Patent licensing in general equilibrium

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Patents and licensing

- A patent grants an entity monopoly rights over its technology for a given period of time
- Licensing: the patent holder gives the patented innovation to others in return for payments based on some policy such as a unit royalty or a fixed fee
- Large literature studying patent licensing, market structure and incentives to innovate (onwards Arrow, 1962; Katz and Shapiro, 1986)

- Free licensing of patents is observed
- Recent example: in 2021, Samsung announced sharing of 505 of its patented technologies with local small and medium enterprises (SMEs) in South Korea
- This paper: a theoretical analysis to understand the optimality of free licensing where the patent holder is a competing firm
- Also explores this issue in the specific context in which there is an "informal" sector consisting of a large number of small firms, where patent enforcement is weak

The benchmark model and results

- A model of an economy where the representative consumer has a constant elasticity of substitution (CES) utility function (Arrow et al., 1961) over two goods that are produced by two firms $1, 2$
- The income of the consumer is the sum of an exogenous component and a positive fraction of the sum of the profits of two firms
- Firm 1 holds a patent for an efficient technology that lowers the marginal cost of production
- We show: (i) for firm 1, giving the patent to firm 2 for free is superior to not licensing, but (ii) licensing by unit royalty is superior to free licensing

Extended model: basic good, informal sector

- To further explore the optimality of free licensing: an extended model with another good (called good 0) that corresponds to the basic good
- Following Matsuyama (2002): a "hierarchial demand" utility function, where the representative consumer exclusively cares about the basic good if its consumption falls below a certain threshold
- The basic good is produced by a competitive fringe (informal sector) where patent enforcement is weak
- It is shown that for firm 1: inclusive free licensing (free license to firm 2 as well as to the fringe) is superior to exclusive royalty licensing (only to firm 2) or no licensing

The benchmark model

- An economy with two goods $1, 2$, one representative consumer
- Two firms $1, 2$, firm i produces good i, firms compete in prices
- $x_1, x_2 =$ amounts of goods $1, 2$
- The representative consumer has a constant elasticity of substitution (CES) utility function (Arrow et al., 1961):

$$u(x_1, x_2) = [\alpha x_1^{\rho} + (1-\alpha) x_2^{\rho}]^{1/\rho} \tag{1}$$

  where $0 < \alpha < 1$, $\rho < 1$ and $\rho \neq 0$.
- The constant elasticity of substitution of the utility function (1): $\sigma \equiv 1/(1-\rho)$, ($\sigma > 0$ and $\sigma \neq 1$)
- If $0 < \rho < 1$: $\sigma > 1$, goods are "good substitutes"; if $\rho < 0$: $0 < \sigma < 1$, goods are "poor substitutes" (e.g., Black et al., 2009).

- Let y = income of the consumer, p_1, p_2 = prices of goods
- The consumer's utility maximization problem: choose $x_1, x_2 \geq 0$ to maximize $u(x_1, x_2)$ subject to the budget constraint $p_1 x_1 + p_2 x_2 \leq y$
- The unique solution to this utility maximization problem:

$$x_1^*(p_1, p_2, y) = y\alpha^\sigma / p_1^\sigma g(p_1, p_2),$$

$$x_2^*(p_1, p_2, y) = y(1-\alpha)^\sigma / p_2^\sigma g(p_1, p_2) \text{ where}$$

$$g(p_1, p_2) := \alpha^\sigma p_1^{1-\sigma} + (1-\alpha)^\sigma p_2^{1-\sigma}$$

Income of the consumer

- The income y of the consumer is the sum of two components: (i) an exogenously given minimum income $\hat{y} > 0$ and (ii) a fraction t of the total profit Π of the economy ($0 \leq t < 1$):

$$y = \hat{y} + t\Pi$$

- The fraction t is a reduced form parameter that captures the interrelation between profits, income and demands for the goods
- It is possible to explicitly model the interdependence between profits and incomes through labour inputs supplied by the consumer (e.g., Murphy et al., 1989) or ownership structure of firms (e.g., Azar and Vives, 2021)

Initial and new production technologies

- Initially both firms $1, 2$ have the same constant marginal cost of production $\overline{c}$
- Firm 1 has a patent on a new technology that lowers the marginal cost from $\overline{c}$ to $\underline{c}$, where $0 < \underline{c} < \overline{c}$

Assumption

The prices p_1, p_2 that firms can set must be in the closed interval $[\underline{p}, \overline{p}]$, where $0 < \underline{p} < \underline{c} < \overline{c} < \overline{p}$.

Duopoly interaction: no licensing versus free licensing

- No licensing: $c_1 = \underline{c}$, $c_2 = \overline{c}$; free licensing: $c_1 = \underline{c}$, $c_2 = \underline{c}$
- If firm 1 sets price p_1, firm 2 sets price p_2 and the income of the representative consumer is y, the operating profits are

$$\pi_1(p_1, p_2, c_1, y) = (p_1 - c_1)x_1^*(p_1, p_2, y),$$

$$\pi_2(p_1, p_2, c_2, y) = (p_2 - c_2)x_2^*(p_1, p_2, y)$$

- Determining equilibrium income:

$$y = \hat{y} + t\Pi = \hat{y} + t(p_1 - c_1)x_1^*(p_1, p_2, y) + t(p_2 - c_2)x_2^*(p_1, p_2, y)$$

Denote the (unique) solution to the equation above by $y^t(\hat{y}, p_1, p_2, c_1, c_2)$

- Profits of firms: $\pi_1(p_1, p_2, c_1, y^t)$, $\pi_2(p_1, p_2, c_2, y^t)$, net payoffs fraction $(1 - t)$ of these profits

Lemma

Suppose firms $1, 2$ *have constant marginal costs* $c_1, c_2 \in (\underline{p}, \overline{p})$. *If the two goods are either poor substitutes* $(0 < \sigma < 1)$ *or not sufficiently good substitutes* $(1 < \sigma < \overline{p}/(\overline{p} - c_i)$ *for* $i = 1, 2)$, *the unique Nash Equilibrium* (NE) *of the duopoly above is* $(p_1 = \overline{p}, p_2 = \overline{p})$.

- No licensing: $c_1 = \underline{c}, c_2 = \overline{c}$
  1. Equilibrium income: $y_{NL} = y^t(\hat{y}, \overline{p}, \overline{p}, \underline{c}, \overline{c})$
  2. Net payoffs: $(1 - t)\pi_1(\overline{p}, \overline{p}, \underline{c}, y_{NL})$, $(1 - t)\pi_2(\overline{p}, \overline{p}, \overline{c}, y_{NL})$
- Free licensing: : $c_1 = \underline{c}, c_2 = \underline{c}$
  1. Equilibrium income: $y_{FL} = y^t(\hat{y}, \overline{p}, \overline{p}, \underline{c}, \underline{c})$
  2. Net payoffs: $(1 - t)\pi_1(\overline{p}, \overline{p}, \underline{c}, y_{FL})$, $(1 - t)\pi_2(\overline{p}, \overline{p}, \underline{c}, y_{FL})$

Proposition

Suppose the two goods are either poor substitutes $(0 < \sigma < 1)$ *or not sufficiently good substitutes* $(1 < \sigma < \overline{p}/(\overline{p} - \underline{c}))$.

1. (i) *For any* $0 < t < 1$, *licensing the new technology to firm 2 for free is superior for firm 1 compared to no licensing and for* $t = 0$, *firm 1 is indifferent between free licensing and no licensing.*
2. (ii) *For any* $0 \leq t < 1$, *receiving the license for free is superior for firm 2 compared to no licensing.*
3. (iii) *For any* $0 < t < 1$, *the representative consumer has higher utility under free licensing compared to no licensing. For* $t = 0$, *the consumer has the same utility under free licensing and no licensing.*

- Free transfer of the new technology makes firm 2 more productive, thus raising the income of the representative consumer, demands for both goods
- Gains from technology transfer for firm 1 even in the absence of any licensing revenue

Licensing by per unit royalty

- Per unit royalty policy: in return for licensing of the new technology, for every unit of good 2 that firm 2 produces, firm 2 pays a unit royalty $r \geq 0$ to firm 1
- $r = 0$ corresponds to free licensing
- Effective marginal cost of firm 2: $\underline{c} + r$
- Since the maximum price is $\overline{p}$ and firm 2 obtains positive profit without a license, any acceptable r must have $\underline{c} + r \leq \overline{p}$, so $r \leq \overline{p} - \underline{c}$.

- If firm 1 sets price p_1, firm 2 sets price p_2 and the income of the representative consumer is y, the profits are

$$\phi_1(p_1, p_2, \underline{c}, y, r) = (p_1 - \underline{c})x_1^*(p_1, p_2, y) + rx_2^*(p_1, p_2, y) \text{ and}$$

$$\phi_2(p_1, p_2, \underline{c}, y, r) = [p_2 - (\underline{c} + r)]x_2^*(p_1, p_2, y)$$

- Determining equilibrium income:

$$y = \hat{y} + t\Pi = \hat{y} + t(p_1 - \underline{c})x_1^*(p_1, p_2, y) + t(p_2 - \underline{c})x_2^*(p_1, p_2, y)$$

  The (unique) solution to the equation above: $y^t(\hat{y}, p_1, p_2, \underline{c}, \underline{c})$

- Profits of firms: $\phi_1(p_1, p_2, \underline{c}, y^t)$, $\phi_2(p_1, p_2, \underline{c}, y^t)$, net payoffs fraction $(1 - t)$ of these profits
- If the two goods are either poor substitutes or not sufficiently good substitutes, for any $0 \leq t < 1$ and $0 \leq r \leq \overline{p} - \underline{c}$, the unique NE of the duopoly above has: $(p_1 = \overline{p}, p_2 = \overline{p})$.

Proposition

If the goods are either poor substitutes or not sufficiently good substitutes, then for any $0 \leq t < 1$*:*

1. *The unique optimal per unit royalty policy for firm* 1 *is to set the royalty* $\overline{r}(t)$ *that makes firm* 2 *just indifferent between no licensing and receiving the license.*
2. *For firm* 1*, the optimal royalty policy is superior to free licensing as well as no licensing for firm; for firm* 2*, receiving license through this policy is inferior to free licensing.*
3. *The representative consumer has same utility under free licensing and licensing with royalty* $\overline{r}(t)$*.*

- Free licensing is superior to no licensing, but once firm 1 has the option of setting royalties for licensing, free licensing is no longer optimal
- Is it possible that even when royalties are allowed, it is still optimal to set no royalties and give licenses for free?
- We consider an extended model to explore this question

A model of hierarchical demand

- Consider another good (good 0) in addition to goods $1, 2$
- Good 0 is the basic good with two features: (i) necessity and (ii) saturation
- For $i = 0, 1, 2$, let x_i be the amount of good i; the representative consumer has a utility function that has the feature of "hiererachial demand" with respect to good 0 (Matsuyama, 2002):

$$U(x_0, x_1, x_2) = \begin{cases} x_0 & \text{if } x_0 \leq \underline{x}_0 \\ \underline{x}_0 + u(x_1, x_2) & \text{if } x_0 > \underline{x}_0 \end{cases} \tag{2}$$

  where $u(x_1, x_2)$ is the CES utility function given in (1).
- The basic good 0 is produced by a competitive fringe of firms
- For goods $1, 2$ as before there are two distinct firms $1, 2$, firm 1 produces good 1 and firm 2 produces good 2; they compete in prices
- The threshold $\underline{x}_0$: the level of minimum requirement of the basic good 0

- If the consumer has income Y and prices are p_0, p_1, p_2, the utility maximization problem is to choose $x_0, x_1, x_2 \geq 0$ to maximize $U(x_0, x_1, x_2)$ subject to $p_0 x_0 + p_1 x_1 + p_2 x_2 \leq Y$.
- If $Y \leq p_0 \underline{x}_0$: optimal to buy only good 0 and not buy goods $1, 2$ at all
- If $Y > p_0 \underline{x}_0$: optimal to buy exactly $\underline{x}_0$ units of good 0 and use the remaining income $Y - p_0 \underline{x}_0$ to buy goods $1, 2$ to maximize $u(x_1, x_2)$:

$$\tilde{x}_1(p_0, p_1, p_2, Y) = x_1^*(p_1, p_2, Y - p_0 \underline{x}_0),$$

$$\tilde{x}_2(p_0, p_1, p_2, Y) = x_2^*(p_1, p_2, Y - p_0 \underline{x}_0)$$

Initial and new technologies

- Initially both firms $1, 2$, as well as all firms in the competitive fringe that produce good 0 have the same constant marginal cost of production $\overline{c}$
- Firm 1 has a new technology that lowers the marginal cost from $\overline{c}$ to $\underline{c}$, where $0 < \underline{c} < \overline{c}$.

New technology and the fringe

- We assume the competitive fringe has weak enforcement of patent rights: if one or more firms in the fringe has the new technology, all remaining firms (both inside the fringe as well as outside) also have the technology
- For this reason no firm in the fringe is willing to make any licensing payment for the new technology

Different options of technology transfer

- No licensing: marginal cost of the fringe stays $\overline{c}$ and the price of good 0 is $p_0 = \overline{c}$; marginal cost of firm 2 is $c_2 = \overline{c}$
- Inclusive free licensing (technology given for free to all firms): marginal cost of the fringe is $\underline{c}$, so $p_0 = \underline{c}$; marginal cost of firm 2 is $c_2 = \underline{c}$
- Exclusive licensing to firm 2 using unit royalty $r \geq 0$: marginal cost of the fringe stays $\overline{c}$, so $p_0 = \overline{c}$; effective marginal cost of firm 2 is $c_2 = \underline{c} + r$.
- Exclusive free licensing (exclusive licensing to firm 2 using royalty $r = 0$): marginal cost of the fringe stays $\overline{c}$, so $p_0 = \overline{c}$ and $c_2 = \underline{c}$

Income of the consumer

- As before the income Y of the consumer is the sum of two components: (i) an exogenously given minimum income, denoted by $\hat{Y} > 0$ and (ii) a fraction t of the total profit Π of the economy, where $0 \leq t < 1$:

$$Y = \hat{Y} + t\Pi$$

- Regardless of the nature of technology transfer, firms in the competitive fringe always make zero profit; so the total profit $\Pi =$ sum of the profits of firms $1, 2$

- p_0 (price of good 0) either $\underline{c}$ or $\overline{c}$; the income Y at least $\hat{Y}$
- We assume $\hat{Y} > \overline{c}\underline{x}_0$, which ensures that the consumer always has sufficient income to buy $\underline{x}_0$ units of the basic good ($Y - p_0\underline{x}_0$ always positive)
- When income is Y and prices of goods $1, 2$ are p_1, p_2, demands of goods $1, 2$:

$$\tilde{x}_1(p_0, p_1, p_2, Y) = x_1^*(p_1, p_2, Y - p_0\underline{x}_0),$$

$$\tilde{x}_2(p_0, p_1, p_2, Y) = x_2^*(p_1, p_2, Y - p_0\underline{x}_0)$$

- We can apply the same reasoning as the benchmark model to determine equilibrium income by replacing $y = Y - p_0\underline{x}_0$ (where $p_0 = \underline{c}$ for inclusive free licensing and $p_0 = \overline{c}$ otherwise)

As before, if good $1, 2$ are either poor substitutes $(0 < \sigma < 1)$ or not sufficiently good substitutes $(1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$, for any licensing option, the resulting duopoly has a unique Nash Equilibrium: $(p_1 = \overline{p}, p_2 = \overline{p})$.

Comparing inclusive free licensing and no licensing

Proposition

Suppose good $1, 2$ *are either poor substitutes or not sufficiently good substitutes.*

1. *For any* $0 \leq t < 1$, *inclusive free licensing (that is, the new technology is given for free to firm* 2 *as well as to all firms in the fringe) is superior for firm* 1 *compared to no licensing and receiving the license for free is superior for firm* 2 *compared to no licensing.*
2. *For any* $0 \leq t < 1$, *the representative consumer has higher utility under inclusive free licensing compared to no licensing.*

Comparing inclusive free licensing and exclusive licensing by royalty

Proposition

Suppose goods $1, 2$ *are either poor subsitutes or not sufficiently good substitutes and* $\hat{Y} > \bar{c}\underline{x}_0$. *Then for any* $0 \leq t < 1$*:*

1. *The unique optimal per unit royalty policy for firm* 1 *is to set a royalty* $\tilde{r}(t)$ *that makes firm* 2 *just indifferent between having a license or not. For firm* 1, *this policy is superior to no licensing as well as exclusive free licensing.*
2. *There exists a threshold* $\kappa(t)$ *(decreasing in* t*) such that for firm* 1, *inclusive free licensing is superior to royalty licensing if* $\hat{Y} < \bar{c}\underline{x}_0 + \kappa(t)$ *and royalty licensing is superior if* $\hat{Y} > \bar{c}\underline{x}_0 + \kappa(t)$.
3. *The utility of the representative consumer is highest at inclusive free licensing: inclusive free licensing gives higher utility than royalty licensing, exclusive free licensing and no licensing.*

- Inclusive free licensing ensures that firms in the fringe operate under the low cost
- Demand of the basic good highly inelastic: a fall in its price implies the consumer spends less on the basic good and devotes more of its income towards the consumption of goods $1, 2$
- The optimality of inclusive free licensing determined by two effects: (i) higher net income and (ii) the general equilibrium effect of higher profits
- These effects compensate for the loss of revenue in royalties when $\hat{Y}$ (the exogenous part of the income of the representative consumer) is relatively small
- Even when $t = 0$, the net income effect on its own can result in the optimality of inclusive free licensing; inclusive free licensing can be optimal even when firm 1 is not large relative to the whole economy