On the measurement of intergenerational mobility in Germany and the Unites States

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Introduction

- This study is a cross-country comparison of intergenerational mobility through a novel welfare-based approach.

- This new approach is a generalization of the Atkinson framework.

- We utilize the concept of M**ultivariate Risk Aversion** (supermodular/submodular functions), to compare the **interdependence** between parents' income and their children

Introduction

The literatures on the **measurement of inequality** and **risk** have been inexorably intertwined since the publication of the seminal articles of

- Atkinson: "On the measurement of inequality"
- Rothschild and Stiglitz: "Increasing risk. I. A definition"

in the **September 1970 issue** of the ***Journal of Economic Theory***.

Introduction

Although **Univariate Risk Aversion** (**Inequality Aversion**) has played a key role to the measurement of income inequality, **Multivariate Risk Aversion** has not been employed to the study of other types of inequality, such as well-being.

We employ for **Multivariate Risk Aversion** measuring inter-generational mobility.
(combines how inequality is transmitted from parents to children along with the inequality of opportunity).

The supermodular dependence order is stronger than the concordance order.
(Müller, A., & Stoyan, D. , 2002)

Intergenerational earning elasticities (IGE)

Cross-country comparisons are based on Intergenerational Earning Elasticities (*IGE)* through the equation below or some variant of it.

$$logY_{i,t} = a_t + \beta logY_{i,t-1} + \varepsilon_{i,t}$$

β *can be interpreted as IGE*, with Y representing "permanent earnings" for individuals from a particular family indexed by i, across two generations, t and $t - 1$.

Submodular/Supermodular Function

Richard (1975) showed that a multivariate risk averse (seeking) decision-maker has a utility function $U(x, y)$

$$\boldsymbol{U_{12}(x, y) = \frac{\partial^2 U(x,y)}{\partial x \partial y} < (>) 0}$$

This is the property of a twice differentiable utility that satisfies the following equation

$$\mathbf{U}(x_1, y_1) + \mathbf{U}(x_0, y_0) < (>)\ \mathbf{U}(x_1, y_0) + \mathbf{U}(x_0, y_1) \ \textbf{ where } x_0 < x_1 \textbf{ and } y_0 < y_1$$

This equation is considered the standard definition of functions that are called submodular (supermodular).

Framework

In Atkinson's one-dimensional approach, the proposed index can be considered as *the proportion of welfare that a society is willing to forego for an equally distributed income* — a pure state of equality.

In our approach, the intergenerational mobility is measured through the fraction of total welfare gained by avoiding perfectly correlated variables of earnings between parents and children.

Multivariate Risk Aversion (2 dimensions)

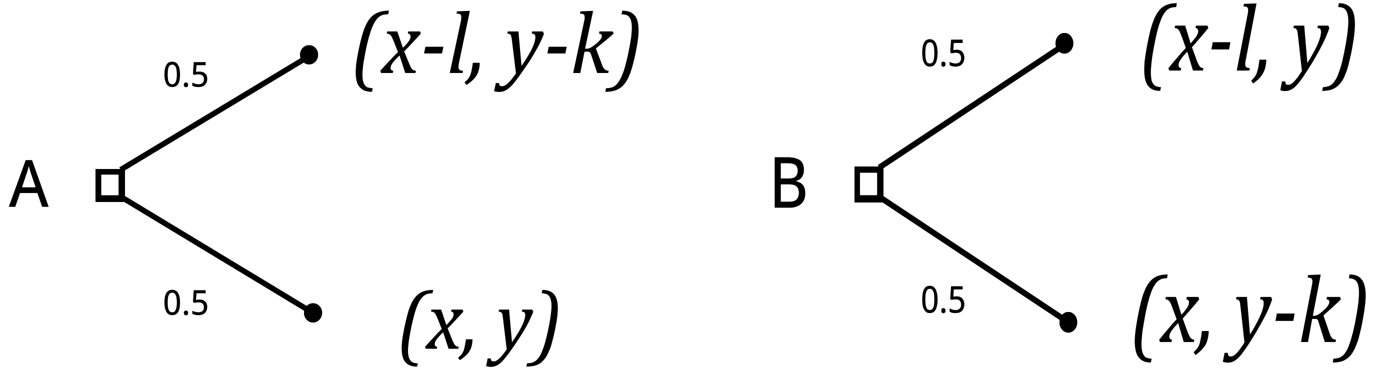


Figure: Agent prefers A or B depending on the sign of U_{12}

Multivariate Risk Aversion (2 dimensions)

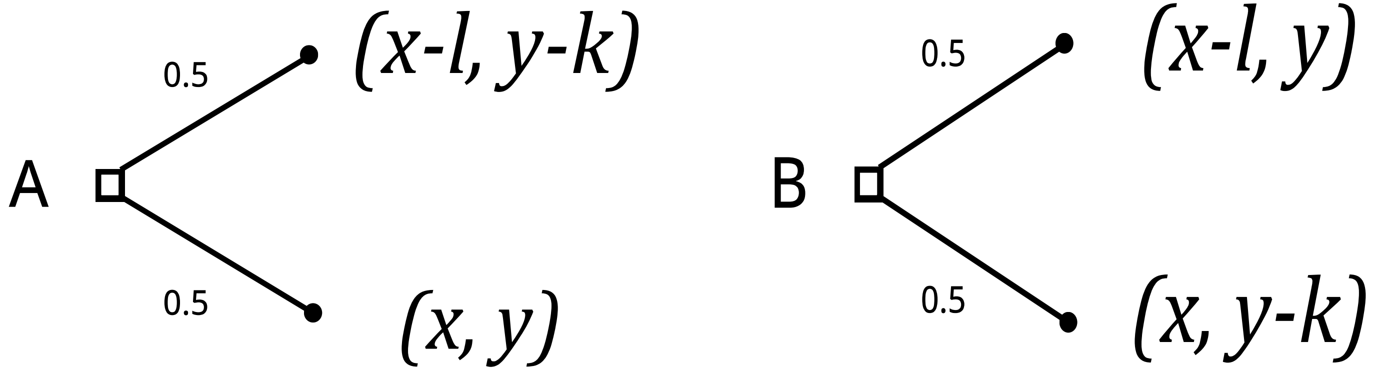


Figure: Agent indifferent between A and B when $U_{12} = 0$

Multivariate Risk Aversion (2 dimensions)

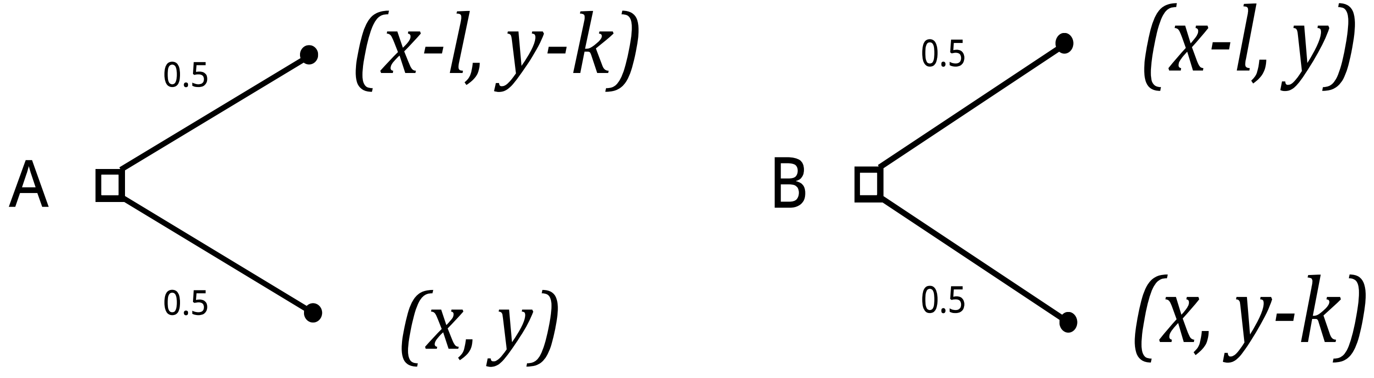


Figure: Agent prefers B when $U_{12} < 0$

Multivariate Risk Aversion (2 dimensions)

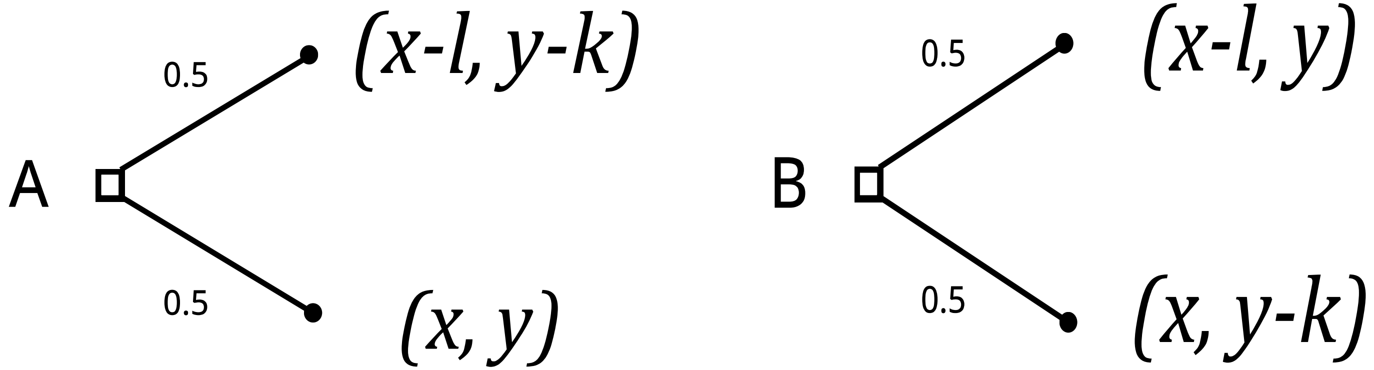


Figure: Agent prefers A when $U_{12} > 0$

Multivariate Risk Aversion (2 attributes)

By following Arrow-Pratt method, we find the Taylor approximation of the equation below and solve for p (correlation premium)

$$U(x-p-l,y)+U(x-p-l,y-k)=U(x-l,y-k)+U(x,y)$$

$$\boldsymbol{p} \approx -\frac{l^2U_{11}(x,y)+k^2U_{22}(x,y)+2lkU_{12}(x,y)}{U_1(x,y)}.$$

Correlation premium can also be defined as the payment that makes an individual indifferent between the two lotteries (the distributions can be discrete or continuous).

What is the amount of money (***m***) to be paid to the decision maker who initially faces lottery A so that he becomes indifferent between A and B?"

Ranking Submodular/Supermodular functions

Index

$$I(x, y) = -\frac{U_{11}(x,y)+U_{22}(x,y)+2U_{12}(x,y)}{U_1(x,y)}$$

Operations for vectors

Suppose

$v_0 = (v_{01}, v_{02}, \dots, v_{0n})$ and $v_1 = (v_{11}, v_{12}, \dots, v_{1n})$,

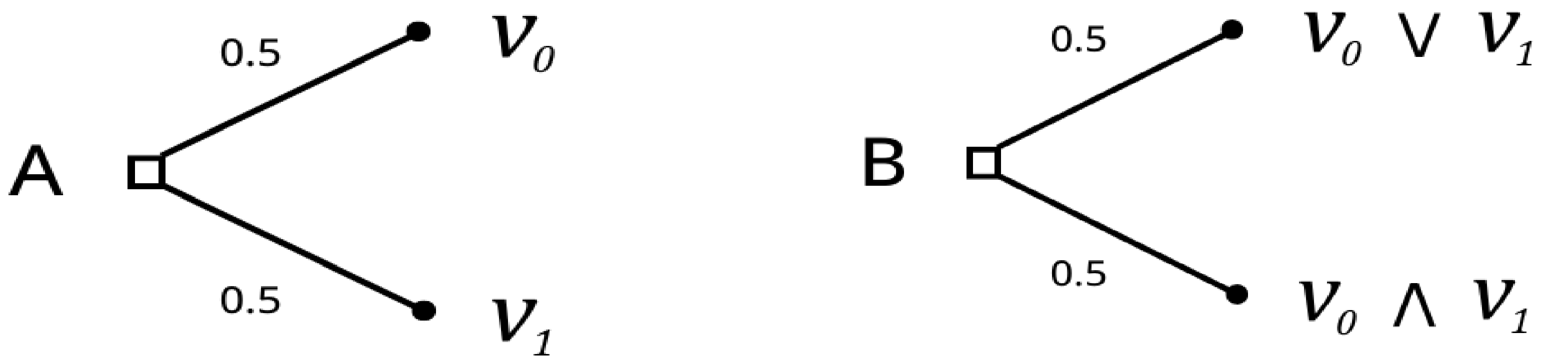
we denote

$$\boldsymbol{v_0} \vee \boldsymbol{v_1} = (max\{v_{01}, v_{11}\}, max\{v_{02}, v_{12}\}, \dots, max\{v_{0n}, v_{1n}\})$$

and

$$\boldsymbol{v_0} \wedge \boldsymbol{v_1} = (min\{v_{01}, v_{11}\}, min\{v_{02}, v_{12}\}, \dots, min\{v_{0n}, v_{1n}\})$$

More attributes



Index

$$I = -\frac{\sum_{i=1}^{n} \mathrm{U}_{ii}(\cdot) + \sum_{i=1}^{n} \sum_{j=1}^{n} 2\mathrm{U}_{ij}(\cdot)}{4\mathrm{U}_{1}(\cdot)}$$

Immobility Aversion

Definition

A function $U(\cdot)$ where all the second cross derivatives of the are negative, exhibits **intergenerational immobility aversion**.

Immobility Aversion

The social evaluation ordering $\underline{\boldsymbol{R}}$ is defined as a binary relation on the set of distributions.

f_A is socially preferred to f_B if

$$\int_0^{\chi_n} \dots \int_0^{\chi_1} U(x_1, x_2, \dots, x_k) \cdot f_A(x_1, x_2, \dots, x_k) dx_1 \dots dx_n >$$

$$\int_0^{\chi_n} \dots \int_0^{\chi_1} U(x_1, x_2, \dots, x_k) \cdot f_B(x_1, x_2, \dots, x_k) dx_1 \dots dx_n$$

Distributional Concerns

When we need to compare multivariate distributions, as in the literature of well-being inequality, there are two basic multidimensional distributional concerns.

Uniform majorization principle (UM)

For all distribution matrices X and Y in $\mathbb{R}^{n\times m}_{++}$, if Y = BX for some $n \times m$ bi-stochastic matrix B, $Y \neq X$, and Y is not a permutation of X.

Correlation increasing majorization principle (CIM)

For all distribution matrices X and Y in $\mathbb{R}^{n\times m}_{++}$, if Y is obtained from X by a correlation increasing transfer (CIT).

Distributional Concerns

The second majorization (Epstein & Tanny, 1980) is also known in the economics of risk as "pairwise more risk" (Richard, 1975)

Correlation increasing majorization principle (CIM)

For all distribution matrices X and Y in $\mathbb{R}_{++}^{n\times m}$, if Y is obtained from X by a correlation increasing transfer (CIT).

It also corresponds to the Atkinson-Bourguignon ordering (Atkinson and Bourguignon, 1982)

Example $v_o, v_1 \in \mathbb{R}^n$

Table 1 Initial social distribution ranking based on the discounted family income

Class	Proportion
Rich $(v_o \vee v_1)$	$0.5 - \pi$
Upper middle (v_o)	π
Lower middle (v_1)	π
Poor $(v_o \wedge v_1)$	$0.5 - \pi$

Table 2 New social distribution

Class	Proportion
Rich $(v_o \vee v_1)$	$0.5 - \frac{\pi}{2}$
Upper middle (v_o)	$\frac{\pi}{2}$
Lower middle (v_1)	$\frac{\pi}{2}$
Poor $(v_o \wedge v_1)$	$0.5 - \frac{\pi}{2}$

Distributional Concerns

In a multidimensional setting,

- **a sequence of UM transfers** would have resulted in the original *threshold social distribution*, where all people in the society enjoy the same level for each variable,

- **a sequence of CI transfers** would have resulted in another *threshold social distribution* with highly correlated attributes without changes for the marginal distributions.

In this new *threshold social distribution*, one family is top-ranked in all dimensions, another second-ranked, and so on. This can be characterized as the **pure state of immobility**.

Index of intergenerational mobility

Definition

The degree of intergenerational mobility is measured as the proportion of welfare that a society is willing to receive additionally for perfectly correlated attributes among the individuals.

The index of intergenerational mobility is given by

$$MI = 1 + \frac{\boldsymbol{m}}{\bar{x}}$$

where **m** (or p) is the correlation premium

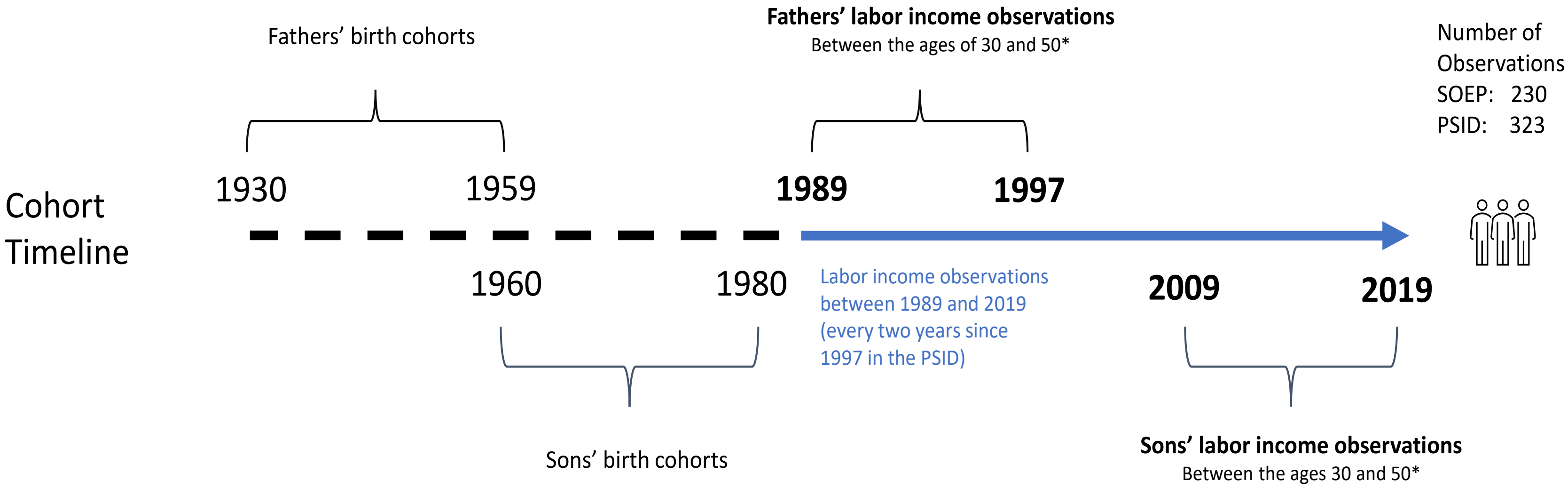
Data

The data used in this research is derived from the Panel Study of Income Dynamics (PSID) and the German Socio-Economic Panel (**SOEP**).

We exclude female household members from our analysis because changes in the labor market over the period of interest in both countries experienced social changes that likely influence the relationship of interest for women.

The data employed spans from 1989 until 2019.

Cohort Structure



*We use more recent data than Stockhausen (2021)

Illustration of the index

Similar to Atkinson (1970), we start our analysis by considering $\varepsilon = 1.5$

$$U(x_1, \ldots x_n) = -\frac{x_1^{-(1-\varepsilon)}}{1-\varepsilon} \ldots \frac{x_n^{-(1-\varepsilon)}}{1-\varepsilon}$$

For the bivariate case, the index is calculated for various values of ε

Results

Table 3 Index illustration

Country	Intergenerational Mobility
USA (PSID)	1.0914
Germany (SOEP)	1.0997

Sensitivity Analysis

Table 3 Index illustration ($\varepsilon = 2$)

Country	Intergenerational Mobility
USA (PSID)	1.1701
Germany (SOEP)	1.1911

Conclusion

- The results indicate higher intergenerational mobility in Germany.

- We highlight once again the need for more cross-country studies in international comparisons of intergenerational mobility with alternative methodologies (Schnitzlein, 2016)