Location choices in duopoly with inventories and dynamic pricing

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Motivation

- We study the **location and pricing decisions of firms** when they have inventories of differentiated products to sell over time
- The firms set prices in each period and, depending on these, a sale occurs or not. The inventories get **reduced stochastically** over time, depending on the prices set and the realization of demand in each period
- Therefore, forward looking firms compete over time with **dynamic capacity constraints**. There is endogenous evolution of inventories (capacities)
- We also endogenize the firms' **location choices** (or product differentiation), based on the size of their initial inventories. We show how inventories affect locations and enrich the set of considerations for locating at the center or away
- This setting of strategic dynamic pricing is important but has not been studied enough in the literature

Empirical relevance and literature

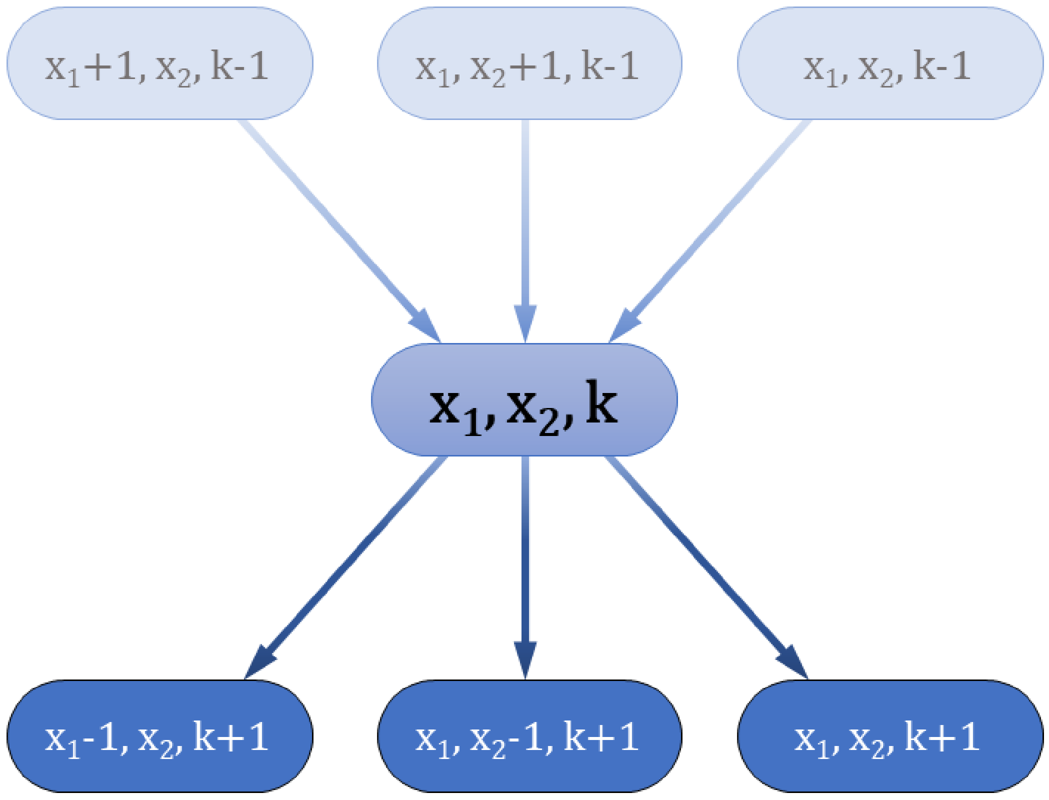
- Numerous economic sectors currently employ dynamic pricing, like the airlines industry, public transport, hospitality, seasonal goods, entertainment and sports events especially with the rapid growth of e-commerce
- Related literature, different strands (in economics and Operations Research):
  - Product differentiation (Hotelling, 1929)
  - Dynamic pricing (Talluri, van Ryzin, 2004)
  - Capacity constraints (Dasgupta and Maskin, 1986a,b)

General framework

- Two sellers, $i = 1{,}2$, with **initial capacity** $(\boldsymbol{X_i})$ **of a single good without replenishment**, which are common knowledge
- We provide firms with a **choice of location** (between an endpoint and the middle of a line segment of unit length)
- Time is discrete in a **finite horizon** and every period (k) one new buyer with **unit demand** enters the market
- Each buyer is characterized by his preference $(u_{i,k})$ relative to the good of each firm i
- We assume that buyers' **preferences** $(\boldsymbol{u_{i,k}}, \boldsymbol{u_{j,k}})$ **are i.i.d. across periods** following a known distribution to the firms
- A buyer that arrives in period (k) observes the posted prices $(p_{i,k}, p_{j,k})$ and decides whether to buy or not
- Firms first **simultaneously choose their locations once** and then, compete for multiple periods on **prices that they simultaneously post each period** before observing the buyers' preferences

General framework

- A sale by any firm is not certain in any period no matter the available time and capacity left to be sold
- In each subsequent period, $(k + 1)$, there are three possible continuations, each with different values for the unit to be sold and different probabilities of sale
- These continuations depend on the posted prices and the preferences of the new buyer that will arrive in the next period
- There are then three new continuations, and so on
- After considering these uncertainties, each firm would like to post a price that maximizes its intertemporal expected profits



Dynamic Programming algorithm

- The model has two principal features: an underlying discrete time dynamic system, and profit functions that are additive over time (Bertsekas, 2005)
- **Dynamic Programming algorithm**: For any initial state $(x_{i,0}, x_{j,0})$, the optimal profit $J^*_{i,0}(x_{i,0}, x_{j,0})$ is equal to $J_{i,0}(x_{i,0}, x_{j,0})$, given by the last step of the following algorithm, which proceeds backward in time from the last period $(k = N-1)$ to the first period $(k = 0)$:

$$J_{i,N}(x_{i,N}, x_{j,N}) = g_{i,N}(x_{i,N}, x_{j,N})$$

$$J_{i,k}(x_{i,k}, x_{j,k}) = \max_{p_{i,k} \in \mathcal{B}_k(x_{i,k}, x_{j,k})} \langle \underset{u_{i,k}, u_{j,k}}{\mathrm{E}} \{ g_{i,k}(x_{i,k}, x_{j,k}, p_{i,k}, p_{j,k}, u_{i,k}, u_{j,k}) + J_{i,k+1}[f_{i,k}(x_{i,k}, x_{j,k}, p_{i,k}, p_{j,k}, u_{i,k}, u_{j,k})] \} \rangle$$

$$for \quad k = 0, 1, \dots, N-1$$

- Or else:

$$J_{i,k}(\alpha) = \max_{p_{i,k} \in \mathcal{B}_k(\alpha)} \left\{ \sum_{\beta} F_{i,\alpha\beta}(p_{i,k}, p_{j,k}) \cdot [g_{i,\alpha\beta}(\alpha, p_{i,k}, p_{j,k}) + J_{i,k+1}(\beta)] \right\}$$

where $g_{i,\alpha\beta}(\alpha, p_{i,k}, p_{j,k})$ is firm i's profit when controls $p_{i,k}$ and $p_{j,k}$ are applied and the transition from state (α) to state (β) occurs with probability $F_{i,\alpha\beta}$

Buyers

- A buyer that arrives in period (k) seeks to maximize his utility:

$$U_{i,k} = r - p_{i,k} - s \cdot u_{i,k}$$

where (r) is the reservation price, (s) the transportation costs (disutility) and $\left(u_{i,k}\right)$ the distance from firm i (preference)

- We assume that reservation price (r) and transportation costs (s) are constant across time
- The buyer observes the posted prices and decides to buy firm i's product when his utility is non-negative and greater than the corresponding utility of the buyer if he had bought the product of firm j:

$$\left[U_{i,k} \geq 0 \Leftrightarrow u_{i,k} \leq \left(r - p_{i,k}\right)/s\right] \cap \left[U_{i,k} \geq U_{j,k}\right]$$

Firms

- We assume that both firms have zero marginal cost, $\left(c_i = c_j = 0\right)$, and the same constant discount factor $\delta \in (0,1)$ so, in the context of dynamic programming, (Bertsekas, 2020), firm i solves in state $\left(x_{i,k}, x_{j,k}\right)$ the following maximization problem:

$$V_{i,k} \coloneqq \max_{p_{i,x_{i,k},x_{j,k},k}} \left\{F_{i,k} \cdot \left(p_{i,k} + \delta_i \cdot V_{i,(i),k+1}\right) + F_{j,k} \cdot \delta_i \cdot V_{i,(j),k+1} + \left(1 - F_{i,k} - F_{j,k}\right) \cdot \delta_i \cdot V_{i,k+1}\right\}$$

where:

$F_{i,k}$	: the probability of a sale by firm i in the current period,
$V_{i,k}$	: current period's expected value,
$V_{i,(i),k+1}$	: next period's expected value if firm i makes a sale in the current period,
$V_{i,(j),k+1}$	: next period's expected value if firm j makes a sale in the current period,
$V_{i,k+1}$	: next period's expected value if no firm makes a sale in the current period.

Solution

- The value of any firm is zero when its capacity is depleted in any period and after the last period no matter its capacity:

$$V_{i,0,x_j,k} = V_{j,x_i,0,k} = V_{i,x_i,x_j,N} = V_{j,x_i,x_j,N} = 0$$

- Note that, next period values do not depend on current period's price:

$$\partial V_{i,k+1}/\partial p_{i,k} = \partial V_{i,(i),k+1}/\partial p_{i,k} = \partial V_{i,(j),k+1}/\partial p_{i,k} = 0$$

- The first order condition provides for the **equilibrium price**:

$$\frac{\partial V_{i,k}}{\partial p_{i,k}} = 0 \quad \Longrightarrow \quad p^*_{i,k} = -\frac{F_{i,k}}{F^{(i)}_{i,k}} + \delta \cdot \left(V_{i,k+1} - V_{i,(i),k+1}\right) + \frac{F^{(i)}_{j,k}}{F^{(i)}_{i,k}} \cdot \delta \cdot \left(V_{i,k+1} - V_{i,(j),k+1}\right)$$

- When we combine the Bellman's equation with the FOC, the **corresponding expected value**:

$$V^*_{i,k} = -\frac{F^2_{i,k}}{F^{(i)}_{i,k}} + \delta \cdot V_{i,k+1} + \frac{F^2_{i,k}}{F^{(i)}_{i,k}} \cdot \left(\frac{F_{j,k}}{F_{i,k}}\right)^{(i)} \cdot \delta \cdot \left(V_{i,k+1} - V_{i,(j),k+1}\right)$$

Solution

- In the first stage of the game firms choose their location, suppose l_i' and l_j'
- In the second (pricing) stage, the locations can be considered as state variables since they are already chosen at the first stage of the game
- The game will certainly be played for the last time in the last period, $(N-1)$, even if both firms have more than one unit capacity
- For the last period of the pricing stage, $(N-1)$, we can calculate the equilibrium prices by solving the 2x2 system (unique reduced state due to unit demand)
- Parameters r, s and δ are known, future expected values are zero, so the system consists of two algebraic equations and its solution:

$$p_{i,N-1}^*\left(x_{i,N-1}=1, x_{j,N-1}=1, l_i', l_j' \,\middle|\, r, s, \delta\right) \;\; and \;\; p_{j,N-1}^*\left(x_{i,N-1}=1, x_{j,N-1}=1, l_i', l_j' \,\middle|\, r, s, \delta\right)$$

- We can now compute the corresponding expected values:

$$V_{i,N-1}^*\left(x_{i,N-1}=1, x_{j,N-1}=1, l_i', l_j' \,\middle|\, r, s, \delta\right) \;\; and \;\; V_{j,N-1}^*\left(x_{i,N-1}=1, x_{j,N-1}=1, l_i', l_j' \,\middle|\, r, s, \delta\right)$$

Solution

- Moving backwards one period, only future expected values change that are no longer zero
- There are 4 states that can lead to the last period's (reduced) state
- Again, we can only consider the states that involve capacities up to two units and these are:

$$\left(x_{i,N-2} = 1, x_{j,N-2} = 1, l_i', l_j'\right) \qquad \left(x_{i,N-2} = 1, x_{j,N-2} = 2, l_i', l_j'\right)$$

$$\left(x_{i,N-2} = 2, x_{j,N-2} = 1, l_i', l_j'\right) \qquad \left(x_{i,N-2} = 2, x_{j,N-2} = 2, l_i', l_j'\right)$$

- We solve separately for each of these four states, and for each the 2x2 system consists of two algebraic equations
- We work in the same way backwards until we reach the first period's state $\left(x_{i,0}', x_{j,0}', l_i', l_j'\right)$

Solution

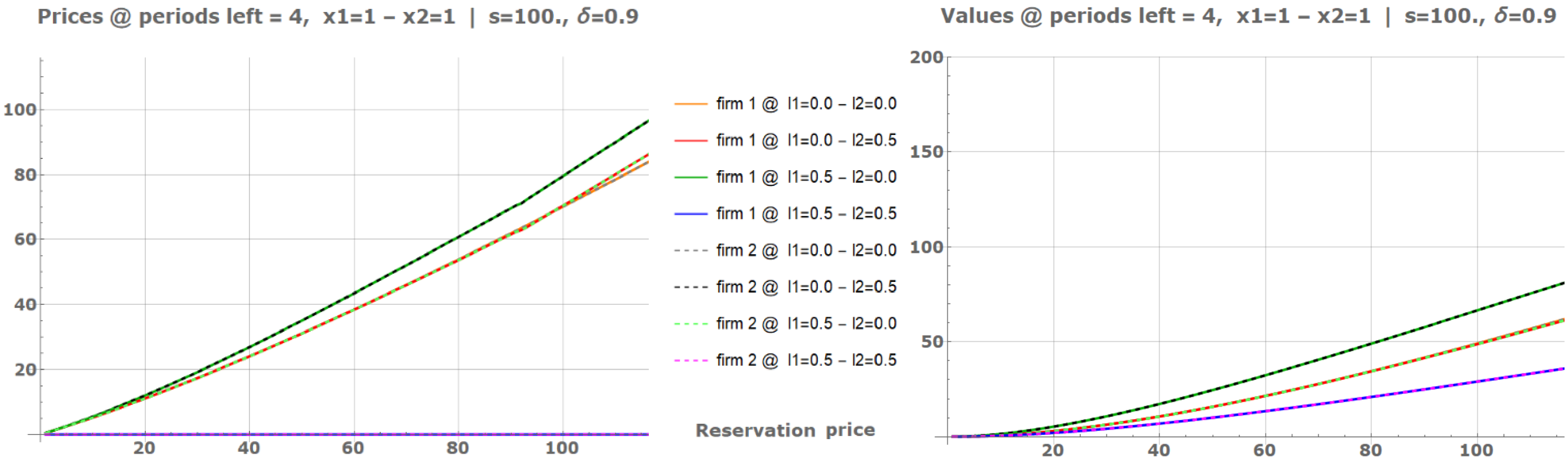
- Same procedure for the other locations' pairs
- We then compare the expected values of the first period among different locations' pairs to obtain the two-stage equilibrium
- In the same way, we can solve for any values of the parameters r, s and δ
- The solution to our problem is a **Markov Perfect Equilibrium (MPE)** because each firm's decision is based solely on the current state of the system and the actions of other firms (and not on any historical information), and where these strategies form a Nash equilibrium in every possible state of the system (Maskin & Tirole, 1988a,b, 2001), (Doraszelski & Escobar, 2010)
- That is, in our problem we have:
  i. Optimality given the current state
  ii. Optimality given the actions of other players
  iii. Consistency across states

Solution

- We assume that buyer's preferences follow a standard uniform distribution, $u_k \sim U(0, 1]$
- We can now obtain an analytical solution
- The formula for the probability of a sale depends on the location of the firms
- Probabilities are constrained by the endpoints of the market but also by the location of the rival firm
- The probability of a sale by a firm can also be interpreted as its "market share", or "market coverage", which is defined through market's structure
- In any state, we can distinguish 8 cases (Puu, 2016), depending on the level of prices
- These cases stem from the different set of probabilities of sale ($\mathcal{P}$) and the corresponding set of constraints ($\mathcal{C}$) on market shares for a well-defined problem
- Every set of constraints ($\mathcal{C}$) in each of the 8 pricing cases represents a different area in the solution domain

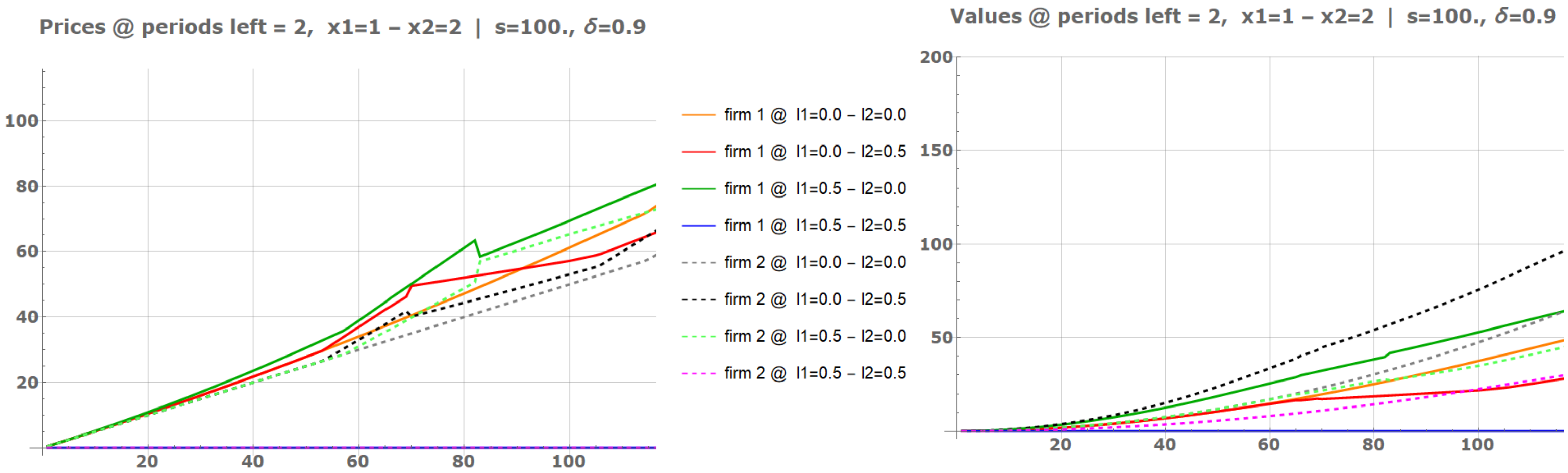
Equilibrium prices and locations equilibria

- If **capacities are equal**, the equilibrium prices in all periods are increasing in reservation price
- The corresponding expected values are increasing in reservation price at a decreasing rate as capacities (and available time) increase because of the discounting of future sales
- In all periods and when capacities are equal, no matter the reservation price, both firms prefer to be in the middle with their rival at an endpoint of the market (‘standard locations equilibria’)

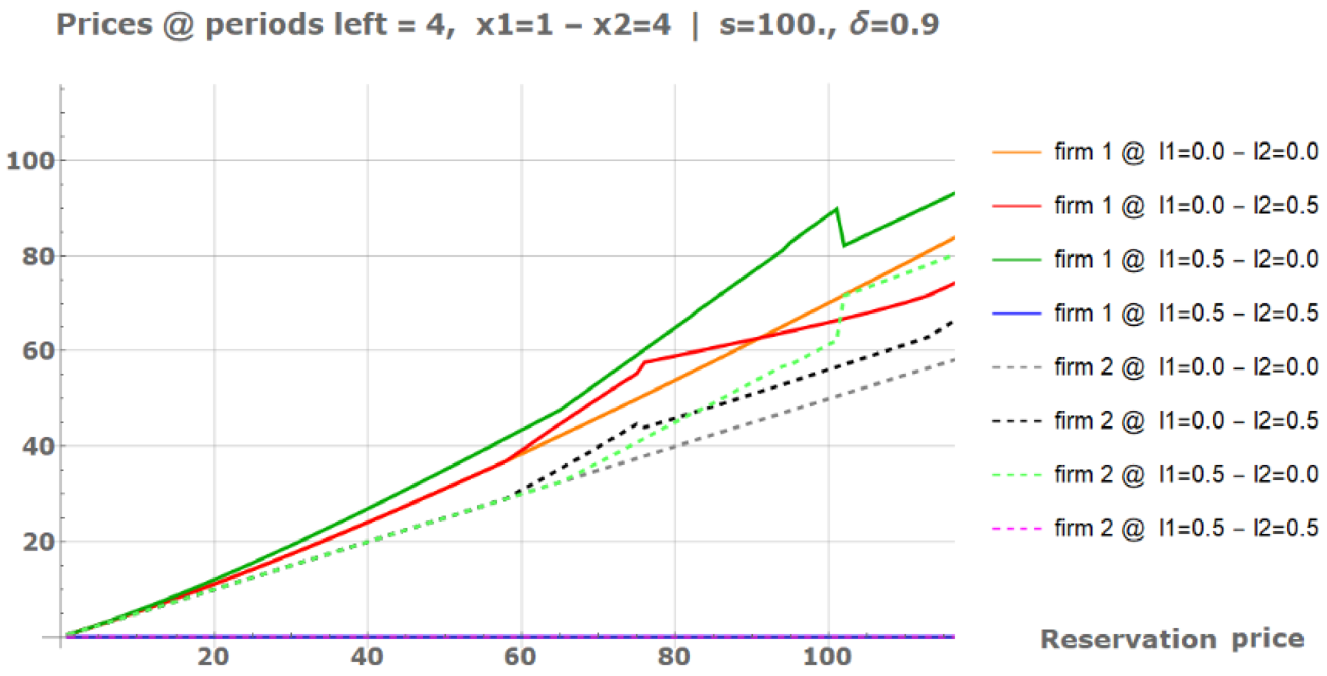


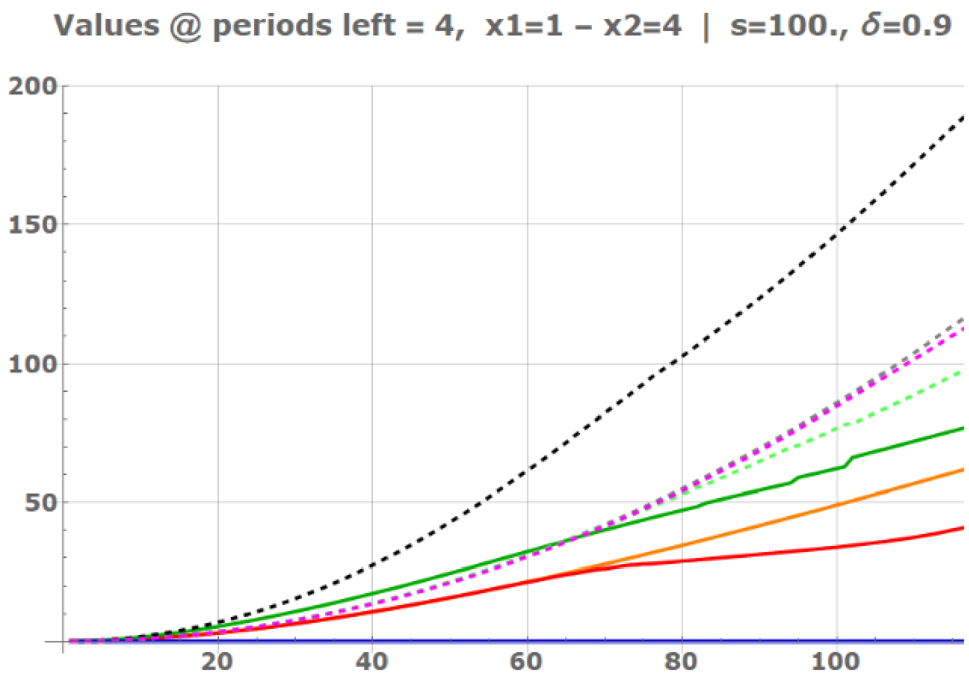
Equilibrium prices and locations equilibria

- If **capacities' asymmetry is low enough** both firms prefer to be located in the middle with their rival at an endpoint of the market ('standard locations equilibria')
- The asymmetry between firms' capacities must be defined with respect to the number of periods left since, for given asymmetric capacities, if the number of periods left is too high the difference between capacities becomes insignificant



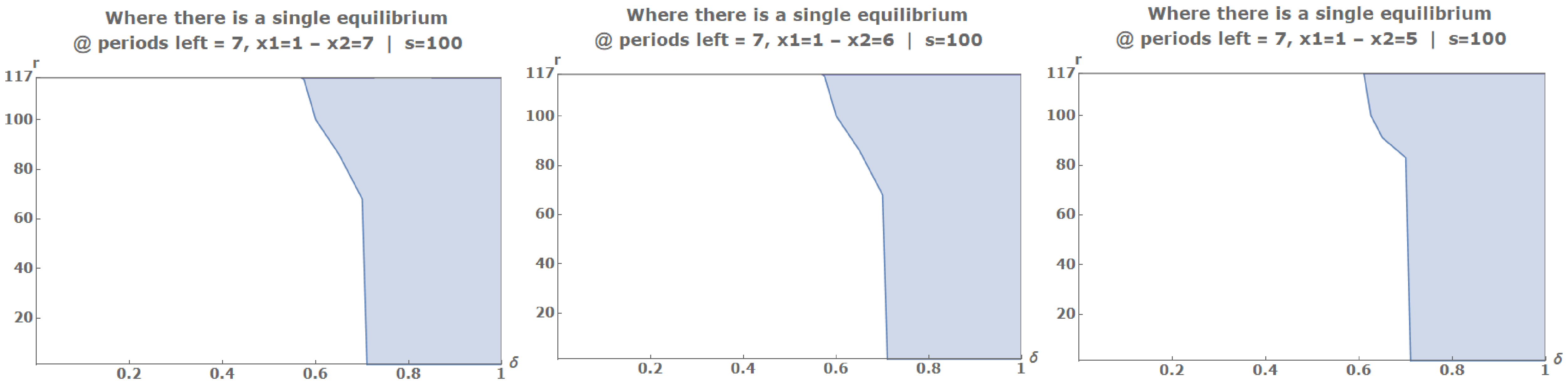
- If **capacities' asymmetry is high enough**, then the high-capacity firm strictly prefers both firms in the middle of the market than being at an endpoint with the rival in the middle
- The high-capacity firm chooses to locate in the middle of the market while the low-capacity firm chooses to locate at an endpoint ('single locations equilibrium')





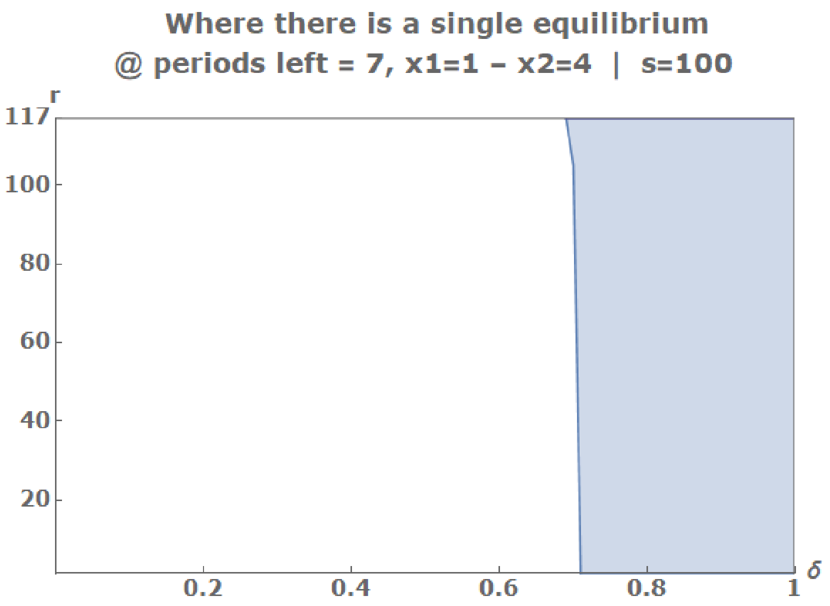
Single locations equilibrium

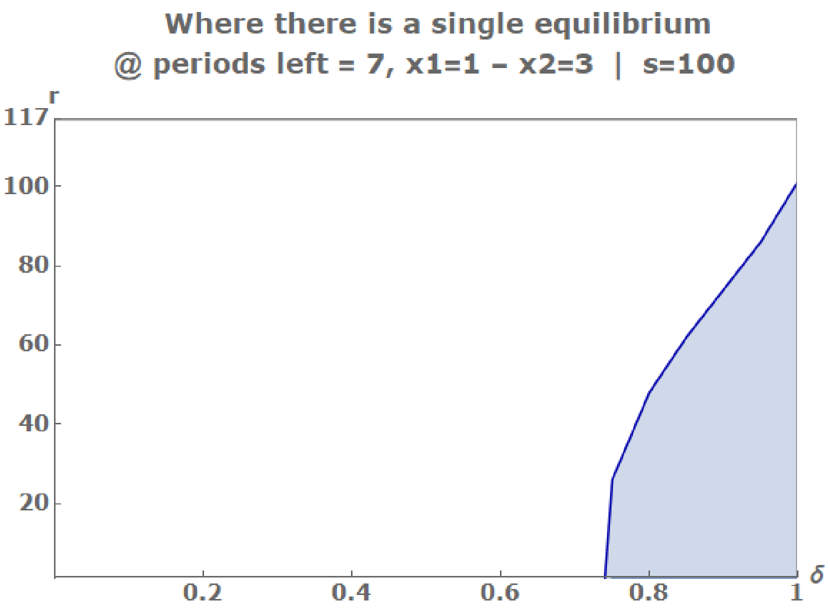
- For high capacities' asymmetry, a 'single locations equilibrium' exists for high δ when the reservation price is high enough and for higher δ when the reservation price is lower

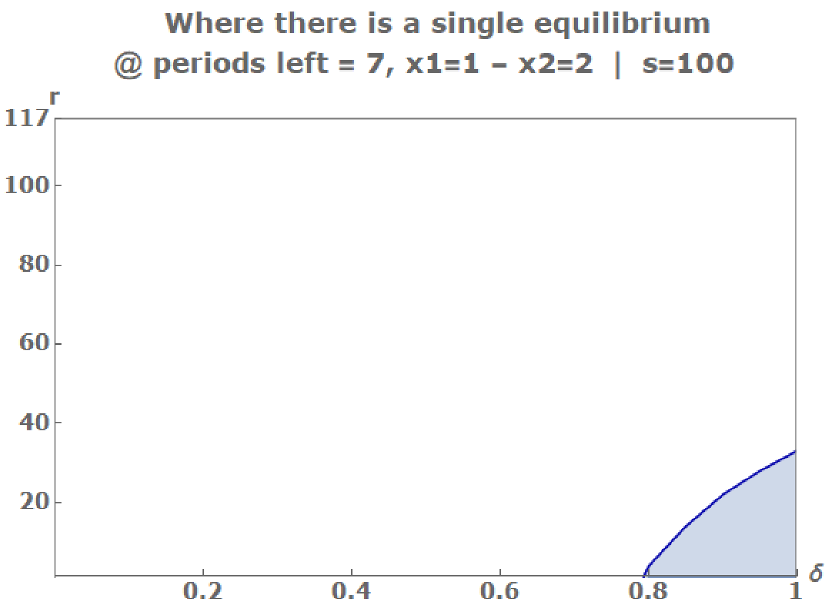


Single locations equilibrium

- For lower capacities' asymmetry (but high enough), a 'single locations equilibrium' exists for δ high enough that is almost the same for all levels of reservation price, and for low capacities' asymmetry only for high δ and low enough reservation price







Equilibrium prices for given locations

- The interaction between locations' asymmetries and capacities' asymmetries produces a rich equilibrium pricing behavior for firms
  - Given the remaining time, the level of competition increases in reservation price and locations asymmetry but decreases in discount factor, firms' distance, and capacities' asymmetry
- Given some asymmetric capacities and the remaining time, the subsequent firms' pricing behavior is significantly affected by which firm is in the middle
  - The firm in the middle has enough market power to dictate pricing strategies and extract more (expected) value from the buyer than its rival at the endpoint
- For high enough reservation price, if the high-capacity firm is at the middle of the market it will always post a lower price than its rival, but if the high-capacity firm is at an endpoint it will post a lower price only when the capacities' asymmetry is high enough
- On the other hand, for low capacities' asymmetry the high-capacity firm will always post a higher price than its rival
  - The capacities' asymmetry must be defined not only with respect to time but also with respect to firms' locations

Results with endogenous locations

- At some states with high enough capacities' asymmetry, only one equilibrium survives: the high-capacity firm at the middle of the market and the low-capacity at an endpoint
    - The high-capacity firm is willing to locate at the middle of the market with its rival and if necessary, sell some units at marginal cost
- The possibility that his rival will sell his unit first is enough for the high-capacity firm to positively consider the chance of losing some of its capacity since he will subsequently sell his remaining units as a monopolist in the middle of the market
    - This happens for a high enough discount factor and reservation price, because then the future expected value is higher
- The range of parameters that this result holds depends on the capacity asymmetry and the high-capacity firm's option value
    - If the latter is low enough, we have only one equilibrium for all levels of reservation price otherwise, only for low enough reservation values