"Downstream market uncertainty and upstream maximizing tariffs in franchising
by

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Abstract: In this paper we examine the form of a standard take-it-or-leave-it contract that an upstream firm (*U*) may offer to a set of potential downstream partners (*D*s) that are monopolists in their market. Examples are franchising with exclusive territories, or selling the right to use a patented invention in order to develop different final goods. The contract offered may be either linear (*l*) or two-part tariff (*t*). Each *D* must incur a start-up cost of different value, and *D*s are distributed according to that value. *D*s are risk-averse while *U* is risk-neutral.

We show that if risk-aversion is high or the probability of success is low, the fixed payment contained in *t* may be negative, due to the cost of uncertainty; in such cases, the input price is lower under *l*. Moreover, when the fixed transfer is positive, a larger number of *D*s accept the *l* rather than the *t* contract, but this result is also reversed when the fixed transfer is negative. While profit is always higher under *t*, the welfare ranking of the two tariffs is ambiguous since one may yield higher expected surplus per market, whereas the other higher expected number of products.

Keywords: Vertical relations, Innovation, Patents. Open source, Risk aversion, Two-part tariff, Linear tariff.

1. Introduction

While there are cases where contracts between an upstream and a downstream firm are negotiated, in other instances the upstream firm sets a fixed contract and attracts all the downstream firms willing to accept it. For instance, Lafontaine (1992) reports that in the franchise business, 42% of surveyed franchisors set take-it-or-leave-it contracts, while Lafontaine and Shaw (1999) shows that 75% of franchisors out of a large dataset offer their franchisees identical contracts. Also, in the plant-breeding industry, the patent owner of a genetically tailored basic plant cannot direct sell his product to consumers. He must sell to specialized breeders the right to develop *plant varieties* that can be directly consumable. The patent holder may sell (usually standard) contracts to the breeders.

In this paper we examine the form of a standard take-it-or-leave-it contract that an upstream firm (*U*) may offer to a set of potential downstream partners. This contract may be either linear (*l*) or two-part tarrif (*t*).[1] Each downstream firm (*D*) produces a different product, so, if successful, it is a monopolist in its market. Success is, however, is uncertain, since the demand of each product may be zero with some positive probability, independent across markets. Before starting business, a *D* must pay a fixed start-up cost, as well as any fixed transfer required by *U* in case of *t*. These costs are paid before the resolution of uncertainty; therefore, the operation may result in loss for the downstream firms. Downstream firms are risk-averse while the upstream firm is risk-neutral.[2]

In the above setting we show that a) in case of *t*, there is a possibility for negative fixed payments, due to the cost of uncertainty, expressed as either high risk of failure or high risk-aversion; b) under both *l* and *t* the input price is above marginal cost (already shown in Rey and Tirole (1986) and Lømo (2020)) and is decreasing on the cost of risk and the distribution of the fixed start-up cost ; c) the standard result that the input price is lower under *t* is reversed when the fixed payment is negative; d) when the fixed transfer is positive, a larger number of *D*s is accepting the contract, thus more products (or franchise locations) are available to consumers; this result is reversed when the cost of risk implies a negative fixed transfer; e) while profit is always higher under *t*, for consumer surplus and/or total welfare things may be otherwise, since for low cost of

[1] While linear contracts are unusual in the franchise industry, they have been proposed and strongly supported in the plant-breeding industry (see Van Overwalle, 2018) as *Open Source* (OS) system of IP protection, where the IP owner's payment consists of only postpaid royalties. Besides its real-life relevance, we proceed to the analysis of linear tariffication also for comparison purposes.

[2] The upstream neutrality is not a necessary assumption for the results to hold. However, it is plausible since the upstream firm deals with a potentially large number of downstream partners.

risk (positive transfer) *t* produces higher quantity per market but *l* yields a larger number of products, and the opposite holds for high cost of risk (negative transfer); However, when the cost of risk is low, even under *t* there is already a sufficiently large number of *D* firms, so the per market advantage of *t* is more important than the advantage of *l*. When the cost of risk is high, the number of products is important but, due to the negative transfer, it is again *t* than produces higher welfare. Nevertheless, there is a range of intermediate values of the cost of risk for which contrary to the standard result, welfare is higher under *l* than under *t*.

Milliou and Petrakis (2024) show that linear tariffs can be welfare-superior to two-part tariffs by inducing more entry in the downstream market. In particular, they consider an upstream supplier that bargains with downstream firms. Downstream firms compete against each other and their number is determined by a zero-profit condition. On the one hand, linear tariffs imply a more severe double margin problem leading to fewer firms entering the market as compared to a two-part tariff (inefficiency effect), but on the other hand, the absence of a fixed fee leads to more firms entering the market under the linear contract. They derive conditions under which a linear contract induces more entry than the two-part tariff and it is welfare superior.[3]

There are substantial differences between the present work and the Milliou and Petrakis (2024) paper. In the latter there is no uncertainty and the downstream firms produce substitute products, therefore competing in the same market. Central in our work is the presence of uncertainty and downstream firms develop new products potentially addressing different need in different markets (exclusive territories). Also, in Milliou and Petrakis (2024) contracts are private and decided by bargaining while *U* only decides whether they will be linear or TPT, whereas in our case contracts are pre-specified take-it-or-leave-it ones. Finally, the number of firms in that paper is determined by free entry, whereas in our work it constitutes an indirect decision of the upstream firm, via its tariff.

Our paper is also closely related to that of Constantatos and Pinopoulos (2023), in that crucial in the latter is also the presence of uncertainty and the existence of a start-up fixed cost. In that paper however, the upstream firm offers contracts to a single type of downstream firms, whereas in the present work downstream firms are differentiated with respect to the fixed cost they need to pay in order to develop their product or market. Thus, this paper completes and extends our

[3] Reisinger and Schnitzer (2012) obtain a similar result by also allowing free entry in the upstream market.

previous contribution by bringing into the analysis the number of firms finally participating in the downstream market.

The rest of the paper is organized as follows. In Section 2, we describe the model. In Section 3, we derive optimal input prices. In Section 4, we perform welfare analysis. In Section 5, we provide a linear demand example. Section 6 concludes the paper.

2. The model

Each downstream market is a monopoly (no competition between downstream firms). In the case of franchise, we assume that the upstream firm imposes exclusive territories. In the case of patents, each downstream developer introduces her own product, which is sufficiently differentiated from other developers' products to be considered as a separate market.

Entering the downstream market is a risky business: with probability of success $z \in (0,1)$ there may be a market with non-convex inverse demand $p(q), p' < 0, p'' \leq 0$, sufficiently large as to provide enough profit to cover any fixed costs incurred; with probability $(1 - z)$ the quantity demanded is zero at any price.

Before downstream production starts, each participating downstream firm (*D*) must pay a fixed cost F. This cost may represent expenses for building a production unit, advertising in the local market, or developing a specific final product that can be directly sold to consumers. In more general terms, F can be considered as the opportunity cost of *D*'s outside option. It must be paid before the state of demand is known and is irrecoverable in case of failure.

There is a large number of potential *D*s and corresponding products (markets). While we assume that all the potential markets are of equal size, *D*s differ with respect to the value of F that each one needs to pay. This may be due to either external reasons (local regulation, access to resources, etc.), or to the fact that they have different outside options. The upstream firm (*U*) is unable to observe the value of each individual *D*'s fixed cost, but knows that all the potential downstream entrepreneurs are distributed according to F and knows the characteristics or that distribution. For simplicity, we assume distribution to be uniform in the interval $[0, \overline{F}]$, with $\overline{F} > 0$ to be determined later on.

Besides F, a *D* has no other cost than what she must pay to *U*. The latter cost may consist of two parts, one fixed (transfer) payment $T \gtreqless 0$ (when required) and a variable cost $w \geq 0$ per unit of output sold; w may be the price of an input purchased form U, or, in the case of innovation, a

royalty. While the variable cost is paid after demand is revealed, the transfer T must be paid in advance, and it is not returned in case of the unlucky market outcome. The (T, w) pair represents a contract offered by *U* on a *take-it-or-leave-it* basis and the only option for *D* is to accept or reject it. Either part of the contract can be zero; contracts with the a priori restriction that $T = 0$ represent linear tariffs (l) whereas in the absence of such restriction the contract represents a two-part tariff (t).

Potential downstream firms are risk averse. Their utility is assumed to take the well-known "mean-variance" form:

$$U_i = \hat{\pi} - T - \lambda\sigma^2(w) - F_i \tag{1}$$

where

$$\hat{\pi}(w; z, \lambda) = \left(p\left(\hat{q}(w)\right) - w\right) \cdot \hat{q}(w)$$

is the expected net profit from a successful R&D, gross of any transfer payments to *U*, λ is the risk-aversion coefficient, common to all developers, and $\sigma^2(w)$ is the maximized-profit's variance between the states of success and failure.

We assume that the upstream firm produces the input at zero cost. Alternatively, in the case of IP, the basic invention has been already obtained and there is no variable or fixed cost in making it available to the *D*s. We assume that *U* is risk-neutral and offers a single contract—whether *l* or *t*—to all its potential customers. *U*'s risk-neutrality can be justified by the multiplicity of developers he is serving, who represent a multiplicity of independent projects.[4]

To sum up, the timing of the two-stage game is as follows. At the first stage, and before the demand-state is revealed, U makes each D a take-it-or-leave-it offer. If a downstream firm accepts, it pays F, and in case of a two-part tariff, it also pays the fixed fee T. At the second stage, and in case of fortunate demand-state, D decides the quantity and buys the necessary input at price w. In

[4] The risk-neutrality assumption is only for convenience; most of our results hold even if the *Ph* is also risk-averse.

case of no demand, neither F nor T are recovered. As usual, we solve the game backwards seeking subgame perfect Nash equilibria.

2. Optimal Input pricing

Under either contract and positive demand, D chooses q to maximize gross operating profits:

$$\max_q \pi_D = [p(q) - w]q.$$

The first-order condition (hereafter, FOC) of the above maximization problem is given by:

$$p - w = -qp'. \tag{2}$$

The second-order condition (hereafter, SOC) is satisfied when:

$$2p' + qp'' < 0. \tag{3}$$

We make here the following Assumption.

Assumption 1. *Downstream marginal revenue is concave, i.e.,* $3p'' + p'''q \leq 0$.

The solution of (2) determines firm D's derived demand for the input, $q(w)$. Using the implicit function theorem, we obtain from (2) that:

$$\frac{dq(w)}{dw} = \frac{1}{2p' + qp''} < 0, \tag{4}$$

which implies that derived demand is downward sloping. Using (4), we have that

$$\frac{d^2q(w)}{dw^2} = -\frac{3p'' + p'''q}{(2p' + qp'')^3} \leq 0. \tag{5}$$

where the negative sign stems from the SOC in (3) and Assumption 1. The expression in (5) implies that the derived demand is not convex.

Commented [XK1]: Not convex

Let $p\big(q(w)\big) \equiv p(w)$. D's operating profit is $\pi_D(w) = (p(w) - w)q(w)$. Recalling that in case of failure demand is zero, expected profit is

$$\hat{\pi}_D(w) \equiv E[\pi_D(w)] = z\pi_D(w) + (1-z)0 = z(p(w) - w)q(w). \tag{6}$$

The profit variance is

$$\sigma_D^2(w) \equiv E\left[\left(\pi_D(w) - \hat{\pi}_D(w)\right)^2\right] = E[\pi_D^2(w)] - [\hat{\pi}_D(w)]^2,$$

which can be written as:

$$\sigma_D^2(w) = z\pi_D^2(w) - z^2\pi_D^2(w) = z(1-z)[(p(w) - w)q(w)]^2. \tag{7}$$

Using (5) and (6), (1) becomes:

$$\hat{v}_D = \begin{cases} \hat{v}_D^T(w) = (\hat{\pi}_D(w) - F) - \lambda\sigma_D^2(w) - T & under\ T \\ \hat{v}_D^L(w) = (\hat{\pi}_D(w) - F) - \lambda\sigma_D^2(w) & under\ L \end{cases} \tag{1a}$$

Downstream returns are decreasing in *w*, but their variance is also decreasing in *w*, which may make the overall impact of *w* on $\hat{v}_D$ ambiguous. Whereas in our context the probability of success z is treated as exogenous, a similar counter-intuitive result could occur as z increases between 0 and 1.[5]

Lemma 1 (Constantatos and Pinopoulos, 2023). *Under either contract type, D's utility is strictly decreasing in w and strictly increasing in z iff:*

$$\lambda < \bar{\lambda} = \frac{1}{2(1-z)\pi_D(w)}. \qquad (dPer \text{ condition})$$

Proof. From $(1a)$, and using (2):

$$\frac{d\hat{v}_D}{dw} = \frac{d\hat{v}_D^T(w)}{dw} = \frac{d\hat{v}_D^L(w)}{dw} = -zq(w)[1 - 2\lambda(1-z)\pi_D(w)]. \tag{8}$$

The above expression is negative when the bracketed term is positive: solving the latter for λ yields the *dPer* condition. In an analogous manner, we have that:

[5] These are cases where the mean–variance utility function might not respect *First Order Stochastic Dominance*. See also Constantatos and Pinopoulos (2023).

$$\frac{d\hat{v}_D}{dz} = \frac{d\hat{v}_D^T(w)}{dz} = \frac{d\hat{v}_D^L(w)}{dz} = \pi_D(w)[1 - \lambda(1-2z)\pi_D(w)].$$

Obviously, the above expression is positive for $1 < z \leq 1/2$. For $0 < z < 1/2$, it is positive when $\lambda < \bar{\bar{\lambda}} \equiv 1/[(1-2z)\pi_D(w)]$, with $\bar{\bar{\lambda}} > \bar{\lambda}$. ■

We assume that no increase in w can make D better-off, i.e., $\lambda \leq \bar{\lambda}$ under any contract. We now turn to the upstream supplier's optimal pricing policy.

3.1. Linear tariff

We consider first the case of a linear tariff and assume for simplicity that he lower bound on F is zero, $\underline{F} = 0$.[6] Setting $w = 0$ imposes an upper bound on F:

$$\overline{F} \equiv \hat{\pi}_D(0) - \lambda\sigma_D^2(0) = zp(0)q(0) - \lambda\sigma_D^2(0). \tag{10}$$

U knows that no D with $F > \overline{F}$ will ever be interested in participating, so when deciding the contract to offer he only considers downstream firms with $F \in [0, \overline{F}]$. In order to avoid trivial cases, we assume that the distribution of Ds according to their fixed cost is uniform with density equal to $1/\overline{F}$ in the $[0, \overline{F}]$ interval, and zero elsewhere.

The critical value of F for which $\hat{v}_D^L(w) = 0$ is

$$F^L = \tilde{F}(w) \equiv \hat{\pi}_D(w) - \lambda\sigma_D^2(w). \tag{11}$$

Using (6) and (7), we have that

$$F^L \equiv \tilde{F}(w) = z(p(w) - w)q(w)[1 - \lambda(1-z)(p(w) - w)q(w)]. \tag{11a}$$

Note that

$$\frac{d\tilde{F}(w)}{dw} \equiv \tilde{F}' = \frac{d\hat{v}_D(w)}{dw} < 0, \tag{12}$$

[6] Interpreting fixed costs as the outside option of each downstream firm, then setting $\underline{F} = 0$ simply implies the presence of some downstream investors with no outside option. No result is affected if we set $\underline{F} > 0$.

where the negative sign stems from the dPer condition (see Lemma 1 and its proof). The expression in (12) implies that market coverage decreases with the input price: a higher w will result in fewer markets served. Using (12), we obtain:

$$\frac{d^2\tilde{F}(w)}{dw^2} \equiv \tilde{F}'' = \frac{d^2\hat{v}_D(w)}{dw^2} = -\frac{z}{2p' + qp''}\left[1 + 2\lambda(1-z)q[2p' + qp''(2p' + qp'')]\right].$$

The sign of the above expression is ambiguous.[7]

Let $\hat{R}(w) \equiv zwq(w)$ represent the upstream firm's revenue from selling input. We then have:

$$\frac{d\hat{R}(w)}{dw} \equiv \hat{R}' = q(w) + w\frac{dq(w)}{dw}. \tag{14}$$

From the above expression, we get:

$$\frac{d^2\hat{R}(w)}{dw^2} \equiv \hat{R}'' = 2\frac{dq(w)}{dw} + w\frac{d^2q(w)}{dw^2} < 0, \tag{15}$$

where the negative sign stems from (4) and (5).

The upstream supplier's target is:

$$\max_w \hat{\pi}_U^L(w) = \hat{R}(w) \cdot F^L \cdot \frac{1}{\overline{F}} = \frac{\hat{R}(w)\tilde{F}(w)}{\overline{F}}.$$

The FOC is equivalent to:

$$\hat{R}'\tilde{F} + \hat{R}\,\tilde{F}' = 0. \tag{16}$$

Note that since $\tilde{F}, \hat{R} > 0$ and $\tilde{F}' < 0$ (see (12)), in equilibrium it must hold that $\hat{R}' > 0$. The above can be rewritten as:

[7] Using (2), we have that $\pi_D = -p'q$ and the dPer condition in Lemma 1 can be rewritten as

$$\lambda < \bar{\lambda} = \frac{1}{2(1-z)p'q^2}.$$

Given the SOC in (3), the sign of $d^2\hat{v}_D(w)/dw^2$ is determined by the sign of the bracketed term in the RHS of (9). It can be checked that when the dPer condition is satisfied, the sign of $d^2\hat{v}_D(w)/dw^2$ can be positive, negative or zero.

$$\hat{R}'\frac{w}{\hat{R}} = -\tilde{F}'\frac{w}{\tilde{F}} \quad \Leftrightarrow \quad \eta_{\hat{R}}(w) = -\eta_{\tilde{F}}(w), \tag{17}$$

where η denotes the elasticity of the index-variable with respect to w. The last expression in (17) shows how the U treats the two margins, intensive and extensive: he sets w in such a way that the elasticity of the per-market revenue equals in absolute value the elasticity of total market coverage. In other words, in equilibrium, a 1% increase in w should cause an equal percentage increase in per-market revenue and reduction in market coverage.

Using (2), (4), (11a) and (14), the equilibrium input price, denoted by w^L, must satisfy:

$$w^L = -\frac{p'q(2p' + qp'')\big(1 + p'q^2\lambda(1-z)\big)}{(3p' + qp'') + p'q^2(5p' + 2qp'')\lambda(1-z)}. \tag{18}$$

Second order condition. The SOC requires:

$$2\hat{R}'\tilde{F}' + \hat{R}''\tilde{F} + \tilde{F}''\hat{R} \leq 0,$$

and since the first term is always negative, it is sufficient that

$$\hat{R}''\tilde{F} + \tilde{F}''\hat{R} \leq 0 \Leftrightarrow \frac{\hat{R}''}{\hat{R}} + \frac{\tilde{F}''}{\tilde{F}} \leq 0$$

This is a weighted sum of 2nd order derivatives, so it is sufficient to see whether $\hat{R}$ and $\tilde{F}$ are concave (or, at least, one concave and one linear). We know that $\hat{R}'' < 0$, so it suffices that $\tilde{F}''$ be non-convex. We assume that $\tilde{F}''$ is negative or not "too positive" so that the SOC above is satisfied. As shown in Section 5 below, the SOC is satisfied for the case of linear demand.

Second order condition. The SOC requires:

$$2\hat{R}'\tilde{F}' + \hat{R}''\tilde{F} + \tilde{F}''\hat{R} \leq 0.$$

The first two terms of the above expression are negative since $\hat{R} > 0$, $\tilde{F}' < 0$ and $\hat{R}'' < 0$. However, the sign of the last term is ambiguous since the sign of $\tilde{F}''$ is ambiguous. We assume that $\tilde{F}''$ is negative or not "too positive" so that the SOC above is satisfied. As shown in Section 5 below, the SOC is satisfied for the case of linear demand.

3.2. Two-part tariff

We now consider the case of a two-part tariff. The lower bound on F is zero, i.e., $\underline{F} = 0$. Setting $w = 0$ and $T = 0$ imposes an upper bound on F which is given in (10). Thus, as in the case of linear tariff, the PDF is $1/\overline{F}$.

The critical value of F for which $\hat{v}_D^T(w) = 0$ is

$$F^T = \tilde{F}(w) - T = \hat{\pi}_D(w) - \lambda\sigma_D^2(w) - T. \tag{19}$$

Using (5) and (6), we have that

$$F^T = \tilde{F}(w) - T = z(p(w) - w)q(w)[1 - \lambda(1 - z)(p(w) - w)q(w)] - T. \tag{19a}$$

The upstream supplier's target is:

$$\max_{w,T} \hat{\pi}_U^T(w) = \left[\hat{R}(w) + T\right] \cdot F^T \cdot \frac{1}{\overline{F}} = \frac{\left[\hat{R}(w) + T\right][\tilde{F}(w) - T]}{\overline{F}}.$$

The FOCs with respect to w and T respectively are equivalent to:

$$\hat{R}'\left(\tilde{F} - T\right) + \left(\hat{R} + T\right)\tilde{F}' = 0. \tag{20}$$

$$\left(\tilde{F} - T\right) - \left(\hat{R} + T\right) = 0. \tag{21}$$

From the FOC with respect to T in (21), we obtain:

$$T = \frac{1}{2}\left[\tilde{F} - \hat{R}\right]. \tag{22}$$

We then have

$$F^T = \tilde{F} - T = \tilde{F} - \frac{1}{2}\left[\tilde{F} - \hat{R}\right] = \frac{\tilde{F} + \hat{R}}{2} > 0. \tag{23}$$

Substituting (22) into the FOC with respect to w in (20), we get:

$$\left(\hat{R}' + \tilde{F}'\right)\left(\tilde{F} - T\right) = 0.$$

Given that $\tilde{F} - T = F^T > 0$, the above implies:

$$\hat{R}' = -\tilde{F}' \tag{24}$$

The above expression shows how *U* treats the two margins: a small increase in *w* must increase the per-market profit as much as it reduces the market coverage – *not* in percentage terms as in the case of a linear tariff. Denote the optimal input price that solves (24) as w^T. Deriving an expression for w^T, in an analogous manner as in (18), is cumbersome and not very useful for comparing input prices; but it is also not necessary as we show in the next subsection.

Second order conditions. Before closing this subsection, note that the SOCs require

$$\frac{\partial^2 \hat{\pi}_U^T}{\partial w^2} = 2\hat{R}'\tilde{F}' + \hat{R}''\left(\tilde{F} - T\right) + \tilde{F}''\left(\hat{R} + T\right) \leq 0,$$

$$\frac{\partial^2 \hat{\pi}_U^T}{\partial T^2} = -2 < 0,$$

$$\frac{\partial^2 \hat{\pi}_U^T}{\partial w^2}\frac{\partial^2 \hat{\pi}_U^T}{\partial T^2} - \left(\frac{\partial^2 \pi^T}{\partial w \partial T}\right)^2 \geq 0.$$

We assume that the above SOCs are satisfied, which is indeed the case when demand is linear.

3.3. Comparisons

Recall that $\hat{R}' > 0$ implying that a small increase in *w* increases *U*'s revenue. Also, $\tilde{F} < 0$ since a small increase in *w* reduces *ceteris paribus* the number of downstream participants and therefore *U*'s profit. The optimal *w*, whether under linear or two-part tariff is found by equilibrating the two margins. From (24) we see that when *U* uses *t*, he simply equalizes the two margins, because the use of *T* allows him to determine the two margins separately. When *U* uses *l*, separate determination of the two margins is impossible, and for this reason the two margins must be weighted according to their importance. The first part of the optimality condition in (17) shows exactly this: $\hat{R}'$ is weighed by $1/\hat{R}$, whereas $\tilde{F}'$ is weighed by $1/\tilde{F}$; which one is larger?

Let $\xi = \lambda(1-z)$ and let us suppose that for some value of ξ, call it ξ_0, the optimal input price is the same under the two tariff schemes. Obviously, $w^L = w^T$ implies $\hat{R} = \tilde{F}$, and this allows us to determine ξ_0 as:

$$\xi_0 = \frac{p-2w}{q(p-w)^2}. \tag{25}$$

It is straightforward to show that: (i) when $\hat{R} < \tilde{F}$ and thus $\xi < \xi_0$, it holds $w^L > w^T$ and $T > 0$; (ii) when $\hat{R} > \tilde{F}$ and thus $\xi > \xi_0$, it holds $w^L < w^T$ and $T < 0$.

Are there admissible values of ξ such that the two contracts coincide? Noting that $\pi_D = -p'q^2$, the dPer condition (Lemma 1) can be written as:

$$\bar{\xi} = -\frac{1}{2p'q^2}.$$

We then obtain the following result.

Proposition 1. *There exists a* $\xi_0 \in \left(0, \bar{\xi}\right)$*, with*

$$\xi_0 = -\frac{1}{(5p' + 2qp'')q^2}, \tag{26}$$

such that $T = 0$ *and*

$$w^L(\xi_0) = w^T(\xi_0) = -\frac{2p'q(2p' + qp'')}{(5p' + 2qp'')}. \tag{27}$$

Proof. Suppose that there exists a ξ_0 such that $w^L = w^T$ and hence $T = 0$. In such case, the optimal input price w^L satisfies $T(w^L) = 0$. Substituting (18) into (22), and after using (2), (4) and (11a), we solve $T = 0$ for ξ and obtain two roots:

$$\xi_1 = -\frac{1}{p'q^2} > 0, \qquad \xi_2 = -\frac{1}{(5p' + 2qp'')q^2} > 0.$$

It is straightforward that $\xi_1 > \bar{\xi}$ so we discard the first root. Using the SOC downstream in (3), it holds $\xi_2 < \bar{\xi}$. Therefore,

$$\xi_0 = -\frac{1}{(5p' + 2qp'')q^2}.$$

Substituting the above into (18), we obtain:

$$w^L(\xi_0) = w^T(\xi_0) = -\frac{2p'q(2p' + qp'')}{(5p' + 2qp'')}.$$

■

From the preceding discussion and Proposition 1, we obtain the following Corollary.

Corollary 1.

(i) For any $\xi \in (0, \xi_0)$, it holds that $w^L > w^T and\ T > 0$.

(ii) For any $\xi \in (\xi_0, \bar{\xi})$, it holds that $w^L < w^T\ and\ T < 0$.

For low values of risk aversion and/or high probability of success, we have the standard results: the input price is higher under the linear tariff and the fixed fee under a two-part tariff is positive. However, for high values of risk aversion and/or low probability of success, the standard results are reversed: the input price is lower under a linear tariff and the fixed fee is negative. In the latter case, we have from (21) that $\left(\hat{R} + T\right) = F^T > 0$: even though *U* ends up subsidizing downstream fixed costs, never covers them entirely.

It is well known that, low-demand purchasers exercise a positive externality to high-demand purchasers, for the presence of the former, lowers the price paid by the latter. In a two-part tariff, the presence of the low-demand agents guarantees positive surplus to the high-demand ones. The peculiar thing in the presence of uncertainty is that in order to accommodate some customers with high *F* (low net willingness-to-pay) *U* is ready not only to forgo the proceeds of the fixed transfer, but to also *pay to all the Ds* part of their fixed cost.

4. Welfare analysis

As welfare in each market, we consider the gross value of the equilibrium quantity of the product, minus the set-up cost and the cost of risk.[8] Total welfare is the summation of each market's net value over all the markets of the participants who decide to accept the contract

[8] The price of the input or any royalty, as well as any fixed payment between the *U* and the *D* are considered pure transfers.

$$W = \int_0^{F^i} \int_0^{q^i(w^i)} [zp(q; w^i) - \lambda\sigma^2(q) - F] dq\, dF\,, \quad i = l, t.$$

Name $V(q) \equiv zp(q; w^l) - \lambda\sigma^2(q)$, a measure of the expected per-market value, gross of the fixed cost. The term V corresponds to the total area under the demand curve minus the cost of risk; while demand is positive only with probability z, the cost of risk is borne with probability one. Thus, total welfare can be written as:

$$W = \int_0^{F^i} \int_0^{q^i(w^i)} [V(q) - F] dq\, dF = F^i z \int_0^{q^i(w^i)} V(q) dq - \frac{{F^i}^2}{2}, \quad i = l, t.$$

Relatively small changes in welfare can be decomposed in two parts, one due to changes in the input price and the other related to the number of markets. The first is equal to the change in value due to a change in final-product quantity generated by a change in *w* times the number of markets served. The second,

$$dW = \frac{\partial W}{\partial w} dw + \frac{\partial W}{\partial F} dF = F^i z V' \frac{\partial q^i}{\partial w} (w^l - w^t) + z\, F^i \int_0^{q^i(w^i)} V(q) dq\ \ (F^l - F^t).$$

Hence, the change in welfare after a change in the input price charged is a weighted average between the change in the per market net value and the change in market coverage.[9] Since $\xi \leq (\geq) \xi_0$ implies $w^l \geq (\leq) w^t$ but also $F^l \geq (\leq) F\hat{}t$, each type of tariff produces higher welfare on the intensive and the other on the extensive margin, according to the situation. Note however that, when $\xi \leq \xi_0$, there are already many downstream participants, so the welfare contribution of some additional ones is relatively small,[10] hence, the weight attached to $(F^l - F^t)$ term is relatively low compared to the one attached to the $(w^l - w^t)$ term. This implies that the two-part tariff produces

[9] Note that $\frac{\partial q^i}{\partial w} < 0$, therefore the 1st term is negative $w^l > w^t$.

[10] I.e., the weight attached to $(F^l - F^t)$ term is relatively low compared to the one attached to the $(w^l - w^t)$ term.

higher welfare. When $\xi \geq \xi_0$, an increase in the number of participants matters more, and it is again the two-part tariff that produces higher welfare. However5, since the evolution of the weights as functions of ξ is non-linear, nothing can be said about the relative welfare between *l* and *t* for intermediate values of ξ without further specification of the model, except perhaps for the obvious fact that at ξ_0 both tariffs produce identical welfare.

Constantatos and Pinopoulos (2023) shows that in situations where the *U* faces a single downstream firm and finds optimal to subsidize part of her fixed cost, while *t* is more profitable than *l*, the latter produces higher welfare. This is so because with a single agent there is only the per-market (intensive margin) effect, which is favored by the lower input price. That paper's analysis concludes that the upstream participation in the start-up cost of the downstream firm is not necessarily a socially desirable action, since it may aim at keeping the input price further away from marginal cost. This effect is also present in this paper, however, the inclusion of the extensive-margin effect related to changes in the ex-ante number of downstream products shows that upstream participation in the payment of *F* may be beneficial since it attracts more *D*s, thus creating a larger number of potential products. As it turns out, if ξ is sufficiently high, the advantage of *t* at the extensive margin is more important than the advantage of *l* at the intensive margin, thus making *t* not only more profitable but also welfare superior.

5. Linear demand example

In this section we consider a linear demand example: $p(q) = 1 - q$, with $p' = -1$, $p'' = 0$ and $p''' = 0$. Downstream equilibrium outcomes in case of success are:

$$q(w) = \frac{1-w}{2}, \qquad p(w) = \frac{1+w}{2}, \qquad \pi_D(w) = \frac{(1-w)^2}{4}.$$

Note that $w < 1$ must hold and that Assumption 1 is satisfied.

Expected profit and downstream variance are respectively:

$$\hat{\pi}_D(w) = z\frac{(1-w)^2}{4}, \qquad \sigma_D^2(w) = z(1-z)\frac{(1-w)^4}{16}.$$

The dPer condition can be written as:

$$\lambda < \bar{\lambda} = \frac{2}{(1-z)(1-w)^2} \quad \Leftrightarrow \quad \xi < \bar{\xi} = \frac{2}{(1-w)^2}.$$

Using (27) and (26), we have that

$$w^L = w^T = \frac{2}{7}, \qquad \xi_0 = \frac{165}{125}.$$

6. Conclusions

In this work we considered an upstream monopolist (*U*) selling to a large number of potential downstream partners (*D*s). Each one of the latter must develop her own exclusive market, by developing either a new product based on some innovation patented by *U*, or a demand for *U*'s products in a given territory (franchise). The development of a new downstream market is uncertain and requires the up-front payment of a fixed cost that is not recoverable in case of failure. All *D*s are risk-averse with similar risk-aversion coefficient, but each one faces a different start-up cost requirement. While *U* cannot observe each individual *D*'s such cost, he knows the distribution of *D*s according to their start-up cost. As a result he sets a single contract, whether linear or two-part tariff, for all the downstream agents on a take-it-or-leave-it basis, and lets *D*s accept or reject it. We compare equilibrium input prices, number of *D*s contracting (market coverage) and welfare between a linear and a two-part tariff contract.

We compare two are distributed and to the same extent, but face different start-up cost requirements; we assume a uniform distribution according or b or product in which she will act as a monopolist. bilateral monopoly in a vertical chain where the downstream part (*D*) has to pay a fixed R&D cost for the development of a new product market. Success is uncertain but in the case that it occurs, the *D* firm is based on some basic innovation owned by the upstream part (Ph). The D faces uncertainty in that its R&D may not be successful and is risk-averse; for simplicity we have assumed that the Ph is risk-neutral. The Ph makes take-it-or-leave-it offers which the D may accept or reject.

We find that for low levels of risk-aversion, linear pricing provides more insurance, thus mitigating part of its inherent inefficiency due to double marginalization. Nevertheless, as long as the optimal transfer under a two-part tariff contract is positive, the equilibrium input price, albeit

above marginal cost (see Rey and Tirole, 1985), is lower than under a linear contract. This result can however be reversed when *D*s are highly risk-averse: the input price is lower under a linear contract. This happens because under a two-part tariff contract *U* provides insurance to *D* by participating to the payment of her start-up cost, but in exchange asks for higher input price. In such a case, while the two-part tariff is still more profitable, a linear contract produces higher output and welfare in any given market.

The social desirability of one or the other tariff is challenged if we consider also that the common-to-all contract determines also the expected number of markets, and thus total ex-ante surplus via the number of products potentially available. As we show, the contract type that requires a lower ex-ante payment from *D* to *U* is the one that attracts the larger number of downstream participants. Therefore, as long as the two-part tariff contract contains a positive payment, it attracts fewer participants. Thus, while the two-part tariff is more efficient in creating per market surplus, its superiority is challenged when the associated smaller number of products is considered. The situation is again reversed for high levels of risk cost: the negative transfer included in the two-part tariff lowers the level of the startup cost that is at risk, thus attracting more participants. In conclusion, for low levels of risk cost, compared to a linear contract, the two-part tariff contract produces higher per market surplus over a lower number of markets, while for high risk-cost, it produces lower surplus per market but over a larger number of markets.

The overall welfare assessment of the two contract types seems ambiguous and it turns out to be so. However, note that when the cost of risk is small, there must be already a large number of l products potentially available, so the marginal benefit of adding an extra one is small; at the same time, we show that the divergence of the input price from marginal cost is relatively high, so a marginal reduction of the input price may create larger additional surplus in each market. It follows that a when the cost of risk is small, a marginal input-price reduction is more valuable than a marginal increase in the number of products, therefore the two-part tariff produces more surplus. When the cost of risk is high, the small number of potential products makes an increase in participation more important than an increase in the per-market surplus, therefore, *the two-part tariff is again welfare superior*. However, for intermediate levels of risk-cost, the linear contract may produce higher welfare. This happens when the cost of risk above the level that makes the two contracts equivalent, but still not too high.

Extensions are to consider a) a different distribution of downstream agents, b) downstream agents differing according to their risk-aversion coefficient, c) downstream firms differing according to their expected market size.

Allowing for risk-averse *U* does not change the results significantly. It can be shown that even if *U* is more risk-averse than any of the *D*s, there are still some levels of downstream risk-aversion for which is optimal for *U* to subsidize part of *D*s startup cost.

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