

# Cross-ownership in duopoly: Are there any incentives to divest?

Rupayan Pal<sup>a</sup> and Emmanuel Petrakis<sup>b</sup>

June 28, 2024

*a* Indira Gandhi Institute of Development Research (IGIDR), India

*b* Department of Economics, University of Crete, Greece

*rupayan@igidr.ac.in, petrakis@uoc.gr*

- 1 Introduction
- 2 Model Setup
- 3 Equilibrium Analysis
  - Simultaneous move: Private Placement
  - Simultaneous move: Alternative Divestment Mechanisms
- 4 Equilibrium Analysis: Sequential Move
- 5 Concluding Remarks

# Introduction (1)

Cross-holdings among competing firms in a sector is widespread and on the rise in recent years:

- Keiretsu is a large cluster of business groups where member companies hold a fraction of shares in other companies (Grabowiecki, 2006).
- Heim et al. (2022) report 10,699 cases of minority acquisitions in rival firms across 63 countries between 1990 and 2013.
- Nain and Wang (2018) report 1068 minority stake acquisitions among rival firms in the US manufacturing sector between 1980 and 2010.
- He and Huang (2017) documents 50% increase in the fraction of U.S. public firms that are cross-held during 1980 and 2014.
- Gilo et al. (2006) and Gilo (2000) report several instances of passive cross-ownership across different sectors. For instance, in the Automobile sector, Toyota bought a 4.94% stake in Suzuki, while Suzuki bought 48-billion-yen worth of shares in Toyota in 2019 (Shiraki and Yamazaki, 2019). In 2018, Renault owned 43% of Nissan, while Nissan had a 15% share in Renault; also, Volkswagen and Suzuki held shares in each other (19.89% and 2.5%) from 2011 to 2015 (Hariskos et al, 2022).

## Introduction (2)

What are the potential implications that firms' cross-holdings may have on the performance of markets, consumer and social welfare?

Theoretical and empirical literature → Cross-holdings among rival firms in a sector could have substantial anticompetitive effects, harming the consumers and the society

Will these observed ownership structures turn out to be stable in the long run?

Note: If firms have divestment incentives, the harm to consumers and the society will be only in the short run and policy measures to correct for potential market inefficiencies may not be justifiable.

## Questions:

- Do firms holding passive shares in their rivals have incentives to divest them by selling their shares in rival firms to outside investors; and if so, under what conditions?
- Do firms have similar divestment incentives under alternative divestment mechanisms?
- Is there any role of the nature of the product market competition?

# Model Setup (1)

- Two firms - 1 & 2
- Strategic variable of firm  $i$  in the product market:  $g_i \in \mathbb{R}^+$ . Higher value chosen for  $g_i$  indicates a "more aggressive play" by firm  $i$
- **Fraction of firm  $j$ 's shares owned by firm  $i$ :  $s_i \in (0, \frac{1}{2}]$**
- Firm  $i$ 's operating profit:  $\pi_i(g_i, g_j)$
  
- Firm  $i$  can resell  $r_i \in [0, s_i]$  of its passive shares in firm  $j$  to an outside investor,  $I_i$ , for a fixed fee  $F_i (\geq 0)$  via a take-it-or-leave-it offer (Equivalent to a private placement mechanism via independent agents)

## Model Setup (2)

Firm  $i$ 's manager maximizes its firm's accounting profit:

$$\Gamma_i(g_i, g_j, r_i, r_j) = \alpha_i F_i + \pi_i(g_i, g_j) + (s_i - \alpha_i r_i) \Gamma_j(g_i, g_j, r_i, r_j) \quad (1)$$

$\alpha_i = 1$ , if outside investor  $I_i$  accepts the offer from the manager of firm  $i$ ;  
 $\alpha_i = 0$ , otherwise.

Eqn. (1)  $\Rightarrow$

$$\Gamma_i(g_i, g_j, r_i, r_j) = \frac{\alpha_i F_i + \pi_i(g_i, g_j) + (s_i - \alpha_i r_i)(\alpha_j F_j + \pi_j(g_i, g_j))}{1 - (s_i - \alpha_i r_i)(s_j - \alpha_j r_j)} \quad (2)$$

# Model Setup (3)

## Assumptions:

For all  $\alpha_i \in \{0, 1\}$ ,  $s_i \in [0, 1/2]$ , and  $r_i \in [0, s_i]$ ;  $i, j = 1, 2$ ;  $i \neq j$ ,

- A1.  $\Gamma_i(g_i, g_j)$  is a twice continuously differentiable function in  $g_i$  and  $g_j$ .
- A2.  $\frac{\partial^2 \Gamma_i}{\partial g_i^2} < 0$  and  $|\frac{\partial^2 \Gamma_i}{\partial g_i^2}| > |\frac{\partial^2 \Gamma_i}{\partial g_i \partial g_j}| > |\frac{\partial^2 \Gamma_i}{\partial g_j^2}|$  (Second-order and stability conditions).

We do not make any a priori assumption regarding the type of goods produced by firms and the nature of firms' product market strategies:

- Goods may be substitutes or complements
- Firms' product market strategies may be Strategic Substitutes or Strategic Complements



## Model Setup (4)

We consider the following three-stage game with observable actions

**Stage 1:** Firm 1 and Firm 2 make simultaneous take-it-or-leave-it offers,  $(r_1, F_1)$  and  $(r_2, F_2)$ , to outside investors,  $I_1$  and  $I_2$ , respectively.

**Stage 2:** Each outside investor accepts or rejects its own offer.

**Stage 3:** Firms engage in product market competition.

# Equilibrium Analysis: Private Placement via independent agents

## Simultaneous Move in Stage 3 (1)

In Stage 3, the FOC of firm  $i$ 's problem

$$\frac{\partial \Gamma_i}{\partial g_i} = 0 \Rightarrow \frac{\partial \pi_i}{\partial g_i} + (s_i - \alpha_i g_i) \frac{\partial \pi_j}{\partial g_i} = 0 \quad (3)$$

- $\frac{\partial \pi_j}{\partial g_i} < (>) 0 \Leftrightarrow \frac{\partial \pi_j}{\partial g_i} > (<) 0$

If goods are substitutes (complements), internalization of rival's operating profit due to cross-holdings induces firms to be less (more) aggressive in the product market compared to that in absence of cross-holdings. The reason is that, compared to the case of no cross-holdings, cross-holdings induced less (more) aggressive play by a firm increases the rival's profit and that overcompensates the respective loss in own profit when goods are substitutes (complements).

# Equilibrium Analysis: Private Placement Simultaneous Move in Stage 3 (2)

If  $\alpha_i = 1, i = 1, 2$ , , firm  $i$ 's equilibrium strategy  $g_i^* = g_i^*(r_i, r_j)$

**Lemma 1:** For all  $i, j = 1, 2, i \neq j$ , it holds that:

- $\frac{\partial g_i^*}{\partial r_i} > 0 (< 0)$ , if firms produce substitute (complement) goods.
- $\frac{\partial g_i^*}{\partial r_j} > 0$ , if (i) goods are substitutes and firms' strategies are strategic complements, or (ii) goods are complements and firms' strategies are strategic substitutes; otherwise,  $\frac{\partial g_i^*}{\partial r_j} < 0$ .

# Equilibrium Analysis: Private Placement Simultaneous Move in Stage 3 (3)

- In stage 2, outside investor  $I_i$ 's valuation of  $r_i$  fraction of passive shares in firm  $j$  is equal to  $r_i \Gamma_j^*(r_i, r_j)$ . Thus, investor  $I_i$  accepts the offer  $(r_i, F_i)$ , if  $r_i \Gamma_j^*(r_i, r_j) \geq F_i$ ; otherwise, it rejects the offer.
- In stage 1, it is optimal for firm  $i$  to set  $F_i = r_i \Gamma_j^*(r_i, r_j)$  and in stage 2 investor  $I_i$  accepts the offer for all  $r_i > 0$ .

# Equilibrium Analysis: Private Placement Simultaneous Move in Stage 3 (4)

$$\Gamma_i^*(r_i, r_j) = \frac{\pi_i(g_i^*(r_i, r_j), g_j^*(r_i, r_j)) + s_i \pi_j(g_i^*(r_i, r_j), g_j^*(r_i, r_j))}{1 - s_i s_j} \quad (4)$$

$$\frac{\partial \Gamma_i^*(r_i, r_j)}{\partial r_i} = \underbrace{\frac{r_i}{1 - s_i s_j} \frac{\partial \pi_j^*}{\partial g_i} \frac{\partial g_i^*}{\partial r_i}}_{\text{Effect via investor's profit (-)}} + \underbrace{\frac{1 - s_i(s_j - r_j)}{1 - s_i s_j} \frac{\partial \pi_i^*}{\partial g_j} \frac{\partial g_j^*}{\partial r_i}}_{\substack{\text{Effect due to rival's strategic response} \\ \text{(-), if strategic complements} \\ \text{(+) , if strategic substitutes}}} \quad (5)$$

# Equilibrium Analysis: Private Placement Simultaneous Move in Stage 3 (5)

**Proposition 1:** The following hold regardless of whether goods are substitutes or complements:

- (a) If strategies are strategic substitutes, each firm has unilateral incentive to divest some fraction of its cross-holdings in the other firm to an outside investor.
- (b) If strategies are strategic complements, it is optimal for a firm not to divest any fraction of its cross-holdings in the other firm to an outside investor.

# Equilibrium Analysis: Private Placement

## Simultaneous Move in Stage 3 (6)

Example 1:  $p_i = a - q_i - \gamma q_j$  and  $C_i = cq_i$ .  $a > c \geq 0$ ;  $\gamma \in (-1, 1)$   $0 < \gamma < 1$  :

Goods are Substitutes |  $-1 < \gamma < 0$ : Goods are Complements

**Cournot Competition:** Strategic variable,  $g_i = q_i$ ,

$$\frac{\partial^2 \Gamma_i}{\partial g_i \partial g_j} = -\gamma(1 + s_2 - r_2) \begin{cases} < 0, \text{ if } 0 < \gamma < 1 \leftarrow \text{Strategic Substitutes} \\ > 0, \text{ if } -1 < \gamma < 0 \leftarrow \text{Strategic Complements} \end{cases}$$

$$\frac{\partial \Gamma_i^*(r_i, r_j)}{\partial r_i} \Big|_{r_i=0} \begin{cases} > 0, \text{ if } 0 < \gamma < 1 \Rightarrow \text{Divest} \\ < 0, \text{ if } -1 < \gamma < 0 \Rightarrow \text{Don't Divest} \end{cases}$$

**Bertrand Competition:** Strategic variable,  $g_i = -p_i$ ,

$$\frac{\partial^2 \Gamma_i}{\partial g_i \partial g_j} = \frac{\gamma(1+s_2-r_2)}{1-\gamma^2} \begin{cases} > 0, \text{ if } 0 < \gamma < 1 \leftarrow \text{Strategic Complements} \\ < 0, \text{ if } -1 < \gamma < 0 \leftarrow \text{Strategic Substitutes} \end{cases}$$

$$\frac{\partial \Gamma_i^*(r_i, r_j)}{\partial r_i} \Big|_{r_i=0} \begin{cases} < 0, \text{ if } 0 < \gamma < 1 \Rightarrow \text{Don't Divest} \\ > 0, \text{ if } -1 < \gamma < 0 \Rightarrow \text{Divest} \end{cases}$$

# Equilibrium Analysis: Private Placement Simultaneous Move in Stage 3 (7)

**Proposition 2:** Suppose that firms' strategies are strategic substitutes. Then, regardless of whether goods are substitutes or complements, it is optimal for firms to divest their respective cross-holdings fully to outside investors.

Proposition 1 and Proposition 2  $\Rightarrow$

- Stability of cross-ownership depends on the nature of firms' strategies – strategic substitutes or strategic complements -, and not on the type of goods – substitutes or complements.
- The result of Stenbacka and Moer (2021) holds true not only for substitute goods, but also in case goods are complements. However, when firms' strategies are strategic complements, Stenbacka and Moer (2021)'s result is completely reversed.



# Equilibrium Analysis: Alternative Divestment Mechanisms (1)

**Competitive bidding:** Firms choose to divest their passive shares of their rivals through some competitive bidding process, e.g., a Dutch auction.

- Firm  $i$ 's maximum possible return from divesting  $r_i$  passive shares of its rival:  $F_i = r_i \Gamma_j^*$
- Let,  $\mu_i \Gamma_j^* =$  the average price of firm  $i$ 's shares in firm  $j$ ;  $0 < \mu_i \leq 1$ .

If  $\mu_i$  is close to 1, Propositions 1 and 2 hold true.

Competitive bidding with many potential investors  $\rightarrow \mu_i$  close to 1

We can safely infer that our main findings hold under this alternative divestment mechanism too.

# Equilibrium Analysis: Alternative Divestment Mechanisms (2)

## Private placement via a common intermediary

→ Divestment through private placement is possible only if total payoff of all agents involved (i.e., payoffs of buyers of divested shares plus payoffs of firms) increases due to divestment

**Proposition 3:** If divestment occurs via a common intermediary,

- (a) when firms' strategies are strategic complements, there are no divestment incentives of cross-holdings, independently whether goods are substitutes or complements.
- (b) when firms' strategies are strategic substitutes and firms are symmetric in all aspects, firms have no incentive to divest, independently whether goods are substitutes or complements. Otherwise, they may or may not have incentives to divest some fraction of their cross-holdings depending on the specific market features.

# Equilibrium Analysis: Sequential move product market competition (1)

**Stage 1:** Firm 1 and Firm 2 make simultaneous take-it-or-leave-it offers,  $(r_1, F_1)$  and  $(r_2, F_2)$ , to outside investors,  $I_1$  and  $I_2$ , respectively.

**Stage 2:** Each outside investor accepts or rejects its own offer.

**Stage 3:** Firm 1 (the Leader) choose the level of  $g_1$

**Stage 3:** Firm 2 (the Follower) choose the level of  $g_2$

Assumption:

$$A3. \text{sign}\left(\frac{\partial^2 \pi_1}{\partial g_2 \partial g_1}\right) = \text{sign}\left(\frac{\partial^2 \pi_2}{\partial g_1 \partial g_2}\right)$$

A3 holds true even when firms face different demand functions and/or their cost functions are different, except in some special cases (Bulow et al., 1985).

# Equilibrium Analysis: Sequential move product market competition (2)

**Lemma 2:** It holds that:

- (a)  $\frac{\partial g_1^L}{\partial r_1} > (<)0$  and  $\frac{\partial g_2^F}{\partial r_2} > (<)0$ , if goods are substitutes (complements).
- (b)  $\frac{\partial g_1^L}{\partial r_2} > 0$  and  $\frac{\partial g_2^F}{\partial r_1} > 0$  if (i) goods are substitutes and firms' strategies are strategic complements, or (ii) goods are complements and firms' strategies are strategic substitutes; otherwise,  $\frac{\partial g_1^L}{\partial r_2} < 0$  and  $\frac{\partial g_2^F}{\partial r_1} < 0$ .

→ Similar to those in the case of simultaneous move competition (Lemma 1)

# Equilibrium Analysis: Sequential move product market competition (3)

In Stage 1: Leader's Problem

$$\frac{d\Gamma_1^L(r_1, r_2)}{dr_1} = \underbrace{\frac{1}{1-s_1s_2} \left( r_1 \frac{\partial \pi_2^F}{\partial g_1^L} \frac{\partial g_1^L}{\partial r_1} \right)}_{\substack{\text{Effect via investor's profit} \\ (-)}} + \underbrace{\frac{1}{1-s_1s_2} \left( -r_1(s_2 - r_2) \frac{\partial \pi_1^L}{\partial g_2^F} \right) \frac{\partial g_2^F}{\partial r_1}}_{\substack{\text{Effect due to rival's strategic response} \\ (-), \text{ if strategic substitutes} \\ (+), \text{ if strategic complements}}$$

$< 0$

# Equilibrium Analysis: Sequential move product market competition (4)

In Stage 1: Follower's Problem

$$\text{Sign} \left( \frac{\partial \Gamma_2^F(r_1, r_2)}{\partial r_2} \Big|_{r_2=0} \right) = -\text{Sign} \left( \frac{\partial g_1^I}{\partial r_1} \frac{\partial g_1^I}{\partial r_2} \right)$$

$$\frac{\partial \Gamma_2^F(r_1, r_2)}{\partial r_2} \Big|_{r_2=0} \begin{cases} > 0, \text{ if firms' strategies are strategic substitutes} \\ < 0, \text{ if firms' strategies are strategic complements} \end{cases}$$

# Equilibrium Analysis: Sequential move product market competition (5)

**Proposition 4:** The following hold in the sequential move game:

(a) The leader has no incentives to divest its passive cross-holdings in the follower to an outside investor, regardless of whether goods are substitutes or complements and firms' strategies are strategic substitutes or strategic complements.

(b) The follower has incentives to divest some fraction of its passive shares in the leader if firms' strategies are strategic substitutes. In contrast, if firms' strategies are strategic complements, the follower has no incentives to divest its cross-holdings, regardless of whether goods are substitutes or complements.

# Concluding Remarks (1)

- In a general duopolistic market context in which goods are either substitutes or complements and firms' strategies are either strategic substitutes or strategic complements, we have examined the firms' incentives to divest their passive shares in their rivals.
- We have considered alternative divestment mechanisms: private placement via independent intermediaries as well as via a common intermediary and competitive bidding.



## Concluding Remarks (2)

- There are always divestment incentives under a private placement mechanism via independent intermediaries **but only if** firms' strategies are strategic substitutes in a simultaneous move product market game.
- This holds independently whether goods are substitutes or complements.
- A similar result is obtained under a competitive bidding mechanism.
- Yet, under a private placement mechanism via a common intermediary, firms do not have divestment incentives when their strategies are strategic substitutes unless there are substantial asymmetries in firms' cross holdings as well as in other market characteristics.

## Concluding Remarks (3)

- Finally, only the follower has divestment incentives in a sequential move product market game when firms' strategies are strategic substitutes, independently whether goods are substitutes or complements.
- In all other cases, no firm has divestment incentives, implying that we can safely analyze the market and societal implications of firms' arbitrary passive cross-holdings with no need to question the stability of the ownership structure.

## Testable implications

- First, divestment activities are more often observed in industries in which firms' strategies are strategic substitutes than those in which they are strategic complements.
- Second, in industries in which firms' strategies are strategic substitutes, divestment activities are less often observed when divestment occurs via a common intermediary than via independent intermediaries or competitive bidding.
- Third, in industries in which firms' strategies are strategic substitutes, leaders are less often engaged in divestment activities than followers.

## Concluding Remarks (5)

There are two maintained assumptions in our analysis

- First, investors buying firms' passive shares have no bargaining power and pay, thus, the highest amount that firms ask for selling those shares.
- Second, these investors have no impact on the firms' managers product market decisions ex-post, i.e., once they have bought the competing firms' shares.

Relaxing these assumptions may lead to interesting new findings regarding the firms' incentives to divest their cross-holdings, an issue which is left for future research.

**Thank You!**