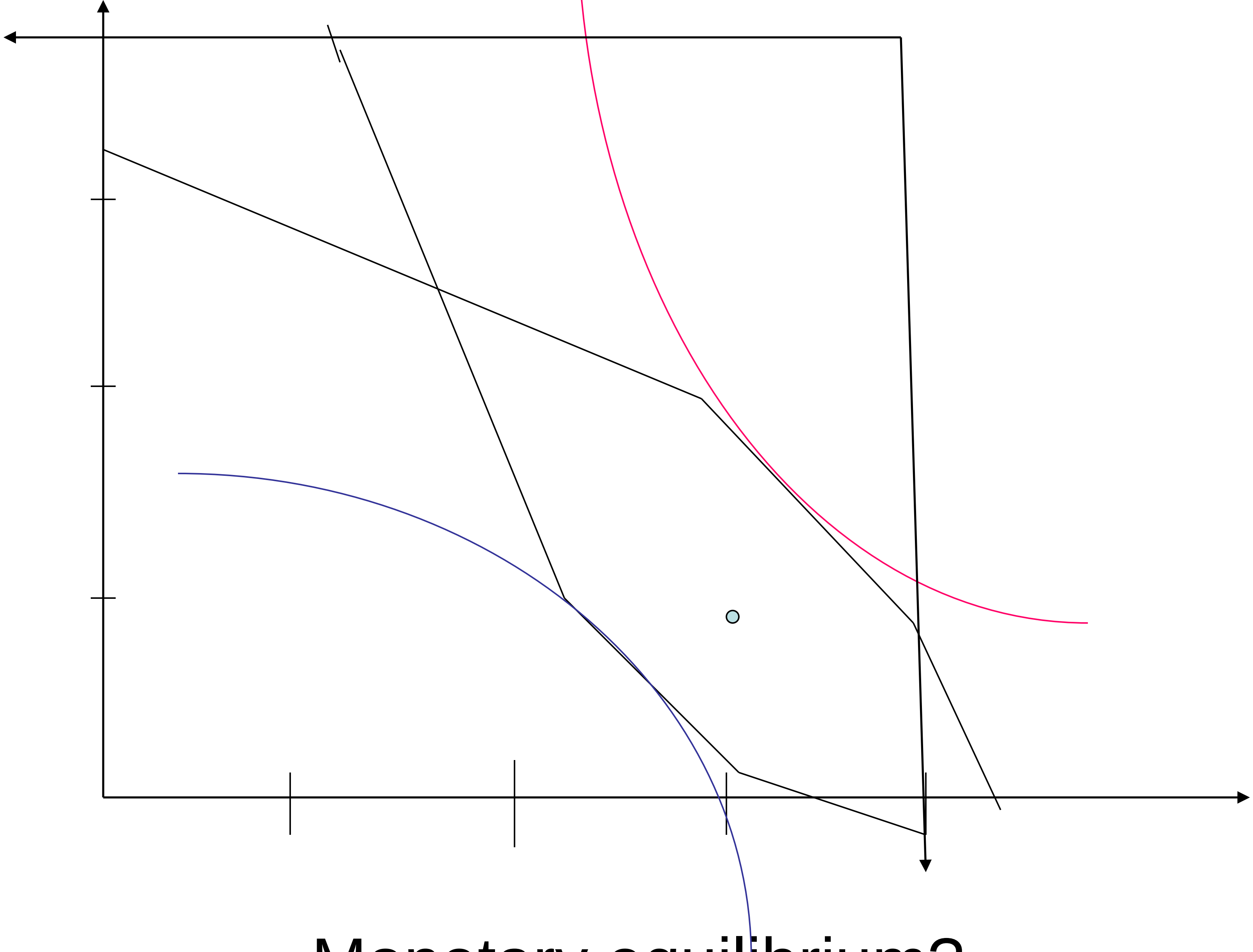
Say $p_1 = p_2 = 1$ and $r = 10\%$



$e_2^h + \frac{m^h}{p_2} + \frac{p_1 e_1^h}{p_2(1+r)}$
$e_2^h + \frac{m^h}{p_2}$
e_2^h
e_1^h
$e_1^h + \frac{m^h}{p_1}$
$e_1^h + \frac{m^h}{p_1} + \frac{p_2 e_2^h}{p_1(1+r)}$



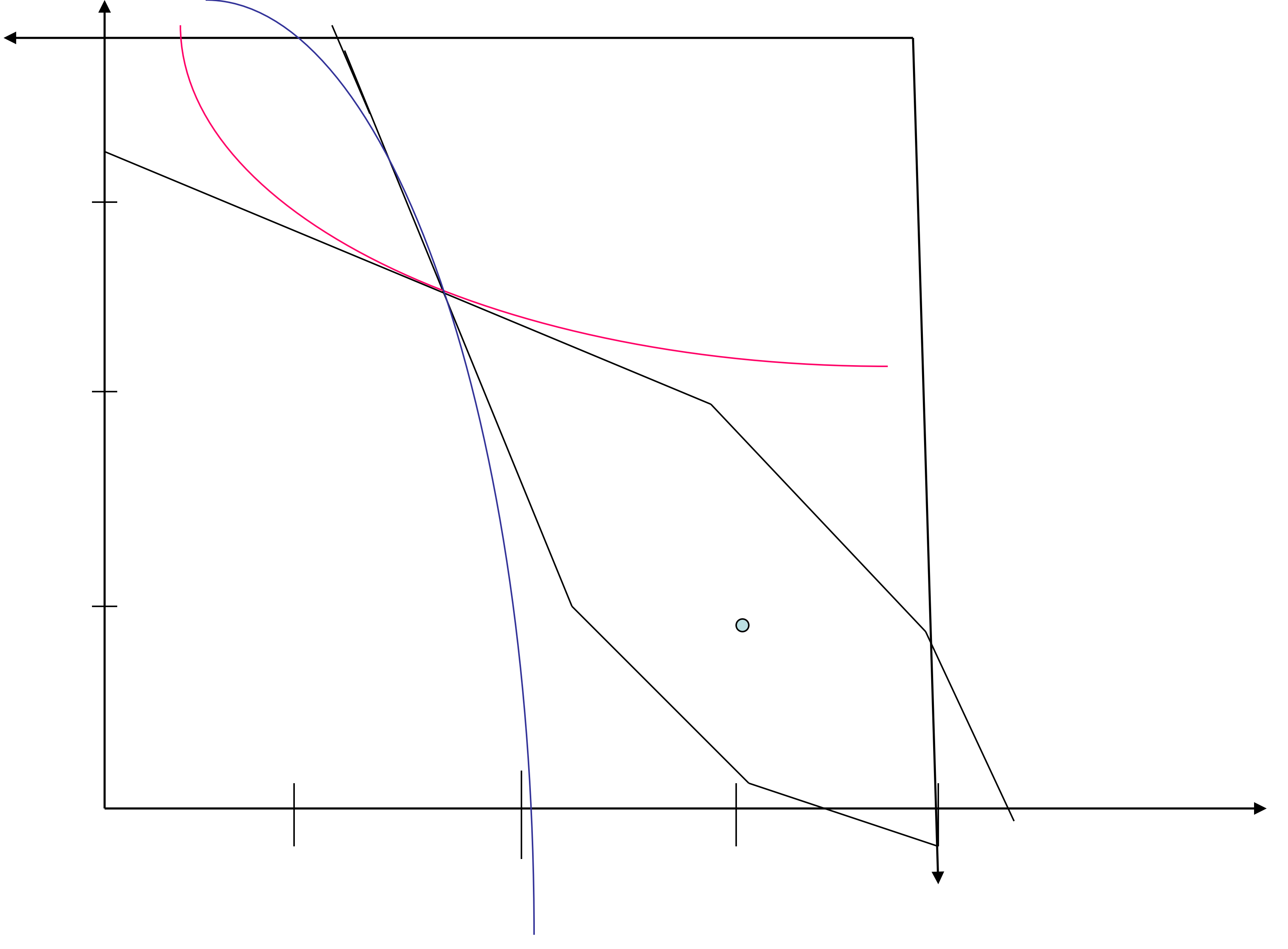
Monetary agent optimization



Monetary equilibrium?

Equilibrium exists? How? (1/2)

- Nobody wants to end up with worthless money in last period.
- But bank is willing to take repayment back at end.
- Prices will rise so some people do not have enough cash to make all the purchases they want. They borrow money, and must repay money + interest to bank. In total, repay inside money M + outside m.
- Bank sucks all money out of system.



Monetary equilibrium! Note marginal conditions
Not Pareto efficient

Monetary Example

- $u^h(x_1, x_2) = \log x_1 + \log x_2$, $h = 1,2$
- $e^1 = (3,1)$
- $e^2 = (1,3)$
- $m^1 = 1$, $m^2 = 1$, $M = 20$

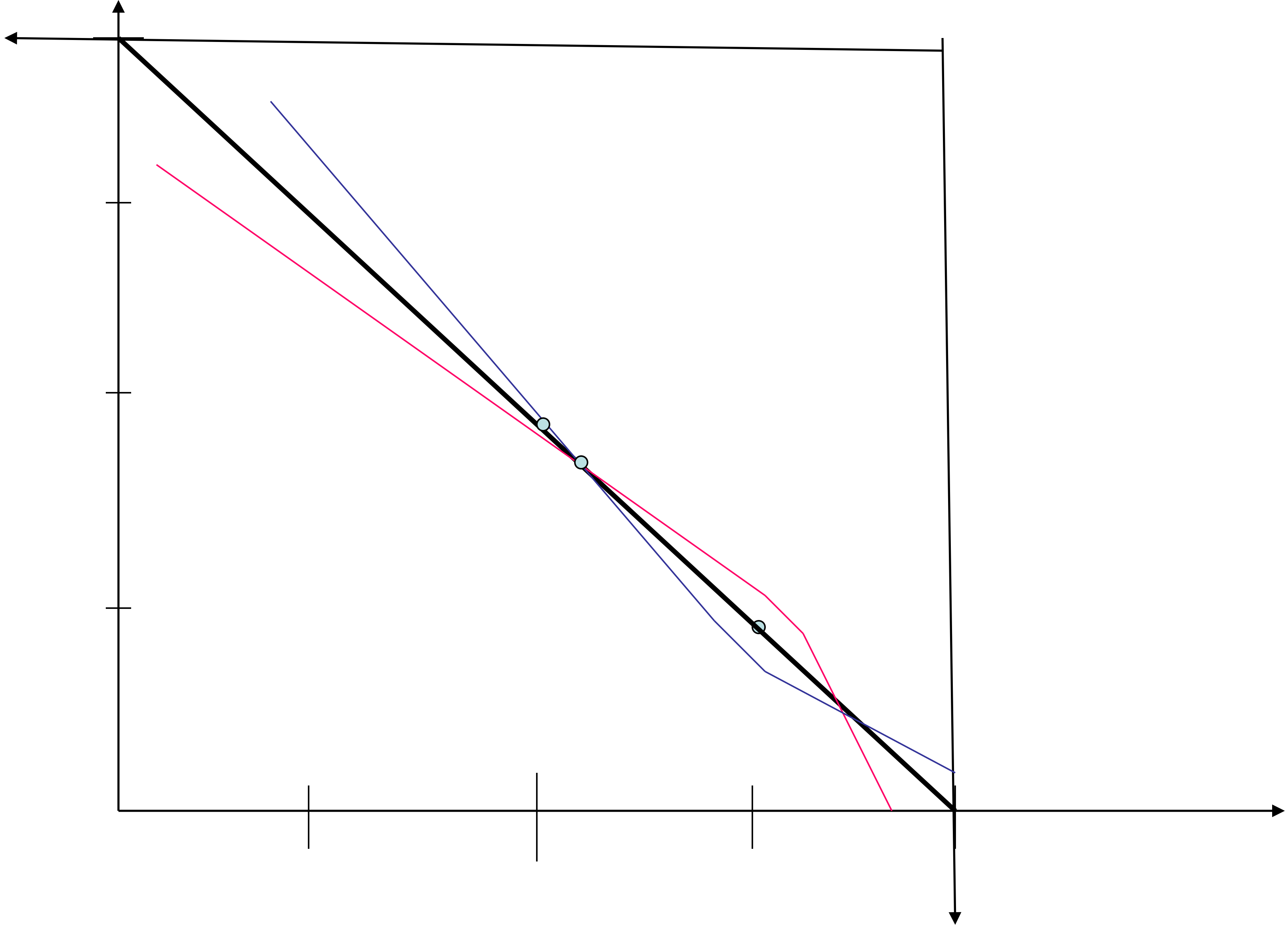
- Equilibrium $(p_1 = p_2, r, (3-\tau,1+\tau), (1+\tau,3-\tau))$
- Equilibrium (12.16, 10%, (2.1,1.9), (1.9, 2.1))

Solving for equilibrium

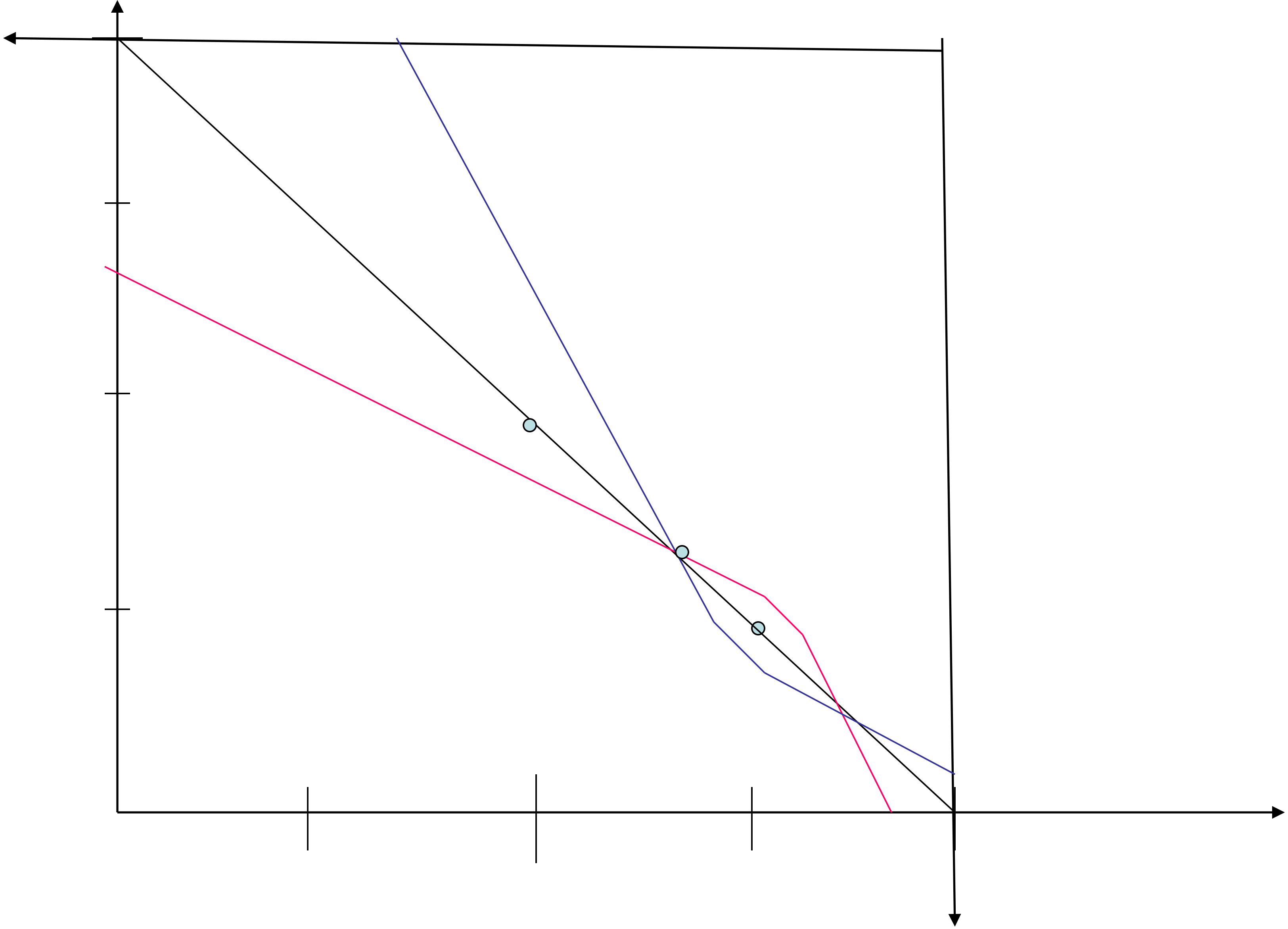
- $x^1 = (3-\tau, 1+\tau) \approx (2.1, 1.9)$; $x^2 \approx (1+\tau, 3-\tau) = (1.9, 2.1)$
- $\tau = .905$, $p = 12.16$, $r = 10\%$
- (a) Agent 1 borrows \$10
- (b) Agent 1 simultaneously
  - Spends \$11 = \$1 + \$10 on good 2 & Sells .905 of good 1
  - Gets 11/12.16 = .905 of good 2, and \$11 from sale good 1
- (c) Repays bank \$11 = \$10(1+r)

- All M + m comes out to bank because r = m/M.

- Get same equilibrium if bank sets r = 10% then endogenously central bank lends \$20 in total.

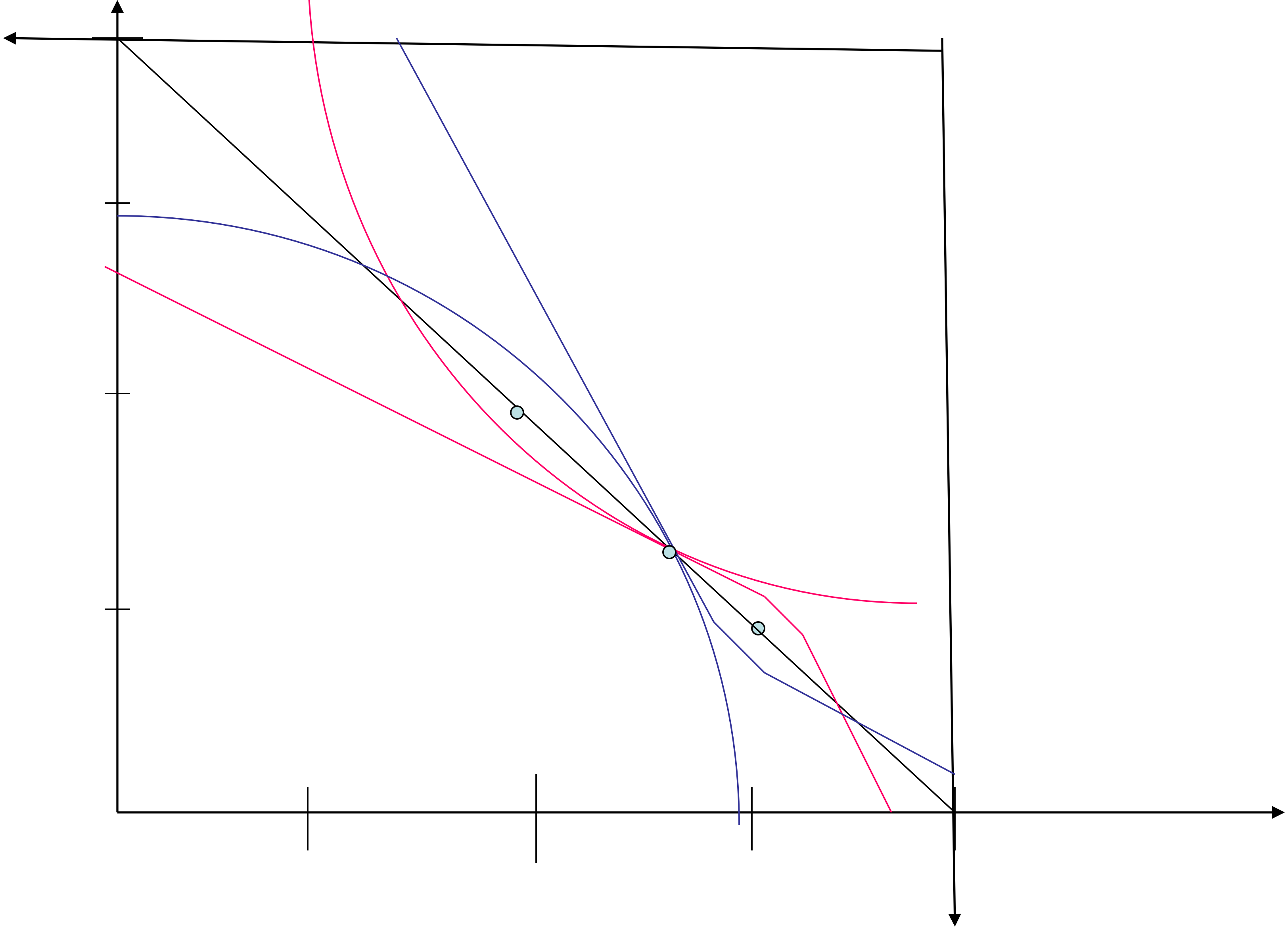


Monetary Equilibrium



Monetary Equilibrium

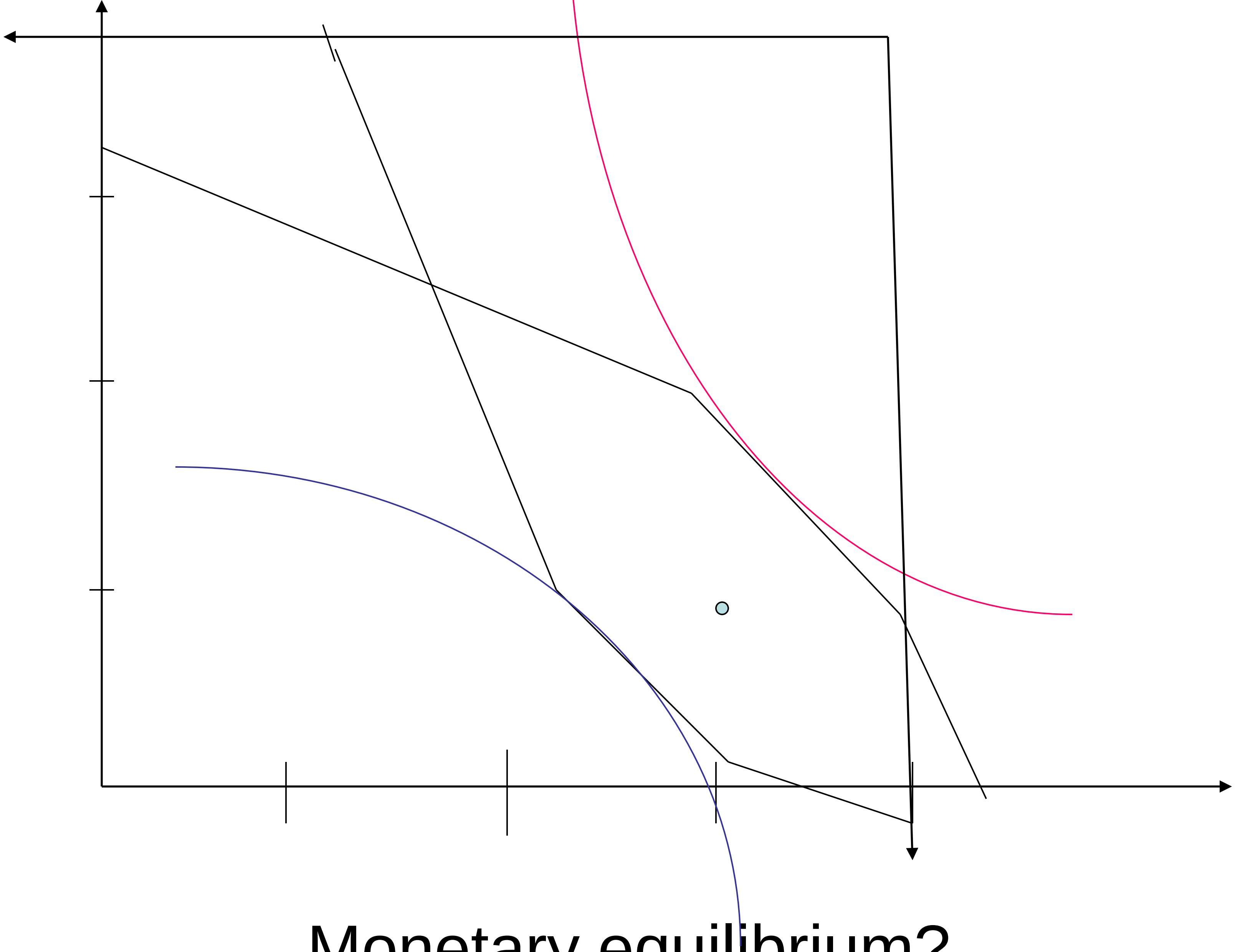
With higher interest rate or smaller M



Monetary Equilibrium

Equilibrium exists? How? (2/2)

- If initial endowments pareto efficient, then equilibrium cannot exist.



Monetary equilibrium?

Equilibrium does exist

- Since $r > 0$ creates trading wedge, must have enough gains to trade to support equilibrium.

- Equilibrium exists as long as enough available gains to trade from initial endowment point $e = (e^h)_{h \in H}$.

- Crucially, equilibrium does not exist if not enough gains to trade.

How to define Distance from Pareto Efficiency

- Debreu 1951 suggested
- Start at allocation $e = (e^h)_{h \in H}$
- Government throws away γ of each good
- Government redistributes what is left and despite waste makes everybody better off.
- $\gamma(x)$ = sup over γ
- But not right definition for gains to trade

Dubey – Geanakoplos 1992

γ-Gains to Trade From $e = (e^h)_{h \in H}$

- Start at allocation $e = (e^h)_{h \in H}$
- Government gathers some goods
- Government throws away $\gamma/(1+\gamma)$ of each gathered good
- Government redistributes what is left and despite waste makes everybody better off.
- $\gamma(x)$ = sup over γ with γ-Gains to Trade

- Throwing away traded goods

Gains to Trade

- Easy to define.
- Easy to compute = ratio of marginal utilities at endowment

Theorem: Existence of Monetary Equilibrium with enough Gains to Trade

- Let economy satisfy usual GE conditions with

  $$\gamma((e^h)_{h \in H}) > r = m/M$$

- Then monetary equilibrium exists.

- Conversely, if $\gamma((e^h)_{h \in H}) \leq r = m/M$

- then equilibrium does not exist if utilities are additively separable.

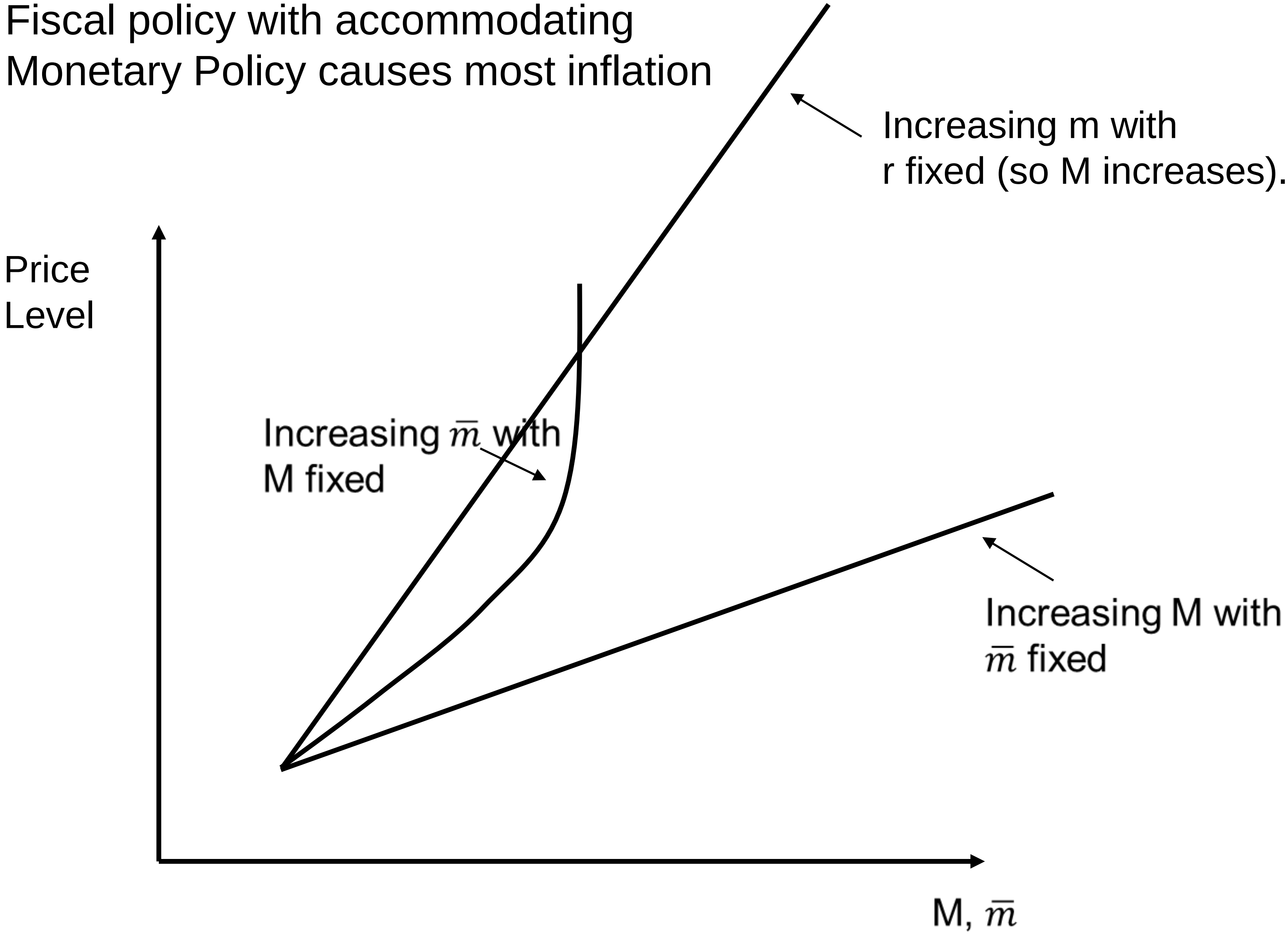
What Model Does

- Given data of m and r fixed by central bank, model predicts price levels and where all cash flows go with heterogeneous agents. Easy to compute.
- Could see effect of bottlenecks on equilibrium price levels with given m,r.
  - Prices up since money chasing fewer trades
  - Also could do supply chains.

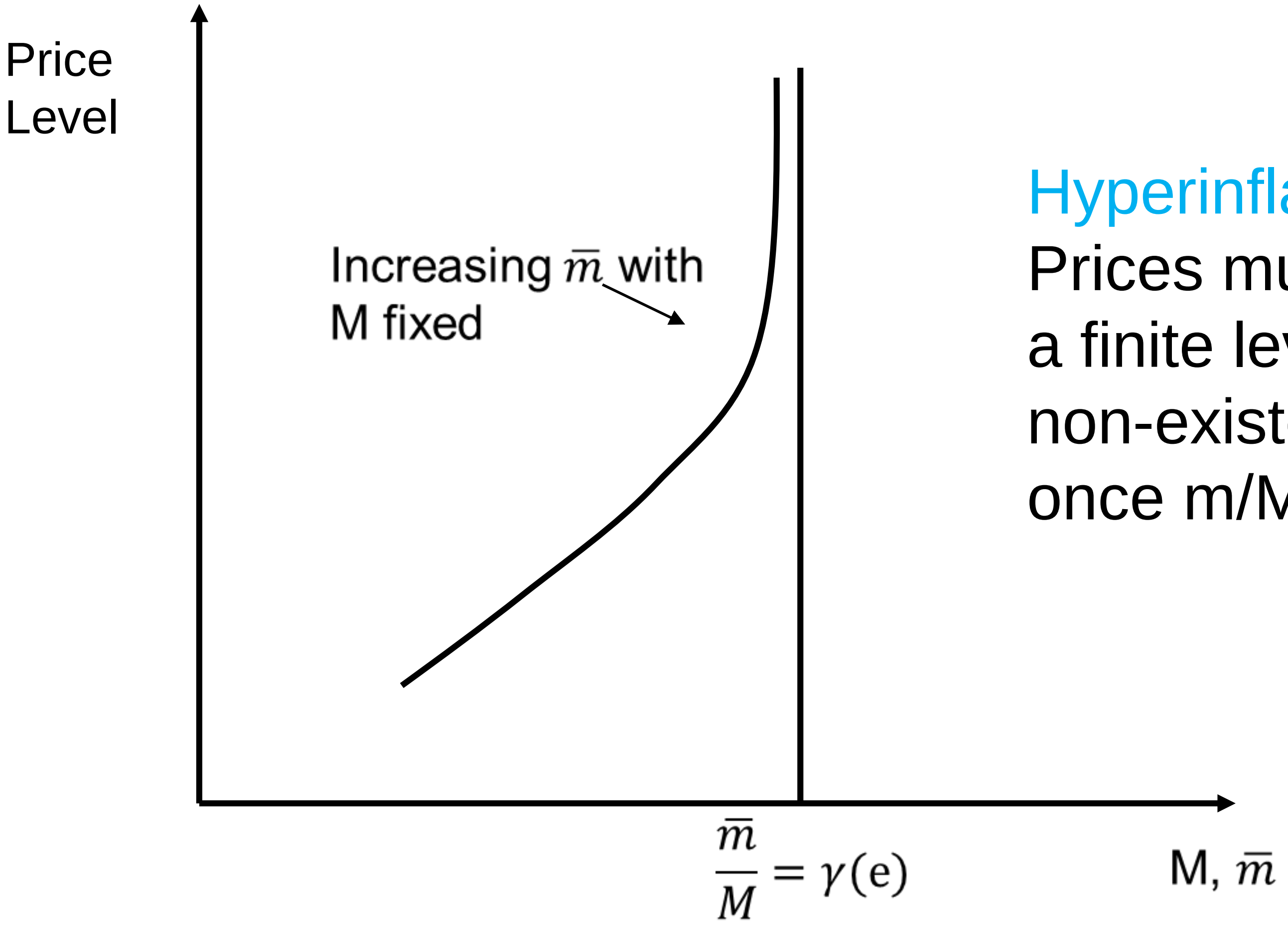
Effects of Monetary-Fiscal Policy

- Central Bank open market operations
- Theorems
  - Raising M by $k with given m raises prices linearly
  - Raising m by $k with r fixed raises prices much faster
  - Raising m by $k with M fixed raises prices, always leading eventually to hyperinflation.

Fiscal policy with accommodating Monetary Policy causes most inflation



Price levels eventually go up much faster in m
Hyperinflation with too much m



Hyperinflation Theorem

Prices must go to infinity at a finite level of m by our non-existence theorem, once m/M gets above γ(e).

Hyperinflation Theorem

- Fix a monetary economy
- $E = ((u^h, e^h, m^h)_{h \in H}, M)$.

- As m^h grows larger, at a finite level, once m/M approaches $\gamma(e)$, prices must go to infinity.

- Proof:
- By our non-existence theorem.

Hyperinflations

- Government spending too much by printing money
- Central Bank worried about inflation does not increase M enough.

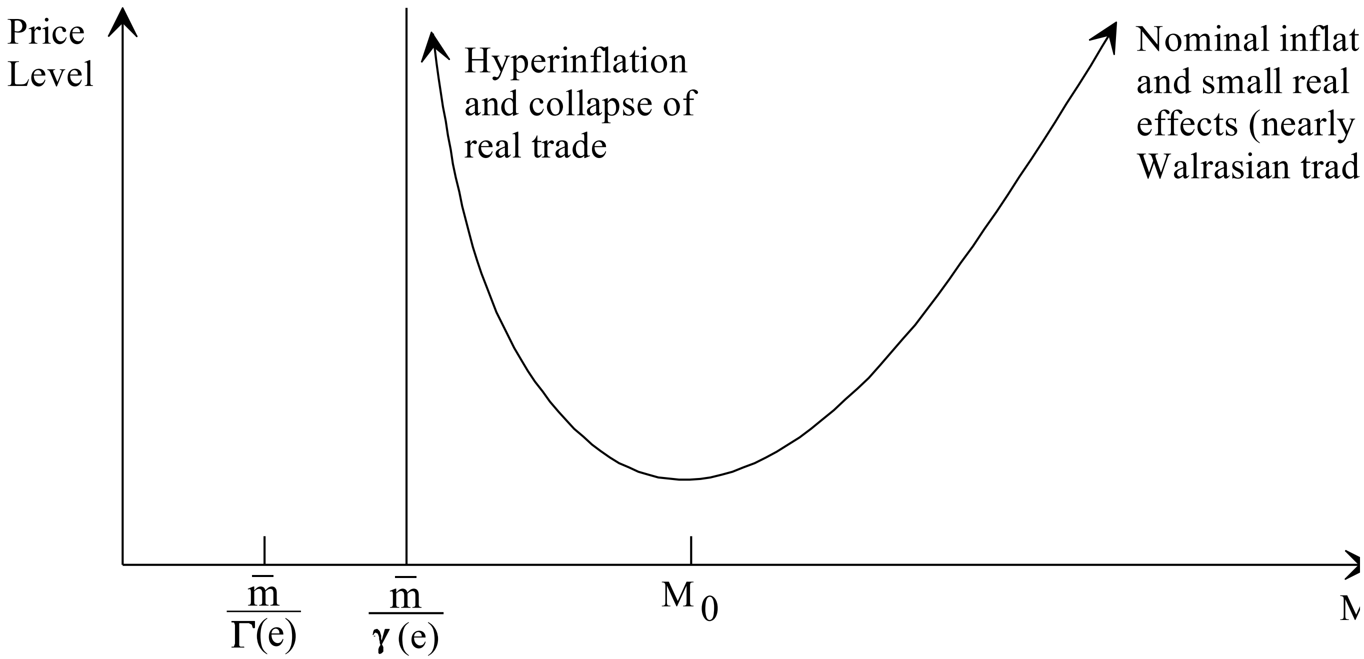


Figure 6. ❍ Fixed

Credit Cards

The credit card economy same as Monetary Economy

$$E = ((u^h, e^h, m^h)_{h \in H}, M)$$

or

$$E = ((u^h, e^h, m^h)_{h \in H}, r)$$

Except that now can pay by credit card.
Credit card is promise to pay later.

Four sub-periods inside one period

- (a) Agents Borrow = Sell bonds to bank
- (b) Agents Trade = buy goods with money and/or credit cards and simultaneously sell goods for money and/or against credit cards
- (c) Agents Repay credit card debt
- (d) Agents Repay banks

Do Credit Cards drive out Money?

- Why pay cash when you can pay by credit card?
- Can money co-exist?
- Is equilibrium existence compromised?
- What happens to price levels?

Credit Card Prices

- People will not sell goods against credit card if they can get same price in cash, because then they have to wait for the cash (plus run a risk of default).
- In a one period model they will charge a higher credit card price.
- In a real multi-period world, credit card price can be the same, but people who pay late are charged interest, which makes up for the interest lost to people who pay on time, plus the defaults.

Endogenous Variables

- Macro: Cash Commodity prices $p_{\ell m}$, $p(m) \in R_+^L$
- Macro: Credit card good prices $p_{\ell c}$, $p(c) \in R_+^L$
- Macro: Interest Rate $r > 0$
- Agent bond sales $q^h_{bm} \in R_+$
- Agent cash commodity sales $(q^h_{\ell m})_{\ell \in L} \in R_+^L$
- Agent credit commodity sales $(q^h_{\ell c})_{\ell \in L} \in R_+^L$
- Agent money expenditures $(q^h_{m\ell})_{\ell \in L}) \in R_+^L$
- Agent credit expenditures $(q^h_{c\ell})_{\ell \in L}) \in R_+^L$
- Agent consumption bundles $x^h \in R_+^L$

Budget Set $B^h(p,r)$

- $\underline{m}^h = q^h_{bm}/(1+r)$ (1a)
- $\Sigma_\ell q^h_{m\ell} \le m^h + \underline{m}^h$ (2b)
- $q^h_{\ell m} + q^h_{\ell c} \le e^h_\ell$ (3ℓb)
- $\underline{x}^h_\ell(m) = q^h_{m\ell}/p_{\ell m}$; $\underline{x}^h_\ell(c) = q^h_{c\ell}/p_{\ell c}$ (4ℓ)
- $\Sigma_\ell q^h_{c\ell} \le \Delta(2) + \Sigma_\ell p_{\ell m} q^h_{\ell m}$ (5c)
- $q^h_{bm} \le \Delta(5) + \Sigma_\ell p_{\ell c} q^h_{\ell c}$ (6d)
- $x^h_\ell \le \Delta(3\ell) + \underline{x}^h_\ell(m) + \underline{x}^h_\ell(c)$ (7ℓ)

Equilibrium $(p,r,(q^h,x^h)_{h\in H})$

- $(q^h,x^h) \in B^h(p,r)$
- $u^h(x^h) \geq u^h(x)$ for all $(q, x) \in B^h(p,r)$
- $\Sigma_h \underline{x}^h_\ell(m) = \Sigma_\ell q^h_{\ell m}$ for all $\ell \in L$
- $\Sigma_h \underline{x}^h_\ell(c) = \Sigma_\ell q^h_{\ell c}$ for all $\ell \in L$
- $\Sigma_h \underline{m}^h = M$

Credit Cards Increase Velocity

- Credit cards allow money to do more work
- Credit card payments can be processed in the dead of night.
- The same dollar can be used by A to repay his credit card debt to B, and then used again by B to pay his bank loan.
- Since "velocity" is doubled, should expect prices to double.

Four sub-periods inside one period

- (a) Agents Borrow
- (b) Agents Trade
- (c) Agents Repay credit card debt
- (d) Agents Repay banks

Credit Cards Improve Trading Efficiency

- WLOG $p_{m\ell} = 1$ for all goods.
- Let $p_{c\ell} = (1+r)^{1/2} p_{m\ell}$ Credit card Premium

- Agent can buy good i for p_{ci} with credit card then pay off by selling $(1+r)^{1/2}$ units of good j for cash. This is better than borrowing $\$1 = p_{mi}$ to buy good with cash, then selling $1+r$ of j for cash to repay bank.
- If borrow $p_{mi}=\$1$, repay by selling $(1+r)^{1/2}$ units of good j on credit card market for p_{cj}. Since $p_{cj}(1+r)^{1/2} = (1+r)^{1/2}(1+r)^{1/2} = 1+r$, bank is repaid.
- Wedge = $(1+r)^{1/2} < 1+r$.

Credit Card Example

- $u^h(x_1, x_2) = \log x_1 + \log x_2$, $h = 1,2$
- $e^1 = (3,1)$
- $e^2 = (1,3)$
- $m^1 = 1$, $m^2 = 1$, $M = 20$

- Equilibrium $((p_{1m} = p_{2m}), (p_{1c} = p_{2c}), r, (3-\tau_m-\tau_c, 1+\tau_m+\tau_c), (1+\tau_m+\tau_c, 3-\tau_m-\tau_c))$

Solving for equilibrium

- Credit card equilibrium exists for M > 1/4
- M = 20, m = 2
- $\tau_m = .49$, $\tau_c = .46$ $\tau_m + \tau_c = .95$
- $p_{1m} = 22.56$, $p_{1c} = 23.66$ r = 10%

- Recall pure monetary equilibrium:
- $\tau = .905$, p = 12.16, r = 10%; ME if M > 1
- Credit cards nearly double prices
- Improve efficiency
- Give money value more generally

Credit Cards

- Improve viability of money (equilibrium exists more often, i.e. with lower M)
- Improve trading efficiency
- Nearly double price levels

- 1970s-80s total inflation was about 100%.

Credit Card Existence Theorem

- If monetary economy satisfies usual conditions and $1+\gamma(e) > (1+m/M)^{1/2}$, then equilibrium exists.

- So credit card equilibrium exists even when monetary equilibrium does not. Credit cards do not drive money out, they increase the viability of money.

Credit Card Efficiency Theorem

- If economy satisfies usual smoothness conditions, then at final equilibrium allocation x, $\gamma(e) = (1+m/M)^{1/2} - 1 < m/M$.

- So credit card equilibrium is more efficient than monetary equilibrium.

Credit Card Inflation Theorem

- Suppose we begin with a credit card economy with money M. Then the credit card equilibrium is identical to the pure monetary equilibrium of the economy with no credit cards but more money
- $M^* = M + (M^2 + mM)^{1/2}$

- So credit cards, like money, cause inflation.

Monetary Reaction Touch of Stagflation Theorem

- Let an economy with money supply M have a pure monetary equilibrium. Suppose credit cards are introduced; prices rise and trading efficiency improves.
- The monetary authority can restore the old cash prices by shrinking the money supply. But it will get the old consumption, wasting the innovation.
- And higher credit card prices.

Example of Monetary Reaction

- Suppose Fed shrinks $M = 20$ to $M = 9.5$?
- Now $(1 + 2/M)^{1/2} = (1+2/9.5)^{1/2} = 1.1$, so

- $\tau_m+\tau_c = .905$
- $p_{1m} = 12.16$ but $p_{1c} = 12.77$
- So can restore old cash prices, but get back old output. Innovation wasted. And get back credit card prices 5% higher than old cash prices.
- A touch of stagflation!

Credit Card Default

- Suppose now that a certain amount of goods are purchased by credit card holders (Mr. Hydes) who default completely.
- What happens?
- Need new variable K to give expected delivery rate.

Credit Card Premium

- Credit card price premium p_c/p_m must rise to compensate defaults. Honest credit card buyers are being robbed by defaulters.
- $p_c/p_m = [(1+r)/K]^{1/2}$

- Credit card price does not rise with $1/K$.
- Kp_c/p_m goes down as K goes down.

Credit Card Default Inflation

- Price level goes even higher.
- Why?
- First because total trade goes down
- Second, because fraction of goods sold against money goes down.
- Why? Higher proportion of goods sold for credit cards! Buy cash-sell credit wedge gets worse so $(1+r)p_m/Kp_c$ goes up, so t_c/t_m up.

Credit Card Default Utility

- As number of defaulters goes up, credit card equilibrium utility for honest Dr. Jeckylls goes down.
- Once there is enough default, the Jeckyll utility goes below what it was in the monetary economy without credit cards.
- Surprise!
- Jeckyll can’t do old money trades because with higher prices his m worth less.

Stagflation

- Now when monetary authority reacts, it can cause output reduction before inflation reduced to old levels. Real stagflation!

Example: Jekyll and Hyde

M	20	20	20	16	12	9
th	0	0	0.03	0.03	0.03	0.03
tm	0.905	0.487597	0.470613	0.467141	0.461369	0.453699
pm	12.16	22.5596	23.37379	19.26612	15.17224	12.12257
tc	0	0.464901	0.463961	0.455679	0.442407	0.425661
pc		23.6609	25.34788	21.14269	16.97349	13.90079
K		1	0.935339	0.934164	0.932189	0.929521
tm+tc-th	0.905	0.952498	0.904574	0.89282	0.873776	0.84936
tm+tc	0.905	0.952498	0.934574	0.92282	0.903776	0.87936
J-utility	1.384035562	1.38573	1.369595	1.369079	1.368093	1.366558

With 6.5% default, credit card equilibrium has double the prices and lower utility than monetary equilibrium.
Contracting M restores old prices at even lower utility.

Stagflation

- So get even less output, and much less utility, and still much higher prices.

- Is that what Fed engineered in 1970s and 1980s?

Multi-period Economies

- Add many periods
- Simpler budget set. All payments simultaneous each period.
  - No artificial periods where only credit repayments made, only banks paid.
  - No sub-periods.
- Agents can deposit with central bank.
  - So not all money is spent at each period.

More kinds of money

- Cash for goods.
- Credit Card purchases of goods.
  - Can pay at t with promise due at any time $n>t$.
- Money market purchases of goods.
  - If own bonds, can pay with them directly for goods.

Multi-Period Questions

- Does monetary equilibrium still exist?
  - How does money survive these alternatives?
  - If can pay by bonds, why need money?
- If huge bank money M injections, or r reductions, come early on, with known end date, what happens to prices? Like Quantitative Easing.
  - Get Liquidity Trap.
- If new money like credit card invented in the middle of time horizon, what do prices do?
  - Inflating forever?
  - If pay back credit and buy goods, still higher prices?
- After Credit Card invention, does new ‘money market’ money cause more inflation?

Multi-Period Monetary Economy

$$(T, (u^h, e^h, f^h, m^h)_{h \in H}, (r, M, B))$$

$u^h : \mathbb{R}_+^{LT} \rightarrow \mathbb{R}$
$e^h \in \mathbb{R}_{++}^{LT}$
$m^h \in \mathbb{R}_+^{T}$
$f_t^h : \mathbb{R}_+^{L} \rightarrow \mathbb{R}_+^{L}$

f_t is a durability function taking goods x that are used at time t-1 into what is left of them at time t.

Notation: Instruments, Flows, Stocks

m	$=$	money
n	$=$	bond promising \$1 at time n
ℓ	$=$	commodity ℓ
$q_{t\alpha\beta}$	$=$	quantity of holdings of instrument α traded against β at time t
b_{tnm}	$=$	quantity of new (primary bonds) promises of n traded against money at t
$b_{tn\ell}$	$=$	quantity of new (primary bonds) promises of n traded against ℓ at t
μ_t	$=$	stock of money held at time t
β_{tn}	$=$	stock of bonds n held at time t
$\bar{b}_{tn}$	$=$	stock of accumulated primary promises of bond n made before or at t

Trade money, commodities, bonds pairwise.

Trading commodities pairwise or commodities vs future commodities destroys money.

Budget Set

$$\sum_{\ell} q_{tm\ell} + \sum_{n>t} q_{tmn} + \bar{b}_{t-1,t} \leq \mu_{t-1} + m_t^h$$

$$q_{t\ell m} + \sum_{n>t} q_{t\ell n} \leq f_{t\ell}^h(x_{t-1}) + e_{t\ell}^h \text{ for all } \ell$$

$$q_{tnm} + \sum_{\ell} q_{tn\ell} \leq \beta_{t-1,n} \text{ for all } n > t$$

$$\mu_t = \Delta(1) + \beta_{t-1,t} + \sum_{\ell} p_{t\ell m} q_{t\ell m} + \sum_{n>t} p_{tnm} q_{tnm} + \sum_{n>t} p_{tnm} b_{tnm}$$

$$x_{t\ell} = \Delta(2) + \frac{q_{tm\ell}}{p_{t\ell m}} + \sum_{n>t} \frac{q_{tn\ell}}{p_{tn\ell}} + \sum_{n>t} \frac{b_{tn\ell}}{p_{tn\ell}} \text{ for all } \ell$$

$$\beta_{t,n} = \Delta(3) + \frac{q_{tmn}}{p_{tmn}} + \sum_{\ell} p_{t\ell n} q_{t\ell n} \text{ for all } n > t;\ 0 \text{ for all } n \leq t$$

$$\bar{b}_{t,n} = \bar{b}_{t-1,n} + b_{tmn} + \sum_{\ell} b_{tn\ell} \text{ for all } n > t;\ 0 \text{ for all } n \leq t$$

Monetary Equilibrium

$$(((q^h, b^h, \mu^h, x^h, \beta^h, \bar{b}^h)_{h \in H}, p), (r, M, B))$$

$$p_{tnm} = \frac{1}{1 + r_{tn}}$$

$$\sum_h \frac{q_{tn\ell}}{p_{t\ell m}} = \sum_h q_{t\ell m}$$

$$\sum_h \frac{q_{tn\ell}}{p_{t\ell n}} + \sum_h \frac{b_{tn\ell}}{p_{t\ell n}} = \sum_h q_{t\ell n}$$

$$\sum_h \frac{q_{tmn}}{p_{tnm}} + \frac{M_{tn}}{p_{tnm}} = \sum_h q_{tnm} + \sum_h b_{tnm} + B_{tnm}$$

$$(q^h, b^h, \mu^h, x^h, \beta^h, \bar{b}^h) \in B^h(p, r)$$

$$(q, b, \mu, x, \beta, \bar{b}) \in B^h(p, r) \Rightarrow u^h(x^h) \geq u^h(x)$$

Existence Theorem

Theorem: Denote the time periods $0,1,...,T,T+1$. Suppose that in period 0, agents can only trade bonds for money with the bank; in period $T+1$ they only repay the bank. Suppose that in every period $t \leq T-2$, the central bank makes available a three period loan/deposit with positive interest rate $r_{t,t+3}$ less than the autarkic gains to trade at time $t+1$. Then no matter what the rest of the policy (r,M,B), so long as r does not allow arbitrage, monetary equilibrium exists.

Liquidity Trap

- Suppose central bank announces M increases for bonds due before time k.
- Agents anticipate that M will go back down to old levels after k.
- Like QE.
- Then price levels will barely rise as M gets higher and higher.

Liquidity Trap Theorem

Theorem: Fix a credit card economy satisfying the assumptions of the existence theorem. Fix a collection of bonds t_n, with $t < n < k < T-2$. Let $M_{tn} \to \infty$ or $r_{tn} \to 0$, for just these markets, holding all other bank policy variables constant. (Temporary policy changes).

Associate with each such economy a monetary equilibrium.

Then prices stay bounded across all those equilibria. In other words, temporary QE has almost no effect after a point; temporary monetary policy becomes completely impotent. Extra money put in reserves and left there.

Credit Card Invention

- Suppose everyone knows credit cards will be created in year 10, and every period more people exogenously slowed to use them.
- Invention of Credit cards causes a big increase in price levels as they spread, and then plateau at a higher level.
- So a temporary inflation.
- Perhaps Fed should not try and undo this; perhaps end of inflation would have come anyway, without Volcker.

References

- Theorems described are from old and ongoing papers by
- Dubey-Geanakoplos 1992
- Dubey-Geanakoplos 2003
- Dubey-Geanakoplos 2006a
- Dubey-Geanakoplos 2006b
- Dubey-Geanakoplos 2024

Credit Card Default Example

- $M = 20 \quad \lambda = .02$
- $\tau_m = .46$, $\tau_c = .45$ $\tau_m + \tau_c = .91$ $u = 1.367$
- $p_{1m} = 23.97$, $p_{1c} = 26.12$ $K = .93$ $r = 10\%$
- $M = 20 \quad \lambda = \infty$ No default
- $\tau_m = .49$, $\tau_c = .46$ $\tau_m + \tau_c = .95$ $u = 1.386$
- $p_{1m} = 22.56$, $p_{1c} = 23.66$ $K = 1$ $r = 10\%$
- $M = 20 \quad \lambda = 0$ No credit card
- $\tau = .905$, $p = 12.16$, $r = 10\%$, $u = 1.384$

Credit Card Monetary Reaction Stagflation

- $M = 20 \quad \lambda = .02$
- $\tau_m = .46$, $\tau_c = .45$ $\tau_m + \tau_c = .91$ $u = 1.367$
- $p_{1m} = 23.97$, $p_{1c} = 26.12$ $K = .93$ $r = 10\%$
- $M = 19.57 \quad \lambda = .02$
- $\tau_m = .45$, $\tau_c = .45$ $\tau_m + \tau_c = .904$ $u = 1.362$
- $p_{1m} = 23.85$, $p_{1c} = 26.26$ $K = .91$ $r = 10\%$
- $M = 20 \quad \lambda = 0$ No credit card
- $\tau = .905$, $p = 12.16$, $r = 10\%$, $u = 1.384$