General Equilibrium, Welfare and Policy when Firms have Market Power*

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Abstract

We consider a market economy in which consumption goods are produced using high- and low-skilled labor. When firms have market power, the economy's surplus, aggregate income, employment, wages, and labor share are smaller than those under perfect competition. A low binding minimum wage alleviates the inefficiency and distributional bias caused by market power, without creating unemployment. Revenues from non-distortionary corporate taxes may be used to fund distributional policies (e.g., unemployment subsidies) and/or public spending enhancing production possibilities (e.g., education/innovation programs). Free entry leads to efficient entry under perfect competition but may lead to either excessive or insufficient entry when firms have market power.

Keywords: General Equilibrium; Market Power; Oligopoly; Monopoly; Wage Markdowns; Labor Share; Income Distribution; Minimum Wages; Unemployment.
JEL Classification: D4, D5, D6, L1, L4.

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List of Symbols	
$m \in \mathbb{N}$	Number of Consumption Goods
$n \in \mathbb{N}$	Number of Firms per Sector
$h \in (0, \infty)$	High-Skilled Labor of a Firm
$l \in (0, \infty)$	Low-Skilled Labor of a Firm
$A \in (0, \infty)$	Technology Parameter
$\alpha, \beta \in (0, 1)$:	Returns to Scale Parameters
c_i	Consumption Good i
c	Composite Good
$\theta \in (1, \infty)$:	Elasticity of Goods' Substitution
$\varepsilon \in (0, \infty)$	Elasticity of Labor Supply
$\lambda \in (0, 1)$	Fraction of High-Skilled Workers
$\eta :=$	$\frac{1}{1 + (1 - \alpha - \beta)\,\varepsilon} \in (0, 1]$
$\gamma :=$	$\frac{(1 + \theta mn - m)\varepsilon}{(1 + \varepsilon mn)\theta} \in (0, 1)$
$p_i \in (0, \infty)$	Price of Good $i \in \{1, ..., m\}$
$v \in (0, \infty)$	Wage of High-Skilled Labor
$w \in (0, \infty)$	Wage of Low-Skilled Labor
W	Surplus
Y	Aggregate Income
Π	Aggregate Profits
H	Employment of High-Skilled Labor
L	Employment of Low-Skilled Labor
LS	Labor Share
SP	Skill Premium
MDv, MDw	Wage Markdowns

1 Introduction

Market power pervades modern economies and is increasingly viewed as a source of significant inefficiencies and inequalities; see Eeckhout (2021) for an account of the severe consequences of the prevalence of market power following technological advancements and the rise of superstar firms over the past 40 years, and Shapiro (2018) for a discussion of the decline in competition in the US over the past 20 years. However, a general equilibrium analysis of a market economy in which firms exercise market power has proven to be elusive. Consequently, economists' benchmark ideas about price formation and performance tend to arise from the analysis of a perfectly competitive economy. Thus, it is a common perception that market economies produce satisfactory outcomes, viewing market power as affecting specific markets, which require suitable intervention.

While partial equilibrium analysis provides basic insights into how market power affects efficiency and surplus distribution, our understanding of the interactions between performance, price formation, and the degree of market power in the entire economy is somewhat lacking. Previous attempts to model a market economy in which firms exercise market power confronted difficult issues related to firms' objectives and the interplay between firms and consumer behavior, rendering the analysis restrictive or controversial. For instance, Gabszewicz and Vial's (1972) analysis of an economy in which firms with market power compete à la Cournot is somewhat inconclusive, because the equilibrium depends on the choice of the numeraire good.

We study a market economy in which firms produce consumption goods with non-increasing returns to scale using high- and low-skilled labor as inputs. Firms are owned by consumers who use their shares of the firms' profits to buy goods. Among consumers, there is a unit mass of workers who supply high- and/or low-skilled labor and care about their leisure and consumption. We assume that the fraction of workers who can supply high-skilled labor is relatively scarce, which is the case of interest. Neither public goods nor production/consumption externalities exist, and uncertainty is absent. Thus, perfect competition generates Pareto optimal allocations.

Consumers care about their consumption of a composite good whose composition is modelled following a related literature in industrial organization and international trade, e.g., Dixit and Stiglitz (1977), Atkeson and Burstein (2008), Allen and Arkolakis (2018).

This composite good naturally identifies the numeraire. Likewise, we postulate workers' preferences for consumption and leisure that give rise to a constant elasticity supply of labor, as is common in the recent literature on the impact of market power in labor markets – see Berger, Herkenhoff and Mongey (2022) and Deb *et al.* (2024) – and on the influence of firms' ownership structure on the equilibrium of oligopolisitic economies – see Azar and Vives (2021).

We consider competitive, monopolistic, and oligopolistic variations in this market economy, and identify the corresponding equilibria. In a competitive economy, firms act as price-takers in all markets. Under imperfect competition, firms exercise market power in the markets for goods and labor; that is, firms internalize the impact of their output and labor decisions on both prices and wages. In the extreme case of a monopolistic economy, a single agent exercises monopoly power in the markets for goods and monopsony power in the labor markets. In an oligopolistic economy, firms compete à la Cournot; that is, each firm chooses its output and its high- and low-skilled labor, internalizing the impact of its choices on prices and wages, given its competitors' choices.

Market power leads to significant deadweight losses and price distortions. Specifically, employment, aggregate (real) income (i.e., the output of the composite good), and surplus are smaller than under perfect competition. Moreover, wage markdowns arise that shift the distribution of income in favor of firms' owners, reducing the labor share. In an oligopolistic economy, the magnitude of these effects increases with the degree of concentration and consumers preference for good variety.

Cost minimization identifies the ratio of high- to low-skilled labor used by a firm, which is independent of its market power. Thus, in our symmetric setting, market power does not affect the skill premium (i.e., the ratio of the wages of high to low-skilled labor), although it affects wage levels. The increase in the skill premium, and hence in wage inequality, observed in recent times, is attributed by Deb *et al.* (2024) to the combined effect of market power exercised by heterogeneous firms and skill-biased technological change.

Contrary to common wisdom, if the number of firms is below a certain threshold, then the surplus, aggregate income, employment, wages, labor share, and aggregate profits are larger in a monopolistic economy than in an oligopolistic economy, whereas wage markdowns are smaller. Moreover, the monopolistic equilibrium allocation Pareto dominates the oligopolistic equilibrium allocation. These results arise because the monopolist's interests are better aligned with those of society than with those of Cournot competitors. While the monopolist trades off the impact of its labor and output decisions on both the surplus and its distribution, and hence is concerned with horizontal complementarities across all sectors of the economy, a Cournot competitor is exclusively concerned with its own profit and ignores these horizontal complementarities. As the number of firms increases, intensified competition leads the oligopolistic allocation towards the competitive allocation, reducing the impact of firms' incentives and eventually reversing the inequalities noted above. A version of this result was derived by Azar and Vives (2021), who showed that the sign of the effect on the wage markdown of a change in economy-wide common ownership of firms is determined by a condition identical to the one we identify. In their setting, common ownership helps align the interests of Cournot competitors with those of society.

The surge in market power in recent times, and the findings about its negative impact on performance and distribution, suggest the need for inquiry into alternative policy interventions. Certainly, antitrust and competition policies can effectively reduce inefficiencies, although their implementation and effects require time. We examine the impact of minimum wages on the economy's equilibrium. The effectiveness of price cap regulation in countering the inefficiencies created by market power is a recurrent topic in industrial organization – see Lemus and Moreno (2017). In the labor literature, market power is a recurrent topic – see the survey of Bhaskar, Manning and To (2002), and Jarosch, Nimczik and Sorkin's (2024) empirical study on the effect of concentration on wages. The insight that a minimum wage can increase employment goes back to Robinson (1969) under monopsony, and to Manning (2013) under monopsonistic competition. More recent studies on the impact of minimum wages in partial equilibrium include Azar *et al.* (2024), who show that their positive effect is larger in highly concentrated labor markets, and Tong and Ornaghi (2022), who study their effectiveness in a sector in which firms exercise market power in both product and labor markets. In a general equilibrium setting, Berger, Herkenhoff and Mongey (2024) study the efficiency and distributional effects of minimum wages in a model calibrated to the US economy.

In a competitive economy, a minimum wage above the equilibrium wage of low-skilled labor reduces the demand for this type of labor, and owing to the complementarity of inputs, it also reduces the demand for high-skilled labor. Hence, the surplus, aggregate income, aggregate profits, employment, wage of high-skilled labor, and skill premium decrease. The labor share remains constant because in a competitive economy the wage of each type of labor equals the value of its marginal productivity. Moreover, unemployment of low-skilled workers arises. For sufficiently high minimum wages, unemployment of high-skilled workers also arises. The magnitude of these effects increases with the minimum wage. Thus, minimum wages have negative welfare affects, except for employed low-skilled workers.

In a monopolistic economy, the effects of variations in the minimum wage differ in intervals of low, intermediate, high, and extreme minimum wages. The surplus and aggregate income increase with the minimum wage in the interval of low minimum wages, whereas they decrease in the intervals of intermediate and extreme minimum wages. In the interval of low minimum wages, the employment and wage of high- and low-skilled labor increase with the minimum wage: The marginal cost of low-skilled labor decreases below its marginal revenue, leading the monopolist to increase its low-skilled labor, which in turn increases the productivity of high-skilled labor. Thus, increasing the minimum wage in the interval of low minimum wages has a positive effect on economic performance without creating unemployment. With a minimum wage above the interval of low minimum wages, unemployment among low-skilled workers arises. With extreme minimum wages unemployment of high-skilled workers also arises. In the interval of high minimum wages, the employment of high-skilled workers increases with the minimum wage, but the effect on the employment of low-skilled workers, aggregate income and surplus is ambiguous. Increasing the minimum wage in the interval of (extreme) intermediate minimum wages has a negative effect on performance because it increases the monopolist marginal cost of (high- and) low-skilled labor above its marginal revenue, thereby leading to a reduction in the employment of this type of labor. Thus, the maximum surplus and/or aggregate income may be reached either at the largest low or at the largest high minimum wage, depending on the parameters.

The markdown on the wage of low-skilled labor decreases with the minimum wage in the interval of low minimum wages and disappears for larger minimum wages. The markdown on the wage of high-skilled labor decreases with the minimum wage in the interval of high minimum wages and disappears for extreme wages. The skill premium decreases in the intervals of low and intermediate minimum wages and disappears for larger minimum wages. However, unemployment among low-skilled workers adds another source of labor income inequality. The labor share increases with the minimum wage, reaching its value at the competitive equilibrium for extreme minimum wages.

The welfare effects of minimum wages in a monopolistic economy are complex. Obviously, profits, and hence the welfare of owners, decrease with the minimum wage. In the interval of low minimum wages, the welfare of high- and low-skilled workers increases with the minimum wage because both their wages increase and there is full employment. With higher minimum wages, unemployed workers become worse off than they are in the absence of a minimum wage. The wage and labor income of high-skilled workers decrease with the minimum wage in the interval of intermediate minimum wages but increase in the interval of high minimum wages.

An oligopolistic economy with a minimum wage may have multiple equilibria. We focus on the symmetric equilibrium, which is the most efficient, and hence provides a best-case scenario to assess the impact of minimum wages. We show that minimum wages have qualitatively analogous effects on the symmetric equilibrium of an oligopolistic economy to those in a monopolistic economy. The differing effects relate to the thresholds identifying the intervals of low, intermediate, high, and extreme minimum wages, which involve all parameters that determine firms' market power. In addition, although the markdowns on the wages of high- and low-skilled workers decrease with the minimum wage, they remain positive. Likewise, the labor share, which is maximal at the highest minimum wage, remains below the competitive labor share. Moreover, as the number of firms increases, a low minimum wage maximizes the surplus and aggregate income. Hence, a minimum wage helps alleviate inefficiencies and distributional distortions of market power.

Corporate taxes (i.e., a proportional tax on firms' profits) neither affect the solution to a firm's profit maximization problem nor affect consumers' decisions because of the lack of income effects in our setting, and therefore, do not affect the economy's performance.

Thus, a corporate tax is an appropriate instrument for pursuing distributional objectives such as providing funding for unemployment subsidies. In addition, a corporate tax may provide funding for public spending. In contrast, taxes on labor income are distortionary because they affect the opportunity cost of leisure, reducing the supply of both types of labor, and hence reducing the surplus, aggregate income, employment and profits.

We consider the impact of the provision of public education aimed at increasing the proportion of high-skilled workers in the labor force. Such programs lead to an increase in surplus, aggregate income and employment of high-skilled labor. Moreover, the wage of low-skilled labor increases, while the wage of high-skilled labor decreases, and therefore the skill premium decreases. However, the effects on the employment of low-skilled labor depends on the parameters. Furthermore, when the program's funding source are the revenues from a corporate tax, the combined net effect of the tax and public spending programs on aggregate profits is below the tax rate. We show that the cost-benefit analysis of such programs is less favorable in economies in which firms have market power. Other public spending programs worth considering are those aimed at improving the technology. We briefly discuss the efficiency and distribution effects of such programs.

The existence of extraordinary rents captured by firms suggests considering whether such rents may persist or disappear in the long run. To this end, we study the equilibria that emerge under free and costly entry. We show that in the free-entry competitive equilibrium, the efficient number of firms enters the economy. However, the free-entry monopolistic equilibrium involves insufficient entry, creating an additional source of inefficiency in the long run. Moreover, monopoly rents do not vanish. In an oligopolistic economy, firms' complex interactions, involving competition for homogeneous and differentiated products, lead to the possibility that either excessive or insufficient entry may arise in the free-entry equilibrium .

Our conclusions regarding the effects of entry are consistent with previous results in the literature, chiefly those of Mankiw and Whinston (1986). Our result on entry efficiency in a competitive economy provides the general equilibrium counterpart of Mankiw and Whinston's (1986) Corollary 1. The possibility of insufficient or excessive entry into an oligopolistic economy relates to the presence of economy-wise business-stealing and product-variety effects, as those identified by Mankiw and Whinston (1986). These effects have opposite signs and therefore open the possibility that either phenomenon may arise. We discuss policy effects in the presence of market power and free entry. De Loecker, Eeckhout and Unger (2022) provide a general equilibrium model to study empirically the effects on market power of technological innovations changing factors' productivity and cost of entry.

The remainder of the paper is organized as follows: Section 2 describes the market economy, identifies market-clearing profiles, and establishes some basic properties. Sections 3, 4, and 5 identify equilibria in competitive, monopolistic, and oligopolistic economies, respectively. Section 6 presents a welfare analysis of the outcomes arising in these economies. Section 7 examines policy issues. Section 8 studies the effects of free entry. Section 9 concludes.

2 A Market Economy

We consider a market economy in which there are $m \in \mathbb{N}$ consumption goods. All consumption goods are produced with the same technology using high- and low-skilled labor. Specifically, a firm that uses $h \in \mathbb{R}_+$ units of high-skilled labor and $l \in \mathbb{R}_+$ units of low-skilled labor produces $Ah^{\alpha}l^{\beta}$ units of good, where $A, \alpha, \beta \in \mathbb{R}$ satisfy $A > 0$ and $0 < \beta < \alpha \leq 1 - \beta$. These parameter restrictions justify referring to h and l as high- and low-skilled labor, respectively, and imply that firms have non-increasing returns to scale. There are $n \in \mathbb{N}$ firms producing each good $i \in \{1, ..., m\}$. Assuming that there is an equal number of firms in all sectors renders a symmetric setting in which messy equilibrium calculations are greatly simplified, and the results are easily explained. Also, it allows us to focus on n as a proxy for market power under imperfect competition.

The set of consumers comprises a set of *owners* and a unit mass of *workers.* Owners do not supply labor, but own firms' shares and use their income (i.e., the returns they receive according to their shares of firms' property) to buy consumption goods. Workers supply labor and consume goods. A fraction $\lambda \in (0, 1)$ of the population of workers can supply either high- or low-skilled labor, whereas the remaining fraction $1 - \lambda$ of workers can only supply low-skilled labor. We assume that high-skilled labor is relative scarce, which is the case of interest. Specifically, henceforth we assume that the inequality

$$\frac{(1-\lambda)\,\alpha}{\lambda\beta} > 1 \tag{1}$$

holds. This inequality warrants that high-skilled workers supply high-skilled labor in the equilibria of the economies we consider.

As in Dixit and Stiglitz (1977), consumers care about their consumption of a composite good c, given by

$$c := \left(\frac{1}{m}\right)^{\frac{1}{\theta-1}} \left(\sum_{i=1}^{m} c_i^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where c_i denotes the consumption of good i. The parameter $\theta \in (1, \infty)$ is the elasticity of substitution among goods, which is an inverse measure of consumers preference for variety; that is, consumers value variety more when θ is smaller. The constant term in (2) normalizes the units of measure of the composite good in a way that simplifies the equilibrium price formula.

Workers' preferences for consumption of the composite good and leisure time (or labor, ℓ, its counterpart) are represented by the utility function

$$u(c, \ell) = c - \frac{\varepsilon}{1+\varepsilon}\ell^{\frac{1+\varepsilon}{\varepsilon}},$$

where $\varepsilon \in (0, \infty)$ is the elasticity of a worker's labor supply – see below. (Adopting this quasi-linear utility function facilitates the assessment of the welfare effects of market power, as well as those of alternative policy instruments. Also, it renders irrelevant whether workers share firms' ownership, so long as their interests do not affect firms' incentives.)

Let $(P_1, ..., P_m) \in \mathbb{R}_+^m$ be a profile of prices of the consumption goods. The minimum expenditure required to consume $c \in \mathbb{R}_+$ units of the composite good is calculated by solving the system formed by (2) and the equations

$$\frac{\partial c}{\partial c_1} / \frac{\partial c}{\partial c_i} = \frac{P_1}{P_i}, \ \forall i \in \{2, ..., m\}.$$

The solution to this system gives the compensated demand functions,

$$c_i(P_1, ..., P_m, c) = \left(\frac{1}{m}\right)^{\frac{1}{1-\theta}} \left(\sum_{k=1}^{m} P_k^{1-\theta}\right)^{\frac{\theta}{1-\theta}} P_i^{-\theta} c, \ \forall i \in \{1, ..., m\}. \tag{3}$$

Therefore, the minimum expenditure is

$$\sum_{i=1}^{m} P_i c_i(P_1, ..., P_m, c) = Pc,$$

where

$$P := \left(\frac{1}{m} \sum_{i=1}^{m} P_i^{1-\theta} \right)^{\frac{1}{1-\theta}}, \tag{4}$$

is the effective price of the composite good, best known as the *Dixit-Stiglitz price index.*

We use the Dixit-Stiglitz price index P to convert the *nominal* values of variables describing the economic activity (e.g., prices, wages, incomes, profits, portfolio returns, etc.) into *real* values (i.e., into their values in units of the composite good). We denote the profile of real prices by $p = (p_1, ..., p_m)$, where $p_i := P_i/P$ for all $i \in \{1, ..., m\}$. Equation (4) implies

$$\frac{1}{m} \sum_{i=1}^{m} p_i^{1-\theta} = 1. \tag{5}$$

Real wages are given in units of the composite good per unit of time, whereas real income, and real profits are calculated in units of the composite good. To avoid endless repetition of the term "real," we will henceforth omit its use, reserving it for instances requiring attention to this attribute to properly assess the implications of a claim or assumption.

A worker whose non-labor income is $I \in \mathbb{R}_+$ chooses her consumption of the composite good and supply of labor at the *wage* $s \in \mathbb{R}_+$ by solving the problem

$$\max_{(c,\ell) \in \mathbb{R}_+^2} u(c, \ell), \text{ subject to: } c \leq s\ell + I.$$

Thus, the worker's demand for the composite good and labor supply are, respectively,

$$c\,(s, I) = s^{1+\varepsilon} + I, \text{ and } \ell\,(s, I) = s^{\varepsilon}.$$

Hence, a worker's labor supply strictly increases with the wage s, and is independent of his non-labor income I. The parameter ε is the elasticity of the worker's labor supply, which is a convex (concave) function of the wage when $\varepsilon \geq 1$ ($\varepsilon \leq 1$). A *worker's indirect utility* is therefore given for $(s, I) \in \mathbb{R}_+^2$ by

$$u(c\,(s, I)\,, \ell\,(s, I)) = \frac{s^{1+\varepsilon}}{1+\varepsilon} + I, \tag{6}$$

which increases with the worker's non-labor income I and the wage s.

Let us denote the wages of high- and low-skilled labor by v and w, respectively. The aggregate supply of high- and low-skilled labor is given for $(v,w) \in \mathbb{R}^2_+$ by

$$\left(H^S(v,w), L^S(v,w)\right) = \begin{cases} \left(\lambda v^{\varepsilon}, (1-\lambda) w^{\varepsilon}\right) & \text{if} \quad v > w \\ \{(x, v^{\varepsilon} - x),\, x \in [0, \lambda v^{\varepsilon}]\} & \text{if} \quad v = w \\ (0, w^{\varepsilon}) & \text{if} \quad v < w. \end{cases} \tag{7}$$

When $v > w$, the ratio v/w identifies the *skill premium* for high-skilled labor.

Since owners' and workers' consumption of composite good is just their real income, using the formulae for the compensated demand functions (3) and equation (5) we calculate the *market demand for consumption good* $i \in \{1, ..., m\}$ when the prices are $p = (p_1, ..., p_m) \in \mathbb{R}^m_+$ and consumers' total income, i.e., *aggregate (real) income*, is Y as

$$c_i(p, Y) = \frac{1}{m} p_i^{-\theta} Y. \tag{8}$$

An *economic activity* is a profile $(\boldsymbol{h}, \boldsymbol{l}, f, g)$, where $(\boldsymbol{h}, \boldsymbol{l}) \in \mathbb{R}^{mn}_+ \times \mathbb{R}^{mn}_+$ is a *profile of firms' labor inputs*, and (f, g) are mappings $f : [0, \lambda] \to \mathbb{R}_+$, $g : [0, 1-\lambda] \to \mathbb{R}_+$, satisfying

$$\int_0^{\lambda} f(x) dx = H(\boldsymbol{h}) \text{ and } \int_0^{1-\lambda} g(x) dx = L(\boldsymbol{l}),$$

where

$$\left(H(\boldsymbol{h}), L(\boldsymbol{l})\right) := \left(\sum_{i=1}^{m} \sum_{j=1}^{n} h_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} l_{ij}\right), \tag{9}$$

identifying the *labor supplied by high- and low-skilled workers.* The output of composite good (i.e., the *aggregate income*) resulting from an economic activity is

$$Y(\boldsymbol{h}, \boldsymbol{l}) := \left(\frac{1}{m}\right)^{\frac{1}{\theta-1}} \left(\sum_{i=1}^{m} \left(\sum_{j=1}^{n} A h_{ij}^{\alpha} l_{ij}^{\beta}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}. \tag{10}$$

In the economy, the utility an owner is simply his consumption of the composite good, while the utility a worker is his consumption of the composite minus the disutility (in units of the composite good) he derives from supplying labor. Hence, the *surplus*, i.e., the total welfare, generated by an economic activity is the aggregate income minus the workers' total disutility of labor, i.e.,

$$Y(\boldsymbol{h}, \boldsymbol{l}) - \frac{\varepsilon}{1+\varepsilon} \left(\int_0^{\lambda} f(x)^{\frac{1+\varepsilon}{\varepsilon}} dx + \int_0^{1-\lambda} g(x)^{\frac{1+\varepsilon}{\varepsilon}} dx\right). \tag{11}$$

Proposition 1 identifies the solution to the *surplus maximization problem.* Write

$$\eta := \frac{1}{1 + (1 - \alpha - \beta)\,\varepsilon} \in (0, 1].$$

Proposition 1. *The maximum surplus is*

$$W^* := \frac{Y^*}{(1 + \varepsilon)\,\eta}, \tag{12}$$

where

$$Y^* = \left(A^{1+\varepsilon} (mn)^{(1-\alpha-\beta)(1+\varepsilon)} \alpha^{\alpha\varepsilon} \beta^{\beta\varepsilon} \lambda^{\alpha} (1 - \lambda)^{\beta}\right)^{\eta}. \tag{13}$$

Realizing this surplus requires that each high-skilled worker supplies mnh^*/λ units of high-skilled labor and each low-skilled worker supplies $mnl^*/(1 - \lambda)$ units of low-skilled labor, where

$$h^* := \left(\frac{A^{\varepsilon}}{mn} \left(\alpha^{1+(1-\beta)\varepsilon} \beta^{\beta\varepsilon}\right)^{\frac{\varepsilon}{1+\varepsilon}} \left(\lambda^{1+(1-\beta)\varepsilon} (1 - \lambda)^{\beta\varepsilon}\right)^{\frac{1}{1+\varepsilon}}\right)^{\eta} \tag{14}$$

and

$$l^* := \left(\frac{A^{\varepsilon}}{mn} \left(\alpha^{\alpha\varepsilon} \beta^{1+(1-\alpha)\varepsilon}\right)^{\frac{\varepsilon}{1+\varepsilon}} \left(\lambda^{\alpha\varepsilon} (1 - \lambda)^{1+(1-\alpha)\varepsilon}\right)^{\frac{1}{1+\varepsilon}}\right)^{\eta}. \tag{15}$$

If $\alpha + \beta < 1$, then the unique profile of firms' labor inputs that generates the surplus W^* is $(h_{ij}, l_{ij}) = (h^*, l^*)$ for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, whereas if $\alpha + \beta = 1$, then any profile of firms' labor inputs such that $\sum_{j=1}^{n} (h_{ij}, l_{ij}) = (nh^*, nl^*)$ for all $i \in \{1, ..., m\}$ generates the surplus W^*.

Proof. Maximizing the surplus requires, given the total labor used in production activities, minimizing workers' aggregate disutility of supplying such labor, and maximizing the output of composite good (i.e., aggregate income) that can be produced with such labor. Because a worker's disutility of labor is a convex function, cost-minimization requires splitting evenly among (high-) low-skilled workers, the total amount of (high-) low-skilled labor to be supplied. Moreover, the concavity and symmetry of the formula identifying the composite good (2) implies that output-maximization involves producing the same amount of each consumption good. Further, if goods are produced with decreasing returns to scale, then it requires allocating the same high- and low-skilled labor to each firm, while if goods are produced with constant returns to scale, then there are multiple optimal allocations of labor to firms.

Hence, in either case there is a solution to the surplus maximization problem that involves a symmetric allocation of labor to firms, $(h_{ij}, l_{ij}) = (h, l) \in \mathbb{R}^2_+$ for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, and symmetric supply of labor by workers of each type; that is, each high-skilled worker should provide mnh/λ units of high-skilled labor, and each low-skilled worker should provide $mnl/(1-\lambda)$ units of low-skilled labor. Furthermore, (h, l) must solve the problem

$$\max_{(h,l)\in\mathbb{R}^2_+} mnAh^\alpha l^\beta - \frac{\varepsilon}{1+\varepsilon}\left(\lambda\left(\frac{mnh}{\lambda}\right)^{\frac{1+\varepsilon}{\varepsilon}} + (1-\lambda)\left(\frac{mnl}{1-\lambda}\right)^{\frac{1+\varepsilon}{\varepsilon}}\right).$$

Since the objective function in this problem is concave, the system of first order conditions

$$\alpha A h^{\alpha-1} l^\beta = \left(\frac{mnh}{\lambda}\right)^{\frac{1}{\varepsilon}}, \tag{16}$$

$$\beta A h^\alpha l^{\beta-1} = \left(\frac{mnl}{1-\lambda}\right)^{\frac{1}{\varepsilon}}, \tag{17}$$

identifies the solution, which is the formulae (14) and (15). Using the formulae (10) and (11) we calculate the aggregate income Y^* and surplus W^* generated by this profile of firms' labor inputs, and supply of labor by high- and low-skilled workers. □

Next, we identify the profiles of firms' labor inputs and the price system that form a market-clearing profile. We assume throughout that firms maximize (real) profits. (While this objective is perfectly aligned with the interests of owners, it may be misaligned with those of workers. A reasonable interpretation of this assumption is that owners are large investors who control firms, while workers are, at best, small investors who exercise no influence over firms' objective.) Hence, when prices and wages are positive, a firm producing a positive output (which requires using positive amounts of both types of labor) supplies its output in full to the market, since otherwise it can increase its profit by reducing the labor it uses.

Definition 1. A *market-clearing profile* is a collection $(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$ such that all markets clear. Formally:

(i) Good markets clear, i.e., for all $i \in \{1, ..., m\}$,

$$\sum_{j=1}^{n} A h_{ij}^\alpha l_{ij}^\beta = \frac{1}{m} p_i^{-\theta} Y(\boldsymbol{h}, \boldsymbol{l}). \tag{18}$$

(ii) Labor markets clear, i.e.,

$$(H(\boldsymbol{h}), L(\boldsymbol{l})) \in (H^S(v,w), L^S(v,w)). \tag{19}$$

We denote by M the set of all market-clearing profiles. Naturally, for a market-clearing profile to be a market equilibrium, the inputs of each firm must solve the firm's profit maximization problem. We consider alternative settings involving differing assumptions about firms' market power in the markets for goods and labor. In each of these settings, the equilibrium conditions give rise to alternative market-clearing profiles.

Let $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M$. It is easily verified that the aggregate income $Y(\boldsymbol{h}, \boldsymbol{l})$ is indeed the sum of the *aggregate labor income,* which by (19) is equal to $vH(\boldsymbol{h}) + wL(\boldsymbol{l})$, and the *aggregate profits,* $\Pi(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$, since

$$\begin{aligned}
\Pi(\boldsymbol{h}, \boldsymbol{l}, p, v, w) &= \sum_{i=1}^{m}\sum_{j=1}^{n}(p_i A h_{ij}^{\alpha} l_{ij}^{\beta} - v h_{ij} - w l_{ij}) \qquad (20)\\
&= \sum_{i=1}^{m} p_i \sum_{j=1}^{n} A h_{ij}^{\alpha} l_{ij}^{\beta} - v \sum_{i=1}^{m}\sum_{j=1}^{n} h_{ij} - w \sum_{i=1}^{m}\sum_{j=1}^{n} l_{ij}\\
\text{(by (18))} \quad &= \frac{1}{m}\sum_{i=1}^{m} p_i^{1-\theta} Y(\boldsymbol{h}, \boldsymbol{l}) - (vH(\boldsymbol{h}) + wL(\boldsymbol{l}))\\
\text{(by (5))} \quad &= Y(\boldsymbol{h}, \boldsymbol{l}) - (vH(\boldsymbol{h}) + wL(\boldsymbol{l}))\,.
\end{aligned}$$

An informative measure of the distribution of aggregate income between labor income and profits is the *labor share*, which is defined for $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M$ as

$$LS(\boldsymbol{h}, \boldsymbol{l}, p, v, w) := \frac{vH(\boldsymbol{h}) + wL(\boldsymbol{l})}{Y(\boldsymbol{h}, \boldsymbol{l})}. \tag{21}$$

Assume that $v > w$. Then each (high-) low-skilled worker supplies (v^{ε}) w^{ε} units of (high-) low-skilled labor. Hence, we may calculate the surplus using (11) as

$$W(\boldsymbol{h}, \boldsymbol{l}, p, v, w) := Y(\boldsymbol{h}, \boldsymbol{l}) - \frac{\varepsilon}{1+\varepsilon}\left(\lambda\,(v^{\varepsilon})^{\frac{1+\varepsilon}{\varepsilon}} + (1-\lambda)\,(w^{\varepsilon})^{\frac{1+\varepsilon}{\varepsilon}}\right).$$

Substituting $H(\boldsymbol{h}) = \lambda v^{\varepsilon}$ and $L(\boldsymbol{l}) = (1-\lambda)\,w^{\varepsilon}$, we get

$$W(\boldsymbol{h}, \boldsymbol{l}, p, v, w) = Y(\boldsymbol{h}, \boldsymbol{l}) - \frac{\varepsilon}{1+\varepsilon}\,(vH(\boldsymbol{h}) + wL(\boldsymbol{l}))\,,$$

and using (21) we get

$$W(\boldsymbol{h}, \boldsymbol{l}, p, v, w) = \frac{1 + (1 - LS(\boldsymbol{h}, \boldsymbol{l}, p, v, w))\,\varepsilon}{1+\varepsilon} Y(\boldsymbol{h}, \boldsymbol{l}). \tag{22}$$

Proposition 2 provides formulae that identify the good prices and wages of market-clearing profiles in which firms produce a positive output using cost-minimizing inputs.

Proposition 2. *Let* $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M$ *be such that (i)* $\sum_{j=1}^{n} A h_{ij}^{\alpha} l_{ij}^{\beta} > 0$ *for all* $i \in \{1, ..., m\}$*, and (ii) for all* $ij \in \{1, ..., m\} \times \{1, ..., n\}$*, either* $(h_{ij}, l_{ij}) = (0, 0)$ *or* (h_{ij}, l_{ij}) *minimizes the cost of producing* $q_{ij} := A h_{ij}^{\alpha} l_{ij}^{\beta} > 0$ *at wages* (v, w)*. Then*

$$p_i = p_i(\boldsymbol{h}, \boldsymbol{l}) := \left(\frac{1}{m} \frac{Y(\boldsymbol{h}, \boldsymbol{l})}{\sum_{j=1}^{n} A h_{ij}^{\alpha} l_{ij}^{\beta}} \right)^{\frac{1}{\theta}} \text{ for all } i \in \{1, ..., m\}, \tag{23}$$

$$v = v(\boldsymbol{h}) := \left(\frac{H(\boldsymbol{h})}{\lambda} \right)^{\frac{1}{\varepsilon}}, \tag{24}$$

and

$$w = w(\boldsymbol{l}) := \left(\frac{L(\boldsymbol{l})}{1 - \lambda} \right)^{\frac{1}{\varepsilon}}. \tag{25}$$

Furthermore, the skill premium is

$$SP := \frac{v}{w} = \left(\frac{(1 - \lambda)\, \alpha}{\lambda \beta} \right)^{\frac{1}{1+\varepsilon}} > 1. \tag{26}$$

Proof. Let $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M$ satisfy the assumptions in Proposition 2. Equation (23) holds by market-clearing (18), since the supply is positive in the market of every good. Since $L(\boldsymbol{l}) > 0$, low-skilled labor market-clearing (19) implies that $w > 0$. Moreover, $w \leq v$, for if $w > v$ were to hold, then all workers would supply low-skilled labor, and therefore $H(\boldsymbol{h}) > 0$ would imply the existence of an excess demand in the market for high-skilled labor. Also, firms' cost-minimization requires

$$\frac{l_{ij}}{h_{ij}} = \frac{\beta v}{\alpha w}$$

for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$ such that $(h_{ij}, l_{ij}) \neq (0, 0)$, and therefore

$$\frac{L(\boldsymbol{l})}{H(\boldsymbol{h})} = \frac{\beta v}{\alpha w}. \tag{27}$$

If $v = w$, then labor market-clearing conditions (19) require

$$H(\boldsymbol{h}) + L(\boldsymbol{l}) = \lambda v^{\varepsilon} + (1 - \lambda)\, w^{\varepsilon} = v^{\varepsilon} \geq \frac{H(\boldsymbol{h})}{\lambda} \Leftrightarrow \frac{L(\boldsymbol{l})}{H(\boldsymbol{h})} \geq \frac{1 - \lambda}{\lambda},$$

and hence

$$\frac{\beta}{\alpha} \geq \frac{1 - \lambda}{\lambda} \Leftrightarrow 1 \geq \frac{\alpha\,(1 - \lambda)}{\lambda \beta}$$

which contradicts the inequality (1). Hence $v > w$, and therefore labor market-clearing conditions (19) imply $H^S(v,w) = \lambda v^{\varepsilon} = H(\boldsymbol{h})$ and $L^S(v,w) = (1-\lambda)\, w^{\varepsilon} = L(\boldsymbol{l})$, i.e.,

$$v = \left(\frac{H(\boldsymbol{h})}{\lambda}\right)^{\frac{1}{\varepsilon}} \text{ and } w = \left(\frac{L(\boldsymbol{l})}{1-\lambda}\right)^{\frac{1}{\varepsilon}}.$$

Moreover,

$$SP = \frac{v}{w} = \left(\frac{1-\lambda}{\lambda}\frac{H(\boldsymbol{h})}{L(\boldsymbol{l})}\right)^{\frac{1}{\varepsilon}} = \left(\frac{1-\lambda}{\lambda}\frac{\alpha w}{\beta v}\right)^{\frac{1}{\varepsilon}} = \left(\frac{(1-\lambda)\,\alpha}{\lambda\beta}\right)^{\frac{1}{1+\varepsilon}} > 1. \square$$

Thus, when high-skilled labor is relatively scarce, i.e., the inequality (1) holds, the skill premium is the same in every market-clearing profile. Since market equilibria are market-clearing profiles, this implies that the skill premium is the same, regardless of whether firms have market power. As we shall see in Section 7, however, the effect of minimum wages differs depending on firms' market power.

Equation (23) identifying equilibrium prices has an interesting implication: An increase in the output of firm ij leads to an increase of Y (the output of the composite good), and hence, to an increase in the price of all other goods $i' \neq i$. This *horizontal complementarity* across sectors, which is larger the smaller is θ, i.e., the larger is consumers preference for variety, is relevant in explaining the differing impacts of market power in monopolistic and oligopolistic economies.

Let $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M$ such that $(h_{ij}, l_{ij}) = (h, l)$ for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$. Then goods market clearing conditions (18) and equation (5) imply that prices are $p = (1, ..., 1)$. For these market clearing profiles, the *wage markdowns* of high- and low-skilled labor, i.e., the fraction of value of the marginal product from high- and low-skilled labor that is *not* paid as wage, are given by

$$MDv(\boldsymbol{h}, \boldsymbol{l}, p, v, w) := \frac{\alpha A h^{\alpha-1} l^{\beta} - v}{\alpha A h^{\alpha-1} l^{\beta}}, \text{ and } MDw(\boldsymbol{h}, \boldsymbol{l}, p, v, w) := \frac{\beta A h^{\alpha} l^{\beta-1} - w}{\beta A h^{\alpha} l^{\beta-1}}.$$

We will show that in a competitive economy wage markdowns are nil, whereas in economies in which firms have market power wage markdowns are positive. Indeed, wage markdowns are indicators of firms' market power.

3 A Competitive Economy

In a competitive economy firms choose their labor inputs (and hence their output) to maximize (real) profits taking as given the price system, (p, v, w). We provide a formal definition of equilibrium in this setting.

Definition 2. A *competitive equilibrium* (CE henceforth) is a market-clearing profile $(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$ such that for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, (h_{ij}, l_{ij}) solves the problem

$$\max_{(h,l)\in\mathbb{R}^2_+} p_i A h^\alpha l^\beta - vh - wl. \tag{28}$$

Proposition 3 identifies the competitive equilibria.

Proposition 3. *The economy has a CE in which the high- and low-skilled labor of every firm are*

$$(h_{CE}, l_{CE}) = (h^*, l^*)$$

where h^*, l^* are identified in Proposition 1. Moreover, if $\alpha + \beta < 1$, then this is the unique CE, whereas if $\alpha + \beta = 1$, then there are multiple CE. However, in all the CE the surplus and aggregate income are

$$Y_{CE} = Y^* \text{ and } W_{CE} = W^*,$$

where Y^* and $W^*$are identified in Proposition 1, and the employment and wages of high- and low-skilled labor are

$$(H_{CE}, L_{CE}) = (mnh^*, mnl^*)$$

and

$$(v_{CE}, w_{CE}) = \left(\left(\frac{H_{CE}}{\lambda}\right)^{\frac{1}{\varepsilon}}, \left(\frac{L_{CE}}{1-\lambda}\right)^{\frac{1}{\varepsilon}}\right),$$

respectively. Moreover, the aggregate labor income of high- and low-skilled workers and aggregate profits are

$$(v_{CE}H_{CE}, w_{CE}L_{CE}, \Pi_{CE}) = (\alpha, \beta, 1-\alpha-\beta)Y_{CE}.$$

while the labor share and wage markdown are

$$LS_{CE} = \alpha + \beta \text{ and } MDv_{CE} = MDw_{CE} = 0.$$

Proof. It is readily verified that the profile given by $(h_{ij}, l_{ij}) = (h_{CE}, l_{CE})$ for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, $p = (1, ..., 1)$ and $(v, w) = (v_{CE}, w_{CE})$ is indeed a CE.

Let $(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$ be a CE, and assume that goods are produced with decreasing returns to scale, i.e., $\alpha + \beta < 1$. Obviously, prices are positive, for if the price of a good is zero, then there will be an excess demand for the good. Likewise, wages are positive, for otherwise a firm's profit maximization problem has no solution. Moreover, since all firms have the same technology, their cost and supply functions are the same. And since there is an equal number of firms per sector, the market supply of all goods is the same. Further, the market demands for all goods are the same as well. Hence the CE prices of all consumption goods are the same, and therefore the profile of real prices is $p_{CE} = (1, ..., 1)$.

Thus, the first order conditions for profit maximization are

$$\alpha A h^{\alpha-1} l^{\beta} = v \tag{29}$$

$$\beta A h^{\alpha} l^{\beta-1} = w, \tag{30}$$

the same for all firms. Hence, all firms use the same inputs, i.e., $(h_{ij}, l_{ij}) = (h, l)$ for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, where $h > 0$ and $l > 0$. By Proposition 2, wages are identified by equations (24) and (25), i.e.,

$$v = \left(\frac{mnh}{\lambda}\right)^{\frac{1}{\varepsilon}} \text{ and } w = \left(\frac{mnl}{1-\lambda}\right)^{\frac{1}{\varepsilon}}.$$

Replacing these formulae in equations (29) and (30), we obtain equations identical to (16) and (17) identifying the surplus maximizing firms' labor profile, (h^*, l^*), given in Proposition 1. Hence $(h_{CE}, l_{CE}) = (h^*, l^*)$.

If goods are produced with constant returns to scale, i.e., $\alpha + \beta = 1$, then in a CE firms' profits are nil, and the output (and inputs) of individual firms are not determined. Firms producing a positive output must do so at minimum cost. Therefore, by Proposition 2 the CE wages are given by equations (24) and (25). Hence, there is an equilibrium in which all firms use the inputs (h_{CE}, l_{CE}). Moreover, in every CE the output (and inputs) used in every sector are the same and hence equations (29) and (30) imply

$$\sum_{j=1}^{n} h_{ij} = nh_{CE}, \ \sum_{j=1}^{n} l_{ij} = nl_{CE},$$

for all $i \in \{1, ..., m\}$, and therefore employment is $(H_{CE}, L_{CE}) = (mnh_{CE}, mnl_{CE})$.

Thus, whether $\alpha + \beta < 1$ or $\alpha + \beta = 1$, we may readily calculate the aggregate income Y_{CE} using equation (10) and the wages (v_{CE}, w_{CE}) using equations (24) and (25), which in turn allows us to calculate the labor income of high- and low-skilled workers. Then we calculate aggregate profits using equation (20), and the labor share and surplus using the equations (21) and (22). Since wages are equal to the value of the marginal product, as implied by equations (29) and (30), wage markdowns are nil. □

If goods are produced with decreasing (constant) returns to scale, then surplus, aggregate income, employment and wages increase with (are independent of) the number of goods m and firms n. In either case, the CE is independent of the consumers preference for variety parameter, θ. The labor share LS_{CE} increases with the returns to scale parameters, $\alpha + \beta$.

4 A Monopolistic Economy

Let us consider a market economy in which a single agent, a *monopoly*, controls all firms, which may be regarded as *plants*, behaving as a monopoly in the markets for goods and as a monopsony in the markets for labor. Considering this extreme case of market power over the entire economy is unrealistic but provides a useful benchmark and facilitates understanding the source of distortions caused by market power. In addition, it uncovers the potential benefits of institutions or devices coordinating production activities; for example, Azar and Vives (2021) identify the common interests of large investors as one such device. In Section 6 we show that a monopolistic economy may perform better than an oligopolistic economy and identify the source of this result. Note that a monopolistic economy is a market economy, and therefore our results and arguments do not relate to central planning economies, such as that of the Soviet Union, but rather to market-based economies, such as that of China. (Of course, we understand that the Chinese government does not exercise *full* control over firms' production activities, but certainly has tools to shape firms' incentives.)

The monopoly selects its labor profile to maximize its (real) profit in the ensuing market-clearing profile. We refer to the market equilibria of a monopolistic economy as *monopolistic equilibria.* The formal definition follows.

Definition 3. A *monopolistic equilibrium* (ME henceforth) is a market-clearing profile that solves the problem

$$\max_{(\boldsymbol{h},\boldsymbol{l},p,v,w)\in M} \sum_{i=1}^{m} p_i \sum_{j=1}^{n} A h_{ij}^{\alpha} l_{ij}^{\beta} - v \sum_{i=1}^{m}\sum_{j=1}^{n} h_{ij} - w \sum_{i=1}^{m}\sum_{j=1}^{n} l_{ij}. \tag{31}$$

Proposition 4 identifies the monopolistic equilibria.

Proposition 4. *The economy has a ME in which the high- and low-skilled labor used in each plant are*

$$(h_{ME}, l_{ME}) = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{\eta\varepsilon} (h_{CE}, l_{CE}).$$

Moreover, if $\alpha+\beta<1$, then this is the unique ME, whereas if $\alpha+\beta=1$, then there are multiple ME. However, in all the ME the aggregate income and surplus are

$$Y_{ME} = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}$$

and

$$W_{ME} = \left(\eta + \frac{\varepsilon}{1+\varepsilon}\right)\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} W_{CE},$$

respectively, and the employment and wages of high- and low-skilled labor are

$$(H_{ME}, L_{ME}) = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{\eta\varepsilon} (H_{CE}, L_{CE})$$

and

$$(v_{ME}, w_{ME}) = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{\eta} (v_{CE}, w_{CE}).$$

respectively. Furthermore, the aggregate labor income of high- and low-skilled workers and aggregate profits are

$$(v_{ME}H_{ME}, w_{ME}L_{ME}, \Pi_{ME}) = \left(\frac{\alpha\varepsilon}{1+\varepsilon}, \frac{\beta\varepsilon}{1+\varepsilon}, 1 - \frac{(\alpha+\beta)\,\varepsilon}{1+\varepsilon}\right) Y_{ME},$$

while the labor share and wage markdowns are

$$LS_{ME} = \frac{\varepsilon}{1+\varepsilon} LS_{CE} \text{ and } MDv_{ME} = MDw_{ME} = \frac{1}{1+\varepsilon}.$$

Proof. Clearly, a solution to the problem (31) involves producing the same positive amount of each good; otherwise, the monopoly can increase its revenue without affecting its cost by shifting inputs from a sector producing more output than average to a sector producing less output than average. This in turn implies that in a monopolistic equilibrium the profile of prices is $p_{ME} = (1, ..., 1)$. Moreover, if goods are produced with decreasing returns to scale, i.e., $\alpha + \beta < 1$, then in a ME all plants in a sector $i \in \{1, ..., m\}$ must produce the same output; otherwise, the monopoly may reduce the total cost by increasing (decreasing) its output in plants whose output is below (above) average. When goods are produced with constant returns to scale, i.e., $\alpha + \beta = 1$, the monopoly's problem has multiple solutions, including a solution that involves using the same inputs in each of the monopoly's plants, but all solutions involve the same employment, output, labor income of high- and low-skilled workers, and profits. In either case, plants producing a positive output must use cost-minimizing inputs; hence wages are given by equations (24) and (25) in Proposition 2

Thus, we may identify a symmetric profile of labor maximizing the monopoly's profits on the set of market-clearing profiles by solving the problem

$$\max_{(h,l)\in\mathbb{R}^2_+} \pi_M(l,h) := mnAh^\alpha l^\beta - \left(\frac{mnh}{\lambda}\right)^{\frac{1}{\varepsilon}} mnh - \left(\frac{mnl}{1-\lambda}\right)^{\frac{1}{\varepsilon}} mnl. \tag{32}$$

The first order conditions for a solution to this problem are

$$\alpha A h^{\alpha-1} l^\beta = \frac{1+\varepsilon}{\varepsilon}\left(\frac{mnh}{\lambda}\right)^{\frac{1}{\varepsilon}} \tag{33}$$

$$\beta A h^{\alpha} l^{\beta-1} = \frac{1+\varepsilon}{\varepsilon}\left(\frac{mnl}{1-\lambda}\right)^{\frac{1}{\varepsilon}}. \tag{34}$$

The unique solution to this system of equations is given by the formulae (h_{ME}, l_{ME}) in Proposition 4. This is the unique solution to the monopoly's problem (31) if $\alpha + \beta < 1$, whereas if $\alpha + \beta = 1$ there are multiple solutions, all of which satisfy

$$\sum_{j=1}^{n} h_{ij} = nh_{ME}, \ \sum_{j=1}^{n} l_{ij} = nl_{ME}.$$

for all $i \in \{1, ..., m\}$. Hence $(H_{ME}, L_{ME}) = (mnh_{ME}, mnl_{ME})$ in either case. Using these results, we proceed exactly as in the proof of Proposition 3 to calculate the surplus, aggregate income, wages and aggregate labor income of high- and low-skilled labor, and then the aggregate profits, and labor share. As for the wage markdowns, using the formulae for (h_{ME}, l_{ME}) and (v_{ME}, w_{ME}), and noticing that $MDv_{CE} = MDw_{CE} = 0$, we get

$$MDv_{ME} = MDw_{ME} = \frac{1}{1+\varepsilon}. \ \square$$

If goods are produced with decreasing (constant) returns to scale, then surplus, aggregate income, employment and wages increase with (are independent of) the number of goods m and plants n. In either case, the ME is independent of the consumers preference for variety parameter, θ. The labor share LS_{ME} increases with the returns to scale parameters, $\alpha + \beta$, and the elasticity of the labor supply, ε. Wage markdowns decrease with the elasticity of the labor supply, ε. Since the CE labor profile is in the monopolist's feasible set, the aggregate profits in the ME are above those at the CE and are positive even if the goods are produced with constant returns to scale.

5 An Oligopolistic Economy

Let us consider an economy in which firms compete *à la* Cournot by choosing the labor they use (and hence the output they produce), taking as given the labor profile of the other firms in the economy. Note that the case $n = 1$ corresponds to a monopolistic economy when $m = 1$, and to the case of monopolistic competition when there are at least two consumption goods, i.e., $m \geq 2$.

In the game describing firms' interactions in an oligopolistic economy, the payoffs are the firms' profits in the ensuing market-clearing profile. Naturally, a firm choosing its labor inputs in this economy is aware of its market power in the goods and labor markets. We refer to the Counot-Nash equilibria of this game as *oligopolistic equilibria.* The formal definition follows.

Definition 4. An *oligopolistic equilibrium* (OE henceforth) is a market-clearing profile $(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$ such that for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, (h_{ij}, l_{ij}) solves the problem

$$\max_{(h,l)\in\mathbb{R}^2_+} p_i(h, \boldsymbol{h}_{-ij}, l, \boldsymbol{l}_{-ij}) A h^\alpha l^\beta - v(h, \boldsymbol{h}_{-ij}) h - w(l, \boldsymbol{l}_{-ij}) l, \tag{35}$$

where $(h, \boldsymbol{h}_{-ij})$ and $(l, \boldsymbol{l}_{-ij})$ are obtained from $\boldsymbol{h}$ and $\boldsymbol{l}$ by replacing the coordinates ij with h and l, respectively, and $p_i(\cdot)$, $v(\cdot)$, and $w(\cdot)$ are the functions identified by (23), (24), and (25), respectively, in Proposition 2.

Proposition 5 identifies the economy's unique oligopolistic equilibrium. Write

$$\gamma := \frac{(1 + \theta mn - m)\varepsilon}{(1 + \varepsilon mn)\theta} \in (0, 1)\,. \tag{36}$$

Note that $\partial\gamma/\partial n > 0$, $\partial\gamma/\partial\theta > 0$, and $\partial\gamma/\partial\varepsilon > 0$. Moreover, $\partial\gamma/\partial m \geq 0$ if and only if $n\theta - 1 \geq n\varepsilon$.

Proposition 5. *In the unique OE of the economy every firm uses the labor profile*

$$(h_{OE}, l_{OE}) = \gamma^{\eta\varepsilon}(h_{CE}, l_{CE}).$$

Moreover, the aggregate income is

$$Y_{OE} = \gamma^{(\alpha+\beta)\eta\varepsilon} Y_{CE},$$

the surplus is

$$W_{OE} = (1 + (1 - (\alpha + \beta)\,\gamma)\,\varepsilon)\,\eta\gamma^{(\alpha+\beta)\eta\varepsilon} W_{CE},$$

and the employment and wages of high- and low-skilled labor are

$$(H_{OE}, L_{OE}) = \gamma^{\eta\varepsilon}\,(H_{CE}, L_{CE})$$

and

$$(v_{OE}, w_{OE}) = \gamma^{\eta}\,(v_{CE}, w_{CE})\,.$$

respectively. Furthermore, the aggregate labor income of high- and low-skilled workers and aggregate profits are

$$(v_{OE}H_{OE}, w_{OE}L_{OE}, \Pi_{OE}) = (\alpha\gamma, \beta\gamma, 1 - (\alpha + \beta)\gamma)\,Y_{OE},$$

while the labor share and wage markdowns are

$$LS_{OE} = \gamma LS_{CE} \text{ and } MDv_{OE} = MDw_{OE} = 1 - \gamma.$$

Proof. First, we argue that in an oligopolistic equilibrium $(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$ all firms use the same labor, i.e., $(h_{ij}, l_{ij}) = (h, l)$ for all ij. Because all firms pay the same wages, cost minimization implies that they must use high- and low-skilled labor in the same proportion. Moreover, marginal cost, which must account for the effect on wages of increasing the demand for either type of labor, increases with output. Consider any two firms producing good i, ij and ij'; if the output of firm ij is larger than that of firm ij', then the marginal revenue of firm ij is smaller than that of firm ij', and the marginal cost of firm ij is larger than that of firm ij'. Hence either firm ij or firm ij' is not maximizing profits. Thus, the output of all firms that produce good $i \in \{1, ..., m\}$ is the same. Likewise, if the total output in sector i is larger than that in sector i', then the marginal revenue of a firm in sector i is smaller than that of a firm in sector i', and the marginal cost of a firm in sector i is larger than that of a firm in sector i'; therefore, either the firms in sector i or the firms in sector i' are not maximizing profits. Hence, the total output of all goods is the same, and therefore the equilibrium prices are $p_{OE} = (1, ..., 1)$.

We show that the equilibrium is unique. The first order conditions for a solution to the problem (35) are the equations

$$\frac{\partial p_i(\boldsymbol{h}_{-ij}, h, \boldsymbol{l}_{-ij}, l)}{\partial h} A h^{\alpha} l^{\beta} + \alpha p_i(\boldsymbol{h}_{-ij}, h, \boldsymbol{l}_{-ij}, l) A h^{\alpha-1} l^{\beta} = \frac{\partial v(\boldsymbol{h}_{-ij}, h)}{\partial h} h + v(\boldsymbol{h}_{-ij}, h),$$

$$\frac{\partial p_i(\boldsymbol{h}_{-ij}, h, \boldsymbol{l}_{-ij}, l)}{\partial l} A h^{\alpha} l^{\beta} + \beta p_i(\boldsymbol{h}_{-ij}, h, \boldsymbol{l}_{-ij}, l) A h^{\alpha} l^{\beta-1} = \frac{\partial w(\boldsymbol{l}_{-ij}, l)}{\partial l} l + w(\boldsymbol{l}_{-ij}, l),$$

which are versions of the familiar condition "marginal revenue equals marginal cost" applied to high- and low-skilled labor, respectively. Using the symmetry of equilibrium and the formulae for the goods' prices (23) and wages (24) and (25) given in Proposition 2, we calculate the derivatives in the equations above, which yield the system of equations

$$\frac{1 + mn\theta - m}{mn\theta} \alpha A h^{\alpha-1} l^{\beta} = \frac{1 + mn\varepsilon}{mn\varepsilon} \left(\frac{mnh}{\lambda} \right)^{\frac{1}{\varepsilon}}, \tag{37}$$

$$\frac{1 + mn\theta - m}{mn\theta} \beta A h^{\alpha} l^{\beta-1} = \frac{1 + mn\varepsilon}{mn\varepsilon} \left(\frac{mnl}{1 - \lambda} \right)^{\frac{1}{\varepsilon}}. \tag{38}$$

This system's unique solution is the formulae for (h_{OE}, l_{OE}) given in Proposition 5.

Using the formulae (h_{OE}, l_{OE}), we proceed exactly as in the proof of Proposition 3 to calculate the employment, aggregate income, wages and aggregate labor income of high- and low-skilled labor, and then the aggregate profits, labor share and surplus. As for the wage markdowns, using the formulae for (h_{OE}, l_{OE}) and (v_{OE}, w_{OE}), and noticing that $MDv_{CE} = MDw_{CE} = 0$, we get

$$MDv_{OE} = 1 - \gamma = MDw_{OE}. \ \square$$

In an oligopolistic economy, whether goods are produced with decreasing or constant returns to scale, surplus, aggregate income, employment and wages increase with the number of firms, n, and the consumers preference for variety parameter, θ. The labor share LS_{OE} increases with α, β, ε, θ, and n, but may be a non-monotonic function of m.

The wage markdowns decrease with ε, θ and n, but may be non-monotonic functions of m. Aggregate profits are a positive share of the aggregate income even when goods are produced with constant returns to scale. Note that if $m = n = 1$, then the oligopolistic economy is a monopolistic economy; of course, the formulae of propositions 4 and 5 coincide, since $\gamma = \varepsilon/(1+\varepsilon)$ in this case.

6 Welfare Analysis

We discuss the welfare properties of the equilibrium allocations of competitive, monopolistic and oligopolistic economies. Proposition 6 summarizes the direct welfare implications of propositions 3, 4 and 5.

Proposition 6. *(i) The economy's surplus, aggregate income, employment, wages, labor income of high- and low-skilled workers and labor share are larger in the competitive equilibrium than in either the monopolistic or the oligopolistic equilibria. (ii) The aggregate profits are larger in the monopolistic equilibrium than in either the competitive or the oligopolistic equilibria. (iii) The oligopolistic equilibrium approaches the competitive equilibrium as n approaches infinity.*

Proof. Proposition 3 establishes that the maximum surplus is realized in the competitive equilibrium, i.e., $W_{CE} = W^*$, while propositions 4 and 5 establish that neither the monopolistic nor the oligopolistic equilibrium involve the firms' surplus-maximizing labor profile, implying that $W_{ME} < W^*$ and $W_{OE} < W^*$. Incidentally, these results imply the inequalities

$$\left(\eta + \frac{\varepsilon}{1+\varepsilon}\right)\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} < 1 \tag{39}$$

and

$$(1 + (1 - (\alpha+\beta)\gamma)\varepsilon)\,\eta\gamma^{(\alpha+\beta)\eta\varepsilon} < 1. \tag{40}$$

which will be useful for deriving further results. The remaining claims in (i) are direct implications of the inequalities $\varepsilon/(1+\varepsilon) < 1$ and $\gamma < 1$. The claim in (ii) is obvious since the monopoly maximizes profits over the entire set of market-clearing profiles. The claim in (iii) follows from

$$\lim_{n\to\infty} \gamma = 1. \ \square$$

Interestingly, firms' profits in the oligopolistic equilibrium may be larger or smaller than those in the competitive equilibrium, i.e., the sign of $\Pi_{OE} - \Pi_{CE}$ depends on the parameters. Clearly, when goods are produced with constant returns to scale, $\Pi_{OE} > 0 = \Pi_{CE}$. However, if goods are produced with decreasing returns to scale, the inequality $\Pi_{OE} < \Pi_{CE}$ may arise when the elasticity of labor supply ε is large and consumers have a strong preference for variety (i.e., θ is small); that is, when firms' market power in the labor markets is weak, and the product differentiation aspect of competition is strong.

Contrary to common wisdom, when the sign of

$$\mu := \gamma - \frac{\varepsilon}{1+\varepsilon}$$

is negative, the surplus, aggregate income, employment, wages and the labor share are larger, and the wage markdowns are smaller, in the monopolistic equilibrium than in the oligopolistic equilibrium, while when μ is positive these inequalities reverse. Using equation (36) identifying γ we provide a threshold on the number of firms per sector, n, that determines the sign of μ; namely,

$$\mu \lesseqqgtr 0 \Leftrightarrow n \lesseqqgtr \bar{n}(\varepsilon, \theta, m) := \frac{(1+\varepsilon)(m-1)}{m\theta} + \frac{1}{m}.$$

The threshold $\bar{n}(\varepsilon, \theta, m)$ decreases with the preference for variety parameter, θ, increases with the elasticity of labor supply, ε, and increases or decreases with the number of goods, m, depending on whether $1 + \varepsilon - \theta$ is positive or negative. Moreover, since

$$\lim_{n\to\infty} \mu = 1 - \frac{\varepsilon}{1+\varepsilon} > 0,$$

the sign of μ becomes positive for large n.

Remarkably, when $n < \bar{n}(\varepsilon, \theta, m)$ the monopolistic equilibrium allocation is Pareto superior to the oligopolistic equilibrium allocation; both wages and aggregate profits are larger, and hence both workers and owners are better off in the former than in the latter. Note that if $m = 1$, then $n \geq 1 = \bar{n}(\varepsilon, \theta, 1)$. Hence, for the inequality $n < \bar{n}(\varepsilon, \theta, m)$ to hold, there must be at least two consumption goods in the economy, i.e., $m \geq 2$. The efficiencies leading to the monopolistic equilibrium allocation Pareto dominating the oligopolistic equilibrium allocation relate to the horizontal externalities of output decisions across sectors, as we argue below. When there is a single consumption good such efficiencies are absent.

Proposition 7 summarizes these results.

Proposition 7. *If $n < \bar{n}(\varepsilon, \theta, m)$, then the surplus, aggregate income, employment, wages and labor share are larger, whereas the wage markdowns are smaller, in the monopolistic equilibrium than in the oligopolistic equilibrium. Moreover, the monopolistic equilibrium allocation is Pareto superior to the oligopolistic equilibrium allocation.*

Azar and Vives (2021) establish an analogous result in an oligopolistic economy in which firms' objectives are shaped by the composition of their owners' portfolio. Specifically, they show that the sign of the effect on the wage markdown of a change in the economy-wide common ownership of firms is identified by the sign of μ – see their Proposition 6. Hence, common ownership serves as a device inducing firms to partially or fully internalize the horizontal effects of their decisions. A monopolistic economy corresponds to the extreme case in which every owner has a portfolio containing an equal share of every firm. Our simple derivation helps understanding the source of this result and uncovers the Pareto superiority of the monopolistic equilibrium allocation.

To understand the intuition behind this result, let us consider the case $m = 2$ and $n = 1$. In the duopolistic equilibrium of this economy, the output of each competitor is such that the marginal revenue equals the marginal cost. It is easy to identify the conditions under which a monopoly controlling both firms would increase the output of both goods above the output at the duopolistic equilibrium, thus increasing the employment and wages of both types of labor. A marginal increase in the output of good 1, for example, would lead to a decrease in the price of good 1, to an increase in the price of good 2, and to an increase in wages. (Equation (23) makes explicit this horizontal effect.) The effect on profits in the market for good 1 (evaluated at the duopoly equilibrium) is zero. However, the effect on the profits in the market for good 2 depends on the difference between the increase in revenue due to the increase of the price of good 2, and the increase in the cost due to the increase in the wages. The effect on the price of good 2 is large when θ is small, while the effect on wages is small when ε is large. Thus, when θ is sufficiently small and ε is sufficiently large, the effect on profits in the market for good 2 is positive. Specifically, for $m = 2$ and $n = 1$ the inequality in the premise of Proposition 7 becomes $\theta < 1 + \varepsilon$, which holds when θ is near 1, that is, when consumers have a strong preference for variety. When this is the case, the monopolistic equilibrium entails a larger output on both markets as well as larger employment and wages of high- and low-skilled workers.

When $n > 1$, a marginal increase in the monopoly's output in any of its plants generates an additional effect that reduces its revenue since it also reduces the revenue from the other plants producing that good. In addition, an increase in wages increase the cost of a larger number of plants. These negative effects are large when n large, and eventually dominate the positive effect on the revenue generated in the market for the other good. (Specifically, this occurs when $(2n-1)\theta > \varepsilon + 1$.) The threshold $\bar{n}(\varepsilon, \theta, m)$ is increasing with the number of goods m provided $\theta < 1 + \varepsilon$; when this inequality holds, increasing the output of one of the goods has a positive effect on the revenue generated in plants producing other goods, and the impact on total revenue is larger the larger is m, so that a larger number of firms per sector is required for an oligopolistic economy to outperform the monopolistic economy.

It is worth noting that Proposition 7 is not driven by firms exercising market power in the labor markets. On the contrary, when firms act as wage-takers, that is, when the labor markets are competitive, the monopolistic economy always produces a Pareto superior allocation to that of the oligopolistic economy. The discussion in the previous paragraph clarifies this intuition: Assuming that firms are wage-takers amounts to assuming the labor supply is infinitely elastic, in which case the inequality $n < \bar{n}(\varepsilon, \theta, m)$ always holds. The source of this result is the horizontal complementarity across sectors of the economy, which Cournot competitors ignore but the monopoly exploits.

Proposition 7 uncovers effects overlooked by assessments of the competitiveness of an economy based exclusively on indices of concentration that rely on firms' market shares. Measures of competitiveness relying on wage markdowns or price markups, however, are not subject to this caveat.

7 Policy

We inquire into policies that may improve efficiency and distribution, including minimum wages, taxes and public spending. First, we study the effects of a minimum (real) wage. In practice, minimum wages indexed to the Consumer Price Index are common; e.g., 13 states in the US have such policies, and some form of indexation is commonly applied in several countries of the European Union.

The introduction of a minimum wage may lead to a persistent excess supply (i.e., *unemployment*) in either or both labor markets. Hence, the profiles $(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$ identifying the economic activity that may arise in a market economy with a minimum wage $\omega \in \mathbb{R}_+$ need not be market-clearing. While the prices $p = (p_1, ..., p_m)$ must clear the market for goods, i.e., equation (18) must hold for every $i \in \{1, ..., m\}$, the wages $(v, w) \geq (\omega, \omega)$ must induce sufficient supply to cover firms' demand for high- and low-skilled labor, i.e., $(H(\boldsymbol{h}), L(\boldsymbol{l})) \leq (H, L)$ for some $(H, L) \in (H^S(v, w), L^S(v, w))$. We denote by $M(\omega)$ the set of such profiles.

It is easy to see that in an economy with a minimum wage $\omega \in \mathbb{R}_+$ every profile $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M(\omega)$ satisfying the assumptions of Proposition 2 is such that the prices are given by the formula (23), while the wages (v, w) are given by

$$(\bar{v}(\boldsymbol{h}, \omega), \bar{w}(\boldsymbol{l}, \omega)) = (\max\{\omega, v(\boldsymbol{h})\}, \max\{\omega, w(\boldsymbol{l})\}), \tag{41}$$

where $v(\boldsymbol{h})$ and $w(\boldsymbol{l})$ are identified by (24) and (25), in Proposition 2. Thus, the introduction of a minimum wage above the equilibrium wage of low-skilled labor has a real impact on the economy's equilibrium.

We examine the impact of minimum wages in the different settings studied in sections 3, 4 and 5. To facilitate this analysis, we set $A = 1$.The effects of minimum wages in these settings are established in propositions 8, 9 and 10, respectively. The proofs of these propositions involve tedious algebra and calculations, which we omit.

7.1 Minimum Wages in a Competitive Economy

Let us consider the impact of a minimum wage in a competitive economy. A *competitive equilibrium with a minimum wage* $\omega \in \mathbb{R}_+$ is a profile $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M(\omega)$ such that for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, (h_{ij}, l_{ij}) solves the problem of the competitive firm (28). In Section 3 we argue that in a CE the market supply and demand for all goods are the same; hence the prices (and firms' marginal revenue) of all consumption goods are the same. This argument remains valid in the presence of a minimum wage, i.e., $p_{CE}(\omega) = (1, ..., 1)$. In addition, recall that by Proposition 3, in the absence of a minimum wage, the CE wages of high- and low-skilled labor satisfy $v_{CE} > w_{CE}$.

The introduction of a minimum wage $\omega > w_{CE}$ leads firms (workers) to decrease (increase) their demand for (supply of) low-skilled labor, thus implying the emergence of unemployment among low-skilled workers. Firms' reduction of low-skilled labor in turn leads to a decrease in the marginal productivity of high-skilled labor, and hence, to a decrease in the equilibrium wage of high-skilled labor. This is because under perfect competition the wage of each type of labor equals the value of its marginal productivity. If the minimum wage is sufficiently high, then unemployment of high-skilled workers also arises. Specifically, the threshold above which this occurs is

$$\bar{\omega}_{CE} := \left(\frac{mn}{\lambda}\right)^{(1-\alpha-\beta)\eta} \alpha^{(1-\beta)\eta}\beta^{\beta\eta} < v_{CE}.$$

That $\bar{\omega}_{CE}$ is below v_{CE} is easily explained: The minimum wage leads firms to reduce their low-skilled labor, reducing the productivity of high-skilled labor, and eventually creating an excess supply of high-skilled labor for minimum wages below v_{CE}.

Thus, the introduction of a minimum wage $\omega > w_{CE}$ leads to a decrease in the employment of high- and low-skilled labor, surplus, aggregate income and profits. The reduction in the wage and employment of high-skilled labor reduces the labor income of high-skilled workers. Obviously, the skill premium decreases, and eventually disappears. However, the labor share remains unchanged since the competitive firm adjusts its labor to equate the value of the marginal productivity of each type of labor to its wage.

The welfare effects of a minimum wage are therefore clear: Employed low-skilled workers become better off, while unemployed low-skilled workers, high-skilled workers and owners become worse off than in the absence of a minimum wage. (The utility of a worker increases with the wage, as shown in (6). The utility of an unemployed worker is that at a zero wage.)

Proposition 8 summarizes these results.

Proposition 8. *In a competitive economy the surplus, aggregate income, employment and aggregate profits decrease with the minimum wage ω in (w_{CE}, ∞). Moreover, unemployment of low-skilled workers arises and increases with ω, and unemployment of high-skilled workers arises and increases with ω in $(\bar{\omega}_{CE}, \infty)$. Thus, owners, high-skilled workers and unemployed workers are worse off, while employed low-skilled workers are better off, than in the absence of a minimum wage.*

7.2 Minimum Wages in a Monopolistic Economy

Let us consider the impact of minimum wages in a monopolistic economy. A *monopolistic equilibrium with a minimum wage* $\omega \in \mathbb{R}_+$ is a profile $(\boldsymbol{h}, \boldsymbol{l}, p, v, w)$ that solves the variation in the monopoly's problem (31) obtained by replacing M with $M(\omega)$.

The presence of a minimum wage does not affect the validity of the argument in Section 4, which establishes that the solution to the monopoly's problem involves producing the same positive amount of each good, implying that equilibrium prices are all equal to 1, and that the surplus, aggregate income, employment, wages, labor share and wage markdowns are uniquely determined. Hence, we focus on the symmetric equilibrium, which is the unique equilibrium when goods are produced with decreasing returns to scale. (The effects on the equilibrium aggregates are the same for the asymmetric equilibria that arise when goods are produced with constant returns to scale.)

In the monopolistic equilibrium, the cost of a marginal increase in a type of labor is the sum of the wage of the marginal worker plus the cost of the wage increase required for the additional marginal unit of labor to be supplied. For example, the cost of a marginal increase in low-skilled labor may be calculated using equation (34) in the system identifying the solution to the monopoly's problem (32) and equation (25) of Proposition 2 identifying the equilibrium wage, as $(1+\varepsilon)\, w_{ME}/\varepsilon > w_{ME}$.

Let us consider the impact of a minimum wage above, but close to, w_{ME}. (Recall that $w_{ME} < v_{ME}$.) Specifically, let $\omega \in (w_{ME}, (1+\varepsilon)\, w_{ME}/\varepsilon)$, and write $\bar{l}(\omega) := L^S(\omega)/\,(mn) = (1-\lambda)\, \omega^{\varepsilon}/\,(mn) > l_{CE}$. With the minimum wage ω, the marginal cost of low-skilled labor becomes constant and equal to $\omega < (1+\varepsilon)\, w_{ME}/\varepsilon$, for all $l < \bar{l}(\omega)$, i.e., the marginal cost of low-skilled labor is ω if the monopoly's total low-skilled labor is not greater than $L^S(\omega)$. However, for $l = l_{ME}$ the marginal revenue from low-skilled labor equals marginal cost, i.e., equals $(1+\varepsilon)\, w_{ME}/\varepsilon < \omega$; hence it is optimal for the monopoly to increase its low-skilled labor. The increase in low-skilled labor, in turn, leads to an increase in the productivity of high-skilled labor, inducing the monopolist to increase also its high-skilled labor.

If ω is relatively *low,* specifically, $\omega \in (w_{ME}, \omega_{M1})$, where

$$\omega_{M1} := (mn)^{(1-\alpha-\beta)\eta} \left(\frac{\alpha\varepsilon}{1+\varepsilon}\right)^{\frac{\alpha\varepsilon\eta}{1+\varepsilon}} \beta^{\left(1-\frac{\alpha\varepsilon}{1+\varepsilon}\right)\eta} \lambda^{\frac{\alpha\eta}{1+\varepsilon}} (1-\lambda)^{-\left(1-\beta-\frac{\alpha\varepsilon}{1+\varepsilon}\right)\eta},$$

the monopolist's profits $\pi_M(l, \hat{h}(l))$, where $\hat{h}(l)$ is the optimal high-skilled labor given l, increases with l for $l < \bar{l}(\omega)$, has a kink at $\bar{l}(\omega)$, and decreases for $l > \bar{l}(\omega)$. This is because the marginal cost of low-skilled labor has an upward discontinuity at $\bar{l}(\omega)$. Hence, it is optimal for the monopoly to use the entire supply of low-skilled labor at the minimum wage, $L^S(\omega)$. Therefore, there is full employment, and the employment of high- and low-skilled labor increases with ω. Thus, the surplus, aggregate income, wages and labor share increase, whereas the aggregate profits and wage markdown of low-skilled labor decrease. Moreover, the magnitude of these effects increases with ω in the interval (w_{ME}, ω_{M1}). At the threshold ω_{M1}, the solution to the system of first order conditions identifying the solution to the monopoly's problem involves using $\bar{l}(\omega_{M1})$ units of low-skilled labor per plant; i.e., at the profit-maximizing labor profile the marginal revenue from low-skilled labor is equal to ω_{M1}.

For *intermediate* minimum wages $\omega \in (\omega_{M1}, \omega_{M2})$, where

$$\omega_{M2} := \left(\frac{mn}{\lambda}\right)^{(1-\alpha-\beta)\eta} \left(\frac{\alpha\varepsilon}{1+\varepsilon}\right)^{(1-\beta)\eta} \beta^{\beta\eta},$$

the marginal revenue from low-skilled labor falls below ω for some $l < \bar{l}(\omega)$, and therefore, the profit-maximizing low-skilled labor is below $\bar{l}(\omega)$. Thus, unemployment of low-skilled labor arises. Moreover, the monopoly loses its market power in the market for low-skilled labor, i.e., the wage markdown on low-skilled labor is zero. As ω increases, the employment of low-skilled labor decreases, while the supply increases, thus leading to an increase in unemployment of this type of labor. Moreover, the productivity, employment, and wage of high-skilled labor also decrease. Thus, surplus, aggregate income, and profits decrease with ω in $(\omega_{M1}, \omega_{M2})$, whereas the labor share and wage markdowns remain constant. The threshold ω_{M2} is identified by the equality of the profit-maximizing employment of high-skilled labor, and the supply at the minimum wage, $H^S(\omega) = \lambda\omega_{M2}^{\varepsilon}$.

For *high* minimum wages $\omega \in (\omega_{M2}, \omega_{M3})$, where

$$\omega_{M3} := \left(\frac{mn}{\lambda}\right)^{(1-\alpha-\beta)\eta} \alpha^{(1-\beta)\eta}\beta^{\beta\eta},$$

the minimum wage also becomes binding in the market for high-skilled labor. Thus, ω has an impact on the employment of high-skilled labor analogous to that of a low minimum wage on the employment of low-skilled labor. Hence, the monopoly increases its high-skilled labor. The productivity of low-skilled labor, in turn, increases. However, because the marginal cost of both high- and low-skilled labor is the same, there is an input substitution effect leading the monopoly to reduce its low-skilled labor. The net impact of these two opposite effects on the employment of low-skilled labor is ambiguous, as is the sign of the effect on aggregate income and surplus. Nonetheless, the labor share increases, whereas aggregate profits and the wage markdown of high-skilled labor decrease with ω. The threshold ω_{M3} is identified by the equality of marginal revenue from high-skilled labor and the minimum wage.

For *extreme* minimum wages, $\omega > \omega_{M3}$, the monopoly's market power disappears, i.e., wage markdowns are zero. The employment of high- and low-skilled labor, surplus, aggregate income and profits decrease with the minimum wage. The labor share remains constant at $\alpha + \beta$. In this region, the monopolistic and competitive equilibria coincide.

Proposition 9 establishes these results.

Proposition 9. *In a monopolistic economy with a minimum wage ω, the surplus, aggregate income, and employment increase with ω in (w_{ME}, ω_{M1}), but decrease with ω in $(\omega_{M1}, \omega_{M2}) \cup (\omega_{M3}, \infty)$. The employment of high-skilled labor increases with ω in $(\omega_{M2}, \omega_{M3})$. The labor share increases increases with ω in $(w_{ME}, \omega_{M1}) \cup (\omega_{M2}, \omega_{M3})$, and remains constant in $(\omega_{M1}, \omega_{M2}) \cup (\omega_{M3}, \infty)$. The wage markdown of low-skilled labor decreases with ω in (w_{ME}, ω_{M1}) and is zero in (ω_{M1}, ∞), whereas the wage markdown of high-skilled labor remains equal to $1/(1+\varepsilon)$ in (w_{ME}, ω_{M2}), decreases with ω in $(\omega_{M2}, \omega_{M3})$ and is zero in (ω_{M3}, ∞). The skill premium decreases in (w_{ME}, ω_{M2}) and disappears in (ω_{M2}, ∞).*

The welfare effects of minimum wages in a monopolistic economy are complex. The profits and welfare of owners decrease with the minimum wage. Notably, the welfare of workers increases with ω in the interval of low minimum wages. In the interval of intermediate minimum wages, the welfare of employed low-skilled workers increases whereas that of high-skilled workers decreases. In the interval of high minimum wages, the welfare of high-skilled workers and employed low-skilled workers increases. In the interval of extreme minimum wages, the welfare of employed workers increases. For minimum wages above ω_{M1}, an increasing population of unemployed workers becomes worse off than in the monopolistic equilibrium without a minimum wage.

Thus, a policymaker aiming to maximize the aggregate income and/or labor share without creating unemployment must set the minimum wage to ω_{M1} (the largest low minimum wage). However, the largest surplus and labor share may be reached at ω_{M3} (the largest high minimum wage), although at the expense of creating unemployment of low-skilled workers.

7.3 Minimum Wages in an Oligopolistic Economy

Let us consider the impact of a minimum wage in an oligopolistic economy. An *oligopolistic equilibrium with a minimum wage* $\omega \in \mathbb{R}_+$ is a profile $(\boldsymbol{h}, \boldsymbol{l}, p, v, w) \in M(\omega)$ such that for all $ij \in \{1, ..., m\} \times \{1, ..., n\}$, (h_{ij}, l_{ij}) solves the variation in the problem (35) obtained by replacing functions $v(\boldsymbol{h})$ and $w(\boldsymbol{l})$ with the functions $\bar{v}(\boldsymbol{h}, \omega)$ and $\bar{w}(\boldsymbol{h}, \omega)$ identified by (41).

An oligopolistic economy with a minimum wage has a symmetric equilibrium. However, owing to the discontinuity of firms' marginal cost functions at the minimum wage, an oligopolistic economy may have asymmetric equilibria for low and high minimum wages. When goods are produced with decreasing returns to scale, the surplus and aggregate income in asymmetric equilibria are below those in the symmetric equilibrium. Thus, to evaluate the effect of minimum wages in an oligopolistic economy, we focus on the symmetric equilibrium, which provides a reference for the benefits of such a policy in the best-case scenario.

As in a monopolistic economy, the introduction of a minimum wage in an oligopolistic economy modifies the firm's marginal cost of labor, i.e., the right-hand side of equations (37) and (38). Calculating the symmetric oligopolistic equilibrium with a minimum wage, and assessing the effects of variations of the minimum wage, follows the same steps as in a monopolistic economy. Moreover, the qualitative effects of *low, intermediate*, *high,* and *extreme* minimum wages are analogous to those in a monopolistic economy, although the thresholds identifying these regions of minimum wages differ. Specifically, *low* minimum wages are those in the interval (w_{OE}, ω_{O1}), where

$$\omega_{O1} := \left(\frac{\alpha^{\alpha\varepsilon} \gamma^{1+\varepsilon} \left(\beta \left(1 + mn\theta - m\right)\right)^{1+(1-\alpha)\varepsilon} \lambda^{\alpha}}{(mn)^{\alpha\beta(1+\varepsilon)} \varepsilon^{1+(1-\alpha)\varepsilon} \left(1 - \lambda\right)^{(1-\beta)(1+\varepsilon)-\alpha\varepsilon}} \right)^{\frac{\eta}{1+\varepsilon}},$$

intermediate minimum wages are those in the interval $(\omega_{O1}, \omega_{O2})$, where

$$\omega_{O2} := \left(\frac{(\alpha\gamma)^{1-\beta} \left(\beta \left(1 + mn\theta - m\right)\right)^{\beta}}{(mn)^{\alpha+2\beta-1} \theta^{\beta} \lambda^{1-\alpha-\beta}} \right)^{\eta},$$

and high and extreme minimum wages are those in the intervals $(\omega_{O2}, \omega_{O3})$ and (ω_{O3}, ∞), respectively, where

$$\omega_{O3} := \left(\frac{\alpha^{1-\beta} \beta^{\beta} \left(1 + mn\theta - m\right)}{(mn)^{\alpha+\beta} \theta \lambda^{1-\alpha-\beta}} \right)^{\eta}.$$

For low minimum wages, $\omega \in (w_{OE}, \omega_{O1})$, low-skilled labor is in short supply, while for high minimum wages, $\omega \in (\omega_{O2}, \omega_{O3})$, high-skilled labor is in short supply. In either case, the discontinuity of firms' marginal cost for the type of labor in short supply at the minimum wage leads to the existence of a continuum of asymmetric equilibria. The asymmetry arises because the labor in short supply may be divided among firms in many ways.

Proposition 10 provides a summary of these results. Further discussion and illustrative examples are provided in the next section.

Proposition 10. *An oligopolistic economy with a minimum wage ω has a symmetric equilibrium but may also have asymmetric equilibria. In the symmetric equilibrium, the surplus, aggregate income, and employment increase with ω in (w_{OE}, ω_{O1}), and decrease with ω in $(\omega_{O1}, \omega_{O2}) \cup (\omega_{O3}, \infty)$. The employment of high-skilled labor increases with ω in $(\omega_{O2}, \omega_{O3})$. The labor share increases with ω in $(w_{OE}, \omega_{O1}) \cup (\omega_{O2}, \omega_{O3})$, and remains constant below LS_{CE} in $(\omega_{O1}, \omega_{O2}) \cup (\omega_{O3}, \infty)$. The wage markdown of low-skilled labor decreases in (w_{OE}, ω_{O1}) and remains equal to $(m-1)/mn\theta > 0$ in (ω_{O1}, ∞), whereas the wage markdown of high-skilled labor remains equal to $1-\gamma$ in (w_{OE}, ω_{O2}), decreases in $(\omega_{O2}, \omega_{O3})$, and remains equal to $(m-1)/mn\theta > 0$ in (ω_{O3}, ∞). The skill premium decreases in (w_{OE}, ω_{O2}) and disappears when $\omega \geq \omega_{O2}$.*

The welfare effects of minimum wages in an oligopolistic economy are qualitatively analogous to those they have in a monopolistic economy. A policymaker aiming to maximize the surplus, aggregate income and/or labor share without creating unemployment must set the minimum wage to ω_{O1} (the largest low minimum wage). Moreover, because the symmetric oligopolistic equilibrium approaches uniformly the competitive equilibrium, when n is sufficiently large, the minimum wage ω_{O1} also maximizes the surplus and aggregate income.

7.4 Minimum Wages: Examples and Discussion

For the economy in which the parameters are

$$(m, A, \alpha, \beta, \theta, \varepsilon, \lambda) = (2, 1, 3/5, 1/5, 6/5, 1/4, 1/4),$$

the graphs below identify the variables describing the economic activity as functions of the minimum wage in the competitive equilibrium (blue curves), monopolistic equilibrium (red curves), and symmetric oligopolistic equilibrium (green curves) for $n = 1$ (left panels) and $n = 2$ (right panels). These graphs illustrate the results of propositions 8, 9 and 10.

The graphs show that the competitive surplus, aggregate income, and employment of high and low-skilled labor remain uniformly above, while the aggregate profits and unemployment of high- and low-skilled labor remain uniformly below, the corresponding monopolistic and oligopolistic values. The competitive labor share and skill premium also remain uniformly above the monopolistic and oligopolistic values.

The threshold of Proposition 7 for this economy is $\bar{n} = 49/48 > 1$. Hence, for $n = 1$, the surplus, aggregate income and employment in the monopolistic equilibrium are greater than in the oligopolistic equilibrium. When $n = 2$, these inequalities reverse. However, as the graphs show, with a high minimum wage the surplus, aggregate income and employment may be larger in the monopolistic than in the oligopolistic equilibrium; i.e., high minimum wages may be more effective in a monopolistic economy. Indeed, for extreme minimum wages the competitive and monopolistic equilibria coincide.

For these parameter values, the surplus, aggregate income and employment in both the monopolistic and oligopolistic equilibrium increase with the minimum wage in the interval of low minimum wages, reaching their maximum value at the upper bound of the interval of low minimum wages, and then decreasing with the minimum wage for larger minimum wages.

For minimum wages above the upper bound of low minimum wages, unemployment of low-skilled workers arises and increases with the minimum wage. The maximum employment of high-skilled labor is reached at the upper bound of the interval of high minimum wages. Unemployment of high-skilled workers also arises and increases with the minimum wage in the interval of extreme minimum wages.

The wage markdowns and skill premium decrease with the minimum wage, and while they disappear in the monopolistic economy, they remain positive in the oligopolistic economy. The labor share increases and reaches the competitive surplus in the monopolistic economy but remains below the competitive surplus in the oligopolistic economy.

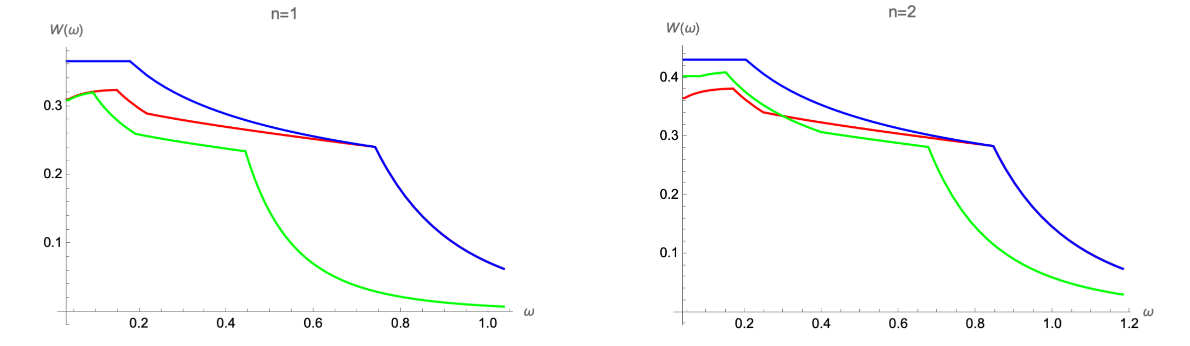
It is possible to identify conditions on the parameter values that determine whether the surplus and/or aggregate income are maximal at the upper bound of the intervals of low or high minimum wages. These conditions involve inequalities of varying complexity. In an oligopolistic economy, for example, the maximum aggregate income is reached at ω_{O1} or ω_{O3} depending on whether the sign of

$$\left(\frac{mn\varepsilon}{1+mn\varepsilon}\right)^{\varepsilon}-\left(\frac{\beta\lambda}{\alpha\left(1-\lambda\right)}\right)^{\frac{\beta}{\alpha}} \tag{42}$$

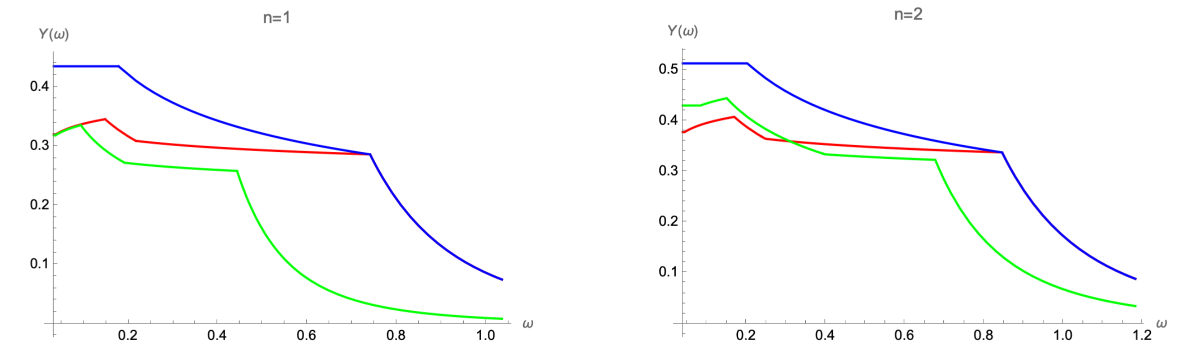
is positive or negative. Thus, the low minimum wage ω_{O1} tends to emerge as the maximizer of aggregate income when the elasticity of labor supply ε is small, high-skilled labor is scarce (i.e., λ is small) and the numbers of goods m and firms n are large. Also, using equation (22) we may calculate the surplus given the aggregate income Y and the labor share LS as $W = Y\left(1+\left(1-LS\right)\varepsilon\right)/\left(1+\varepsilon\right)$. Noticing that the labor share at ω_{O3} is greater than that at ω_{O1} (by Proposition 10), we see that if the maximum aggregate income is reached at ω_{O1}, then the maximum surplus is also reached at ω_{O1}. Nevertheless, when the number of firms per sector n is small, either the aggregate income and/or the surplus may be maximized at the high minimum wage ω_{O3}.

In an oligopolistic economy with a minimum wage ω, the wage markdown of low-skilled (high-skilled) labor decreases in the interval of low (high) minimum wages but does not fully disappear, and the labor share remains below that of a competitive economy. Of course, as the number of firms per sector increases, the wage markdowns and the labor share approach those in the competitive equilibrium. (Recall that γ approaches 1 as n grows large, and therefore the labor share approaches $\alpha+\beta = LS_{CE}$ by Proposition 5.)

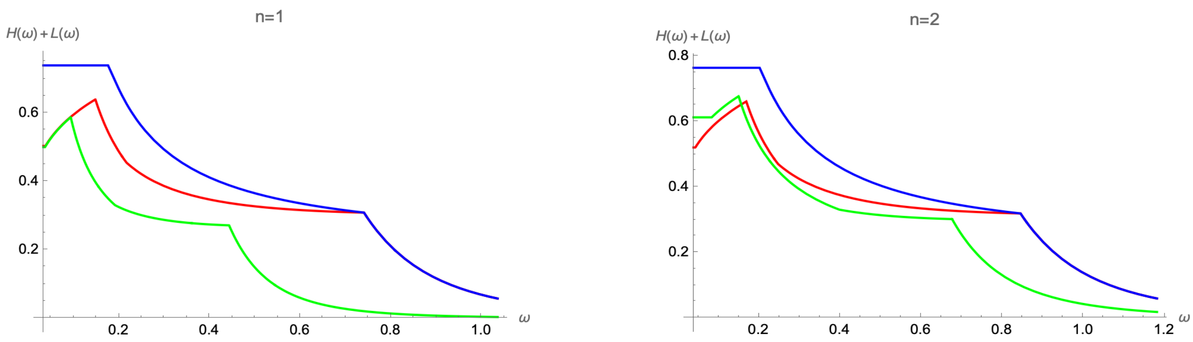
As for the distributional effects of minimum wages in the presence of market power, low minimum wages lead to an increase in wages and employment of both high- and low-skilled workers, and do not create unemployment. Of course, firms' profits decrease. Thus, labor income increases by an amount equal to the sum of the increase in aggregate income and decrease in aggregate profits. Owners are obviously worse off, while workers who own no shares (or small shares) of the firms' property are better off. However, high minimum wages lead to unemployment of low-skilled labor. Unemployed low-skilled workers are obviously worse off in the absence of unemployment benefits.



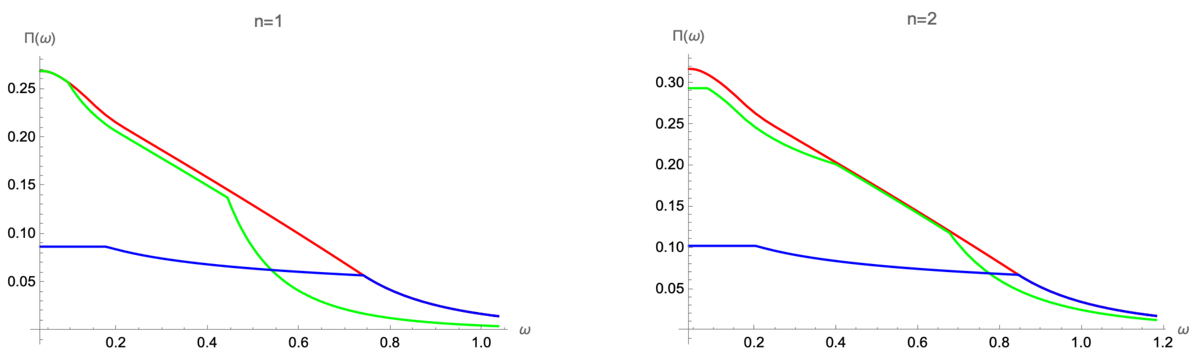
Surplus



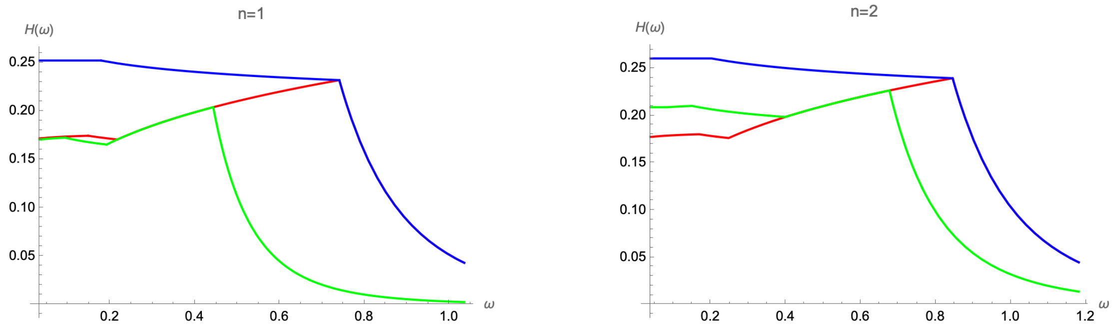
Aggregate Income



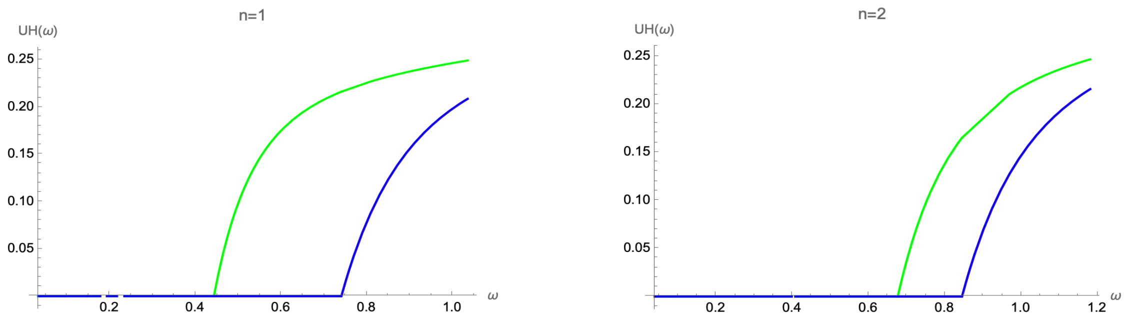
Employment



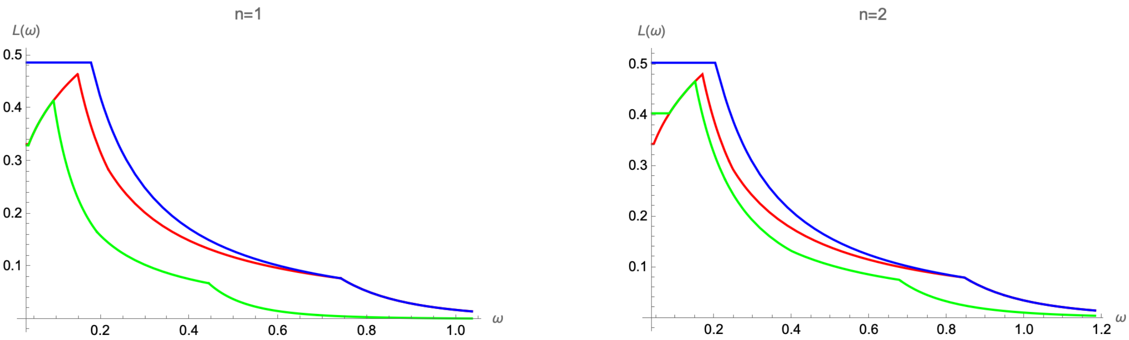
Profits



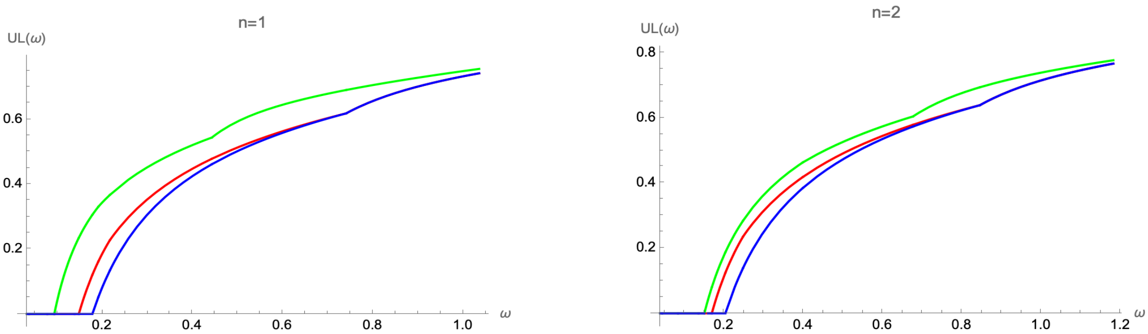
Employment of High-Skilled Labor



Unemployment of High-Skilled Labor



Employment of Low-Skilled Labor



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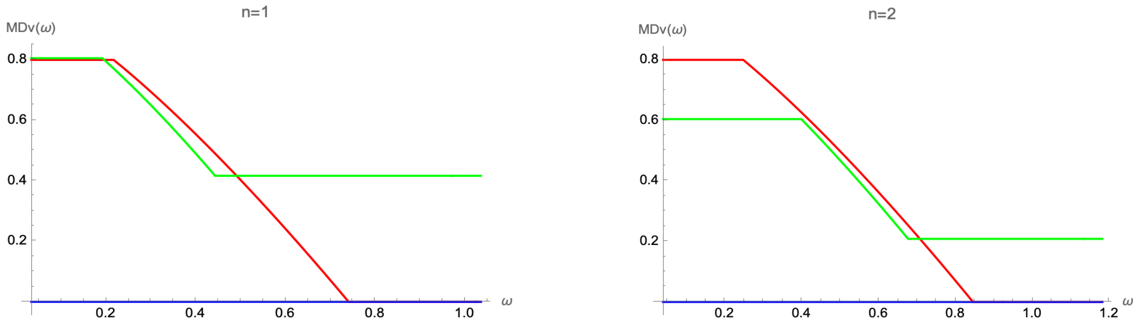
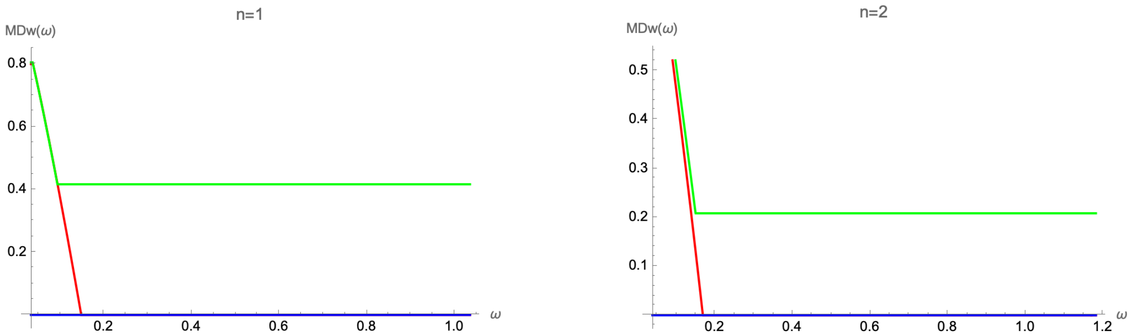
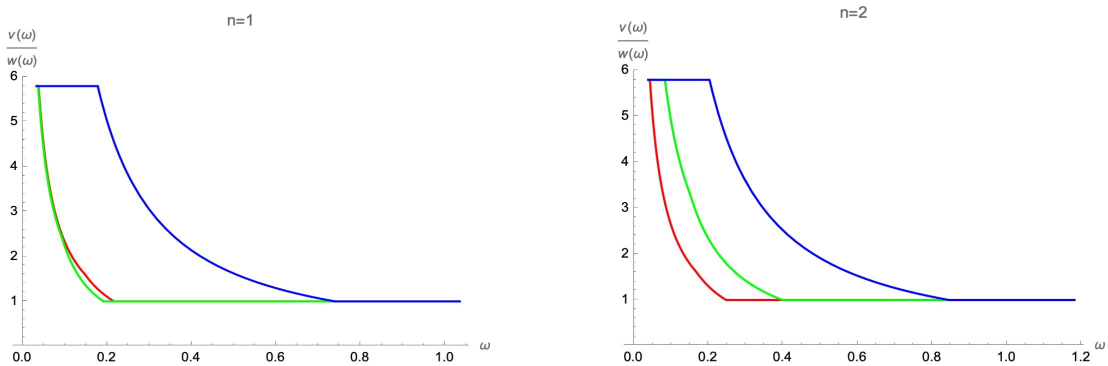


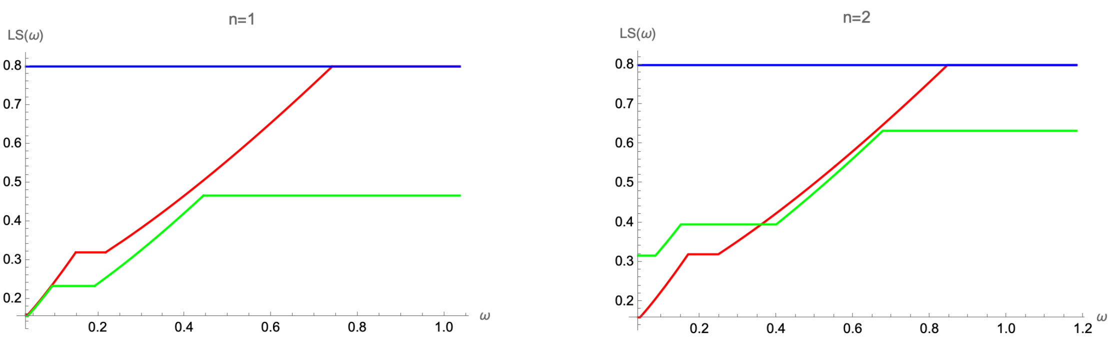
Wage Markdown of High-Skilled Workers



Wage Markdown of Low-Skilled Workers



Skill Premium



Labor Share

7.5 Taxes and Public Spending

Let us consider the impact of taxes and public spending. Clearly, proportional taxes on labor income, which affect after-tax real wages, introduce distortions leading to lower employment, aggregate income, and surplus in all scenarios considered. This is verified by inspecting the equilibrium formulae given in propositions 3, 4 and 5. The distribution effects of such taxes depend on revenue allocation.

However, a corporate tax, i.e., a proportional tax $t \in (0, 1)$ on profits, does not affect the solution to a firm's problem in competitive, monopolistic, or oligopolistic economies, and hence does not affect the equilibrium, but merely affects firms' after-tax profits. Thus, a corporate tax is an appropriate policy tool for pursuing purely distributional objectives. Moreover, in monopolistic and oligopolistic economies, corporate taxes can be used in conjunction with minimum wages to achieve both efficiency gains and distributional objectives. For example, when surplus is maximized at the largest high minimum wage, setting such a minimum wage entails creating unemployment of low-skilled workers. Tax revenues can be used to mitigate the negative effects of unemployment on welfare.

Tax revenues can also be used to provide funding for public spending. The lack of public goods in the present setting reduces the scope of such policies. We consider the benefits of providing *public education* aimed at increasing the fraction of high-skilled workers in the labor force. In addition, we discuss the impact of a program to promote *innovation*, i.e., to improve the technology available to produce consumption goods. Naturally, an assessment of such programs must consider their costs and benefits. Our analysis implicitly assumes that providing education involves using income, and does not consider that it may involve using, e.g., high-skilled labor. Moreover, dynamic issues, which are surely relevant to evaluating the effect of investments in education or innovation, are outside the scope of our static analysis.

Let us first consider the impact of a public education program leading to an increase in the proportion of high-skilled workers in the labor force, assuming that it is funded by the revenues of a corporate tax t. To this end, we examine the effects of a marginal increase in λ on surplus and its distribution.

Using the formulae of Proposition 1, we verify that the maximum surplus and its associated aggregate income are increasing functions of λ since

$$\frac{\partial W^*}{\partial \lambda} = \frac{1}{(1+\varepsilon)\eta} \frac{\partial Y^*}{\partial \lambda}$$

and

$$\frac{\partial Y^*}{\partial \lambda} = \eta Y^* \left(\frac{\alpha}{\lambda} - \frac{\beta}{1-\lambda} \right) > 0,$$

where the inequality follows from (1) implying that high-skilled labor is relatively scarce. Moreover, surplus maximization requires using more high-skilled labor, since

$$\frac{\partial h^*}{\partial \lambda} = \frac{\eta h^*}{(1+\varepsilon)(1-\lambda)\lambda} \left((1-\lambda)(1+\varepsilon) - \beta\varepsilon \right) > 0,$$

where the inequality follows again from (1). However, either more or less low-skilled labor may be required since the sign of

$$\frac{\partial l^*}{\partial \lambda} = \frac{\eta l^*}{(1+\varepsilon)(1-\lambda)\lambda} \left(\alpha\varepsilon - \lambda(1+\varepsilon) \right)$$

is ambiguous.

By Proposition 3, $Y_{CE} = Y^*$, $W_{CE} = W^*$, $H_{CE} = mnh^*$, and $L_{CE} = mnl^*$. Thus, in a competitive economy, the surplus, aggregate income, and employment of high-skilled labor increase with λ. Furthermore, labor income increases by $(\alpha + \beta)$ times the increase in aggregate income; hence, by this aggregate measure, workers are better off. However, the wage of high-skilled labor decreases since

$$\frac{\partial v_{CE}}{\partial \lambda} = -\frac{(1-\eta)(1+\varepsilon)(1-\lambda) + \eta\beta\varepsilon}{\varepsilon(1+\varepsilon)\lambda(1-\lambda)} v_{CE} < 0,$$

whereas the wage of low-skilled labor increases since

$$\frac{\partial w_{CE}}{\partial \lambda} = \frac{\eta\left(\alpha + \lambda(1+\varepsilon)(1-\alpha-\beta)\right) w_{CE}}{\lambda(1-\lambda)(1+\varepsilon)} > 0.$$

This implies that a (high-) low-skilled worker becomes (worse) better off.

When firms have market power, the welfare effects of a marginal increase in λ are qualitatively identical, but of a smaller magnitude than in a competitive economy. This can easily be shown using the formulae of propositions 4 and 5 to get

$$\frac{\partial W_{ME}}{\partial \lambda} = \left(\eta + \frac{\varepsilon}{1+\varepsilon} \right) \left(\frac{\varepsilon}{1+\varepsilon} \right)^{(\alpha+\beta)\eta\varepsilon} \frac{\partial W_{CE}}{\partial \lambda} < \frac{\partial W_{CE}}{\partial \lambda},$$

where the inequality follows from (39), and

$$\frac{\partial W_{OE}}{\partial \lambda} = \left(1 + \left(1 - (\alpha+\beta)\gamma\right)\varepsilon\right) \eta \gamma^{(\alpha+\beta)\eta\varepsilon} \frac{\partial W_{CE}}{\partial \lambda} < \frac{\partial W_{CE}}{\partial \lambda},$$

where the inequality follows from (40), while

$$\frac{\partial Y_{ME}}{\partial \lambda}=\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon}\frac{\partial Y_{CE}}{\partial \lambda}<\frac{\partial Y_{CE}}{\partial \lambda}\text{ and }\frac{\partial Y_{OE}}{\partial \lambda}=\gamma^{(\alpha+\beta)\eta\varepsilon}\frac{\partial Y_{CE}}{\partial \lambda}<\frac{\partial Y_{CE}}{\partial \lambda}.$$

Therefore, the surplus and aggregate income increase by a smaller factor in economies in which firms have market power. Thus, the cost-benefit analysis of such programs is less favorable in economies in which firms have market power.

Let us consider the impact on inequality. By Proposition 2 the skill premium is the same independently of firms' market power and is given by (26). Obviously, an increase of λ leads to a decrease of the skill premium, as can be verified by taking derivative in (26) with respect to λ,

$$\frac{\partial SP}{\partial \lambda}=-\frac{SP}{(1+\varepsilon)\,\lambda\,(1-\lambda)}<0.$$

On this measure of inequality, the effect of an increase of λ is the same whether the economy is competitive, monopolistic or oligopolistic. However, the effect of an increase of λ on the wage levels, and therefore on the wage differential between high- and low-skilled labor, is smaller in a monopolistic or oligopolistic economy than in a competitive economy, since

$$\frac{\partial v_{ME}}{\partial \lambda}=\left(\frac{\varepsilon}{1+\varepsilon}\right)^{\eta}\frac{\partial v_{CE}}{\partial \lambda}<\frac{\partial v_{CE}}{\partial \lambda}\text{ and }\frac{\partial v_{OE}}{\partial \lambda}=\gamma^{\eta}\frac{\partial v_{CE}}{\partial \lambda}<\frac{\partial v_{CE}}{\partial \lambda}.$$

and

$$\frac{\partial w_{ME}}{\partial \lambda}=\left(\frac{\varepsilon}{1+\varepsilon}\right)^{\eta}\frac{\partial w_{CE}}{\partial \lambda}<\frac{\partial w_{CE}}{\partial \lambda}\text{ and }\frac{\partial w_{OE}}{\partial \lambda}=\gamma^{\eta}\frac{\partial w_{CE}}{\partial \lambda}<\frac{\partial w_{CE}}{\partial \lambda}.$$

Of course, the wage differential between high- and low-skilled labor in a competitive economy remains larger than in either a monopolistic or an oligopolistic economy.

Let us next discuss the impact of a program supporting technological innovation, assuming that it is funded using the revenue from a corporate tax t. We consider the case in which technological progress is skilled unbiased, leading simply to increases in the value of the parameter A. By inspecting the formulae of Proposition 1, it is straightforward to verify that an increase in A leads to an increase in the optimal employment of high- and low-skilled labor, (h^*, l^*), as well as the maximum surplus W^* and associated income Y^*.

In a competitive economy, such program leads to an increase in surplus, aggregate income, employment, wages, and profits by Proposition 3. Moreover, by propositions

4 and 5, the qualitative impact is the same but weaker in economies in which firms have market power. In either case, neither the income and surplus distributions nor the labor share, wage markdowns or skill premium are affected. Hence, the benefits of such programs may be assessed simply based on their impact on surplus.

It is also possible to consider skill-biased technological innovation, which the literature suggests is partially responsible for the increase of wage inequality in recent times. We may derive qualitative results on the impact of an increase in the productivity of high-skilled labor on the economy's equilibrium by considering the effect of a marginal increase in the parameter α. Variations in α affect not only the surplus but also its distribution, since they affect the skill premium and wage inequality.

8 Free Entry

Let us reconsider our analysis when new firms can freely enter the economy, incurring a set-up cost of $K > 0$ units of the composite good. We discuss the welfare properties of the free entry equilibria that emerge in competitive, monopolistic, and oligopolistic economies, focusing on the number of firms per sector, n. We investigate whether the number of firms in a free entry equilibrium is *socially optimal*, *excessive,* or *insufficient.*

We denote by $W(n)$, $Y(n)$, and $\Pi(n)$, for the equilibrium surplus, aggregate income, and profits, respectively, as a function of the number of firms per sector. Propositions 3, 4, and 5 identify these formulae for competitive, monopolistic and oligopolistic economies. Treating n as a real number rather than an integer facilitates the analysis. (We consider the integer constraint in the discussion of the results.) We denote by $W'(n)$, $Y'(n)$, and $\Pi'(n)$ the partial derivatives with respect to n.

The *socially optimal number of firms per sector* n^* is a solution to the problem

$$\max_n W(n) - mnK.$$

If $W(n)$ is a strictly concave function of n, then the socially optimal number of firms is the solution to the equation

$$W'(n) = mK. \tag{43}$$

Let us consider a competitive economy. Using equations (12) and (13) of Proposition

2, and Proposition 3, we get

$$Y_{CE}(n) = Y_{CE}(1) n^{\eta(1+\varepsilon)(1-\alpha-\beta)}$$

and

$$W_{CE}(n) = \frac{Y_{CE}(1) n^{\eta(1+\varepsilon)(1-\alpha-\beta)}}{(1+\varepsilon)\,\eta}.$$

Assume that goods are produced with decreasing returns to scale, i.e., $\alpha+\beta < 1$. Because $\eta\,(1+\varepsilon)\,(1-\alpha-\beta) < 1$, W_{CE} is a strictly concave function of n. Thus, the socially optimal number of firms per sector is the solution to equation (43). Taking derivative with respect to n we get

$$W'_{CE}(n) = (1-\alpha-\beta)\,Y_{CE}(1) n^{\eta(1+\varepsilon)(1-\alpha-\beta)-1} = \frac{\Pi_{CE}(n)}{n}. \tag{44}$$

Hence, we may write equation (43) as

$$\frac{\Pi_{CE}(n)}{n} = mK. \tag{45}$$

Solving for n and noticing that $1-\eta\,(1+\varepsilon)\,(1-\alpha-\beta) = \eta\,(\alpha+\beta)$, we get

$$n^*_{CE} = \left(\frac{(1-\alpha-\beta)\,Y_{CE}(1)}{mK}\right)^{\frac{1}{\eta(\alpha+\beta)}}.$$

If goods are produced with constant returns to scale, i.e., $\alpha+\beta = 1$, then $Y_{CE}(n) = Y_{CE}(1)$ and $W_{CE}(n) = W_{CE}(1)$. Therefore, $W_{CE}(n) - mnK$ strictly decreases with n for $n > 1$. Considering that n must be an integer, we conclude that the socially optimal number of firms is either $n = 1$ or $n = 0$ depending on whether $W_{CE}(1) - mK$ is positive or negative.

Let us identify the number of firms per sector in the *free entry competitive equilibrium*, in which the number of firms per sector n must be such that additional firms cannot enter and make profits above K. Thus, the equilibrium number of firms, n_{CE}, is identified by the equation

$$\Pi_{CE}\,(n) = mnK. \tag{46}$$

This equation is identical to that identifying the socially optimal number of firms when goods are produced with decreasing returns to scale, i.e., $\alpha+\beta < 1$. Thus, in this case the number of firms per sector in the free entry competitive equilibrium is *socially optimal,* i.e.,

$$n_{CE} = n^*_{CE}.$$

This result reproduces in our setting Mankiw and Whinston's (1986) Corollary 1, showing that in the absence of a business-stealing effect, the incentives of the marginal entrant and the social planner coincide, as is explicit by equation (44). When goods are produced with constant returns to scale, i.e., $\alpha + \beta = 1$, the competitive equilibrium profits are zero. Therefore, a positive set-up cost would imply that no firm enters the economy; hence, there is *insufficient* when $n^*_{CE} = 1$.

We now consider a monopolistic economy. By Proposition 4, the aggregate income in the monopolistic equilibrium with n firms per sector is

$$Y_{ME}(n) = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1) n^{\eta(1+\varepsilon)(1-\alpha-\beta)}$$

and the surplus is

$$W_{ME}(n) = \frac{(1+\varepsilon)\,\eta + \varepsilon}{(1+\varepsilon)^2\,\eta}\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1) n^{\eta(1+\varepsilon)(1-\alpha-\beta)}.$$

Hence, W_{ME} is a strictly concave function of n. Taking derivative we get

$$W'_{ME}(n) = \left(\eta + \frac{\varepsilon}{1+\varepsilon}\right)(1-\alpha-\beta)\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1) n^{\eta(1+\varepsilon)(1-\alpha-\beta)-1}. \quad (47)$$

Therefore, the socially optimal number of firms per sector is the solution to the equation (43), which is given by

$$n^*_{ME} = \left(\eta + \frac{\varepsilon}{1+\varepsilon}\right)^{\frac{1}{\eta(\alpha+\beta)}}\left(\frac{\varepsilon}{1+\varepsilon}\right)^{\varepsilon} n^*_{CE} < n^*_{CE},$$

where the inequality follows from the inequality (39). Thus, the socially optimal number of firms per sector in a monopolistic economy is less than that in a competitive economy. This is intuitive because a monopolistic plant produces less than a competitive firm and hence creates less surplus.

If the monopoly can freely choose the number of plants per sector n, it will do so with the objective of maximizing its profit net of the set-up cost; i.e., in the *monopolistic equilibrium with free entry* the number of plants per sector, n_{ME}, is a solution to the problem

$$\max_n \Pi_{ME}(n) - mnK. \quad (48)$$

Using the formulae of Proposition 4 we may write

$$\begin{aligned}\Pi_{ME}(n) &= \left(1 - (\alpha+\beta)\frac{\varepsilon}{1+\varepsilon}\right)\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1) n^{(1-\alpha-\beta)(1+\varepsilon)\eta} \\ &= ((1+\varepsilon)\,\eta)^{-1}\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1) n^{(1-\alpha-\beta)(1+\varepsilon)\eta},\end{aligned}$$

which is a strictly concave function of n. Taking derivative with respect to n and simplifying we get

$$\Pi'_{ME}(n) = (1-\alpha-\beta)\left(\frac{\varepsilon}{1+\varepsilon}\right)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1)n^{(1-\alpha-\beta)(1+\varepsilon)\eta-1}. \tag{49}$$

Hence n_{ME} is the solution to the equation

$$\Pi'_{ME}(n) = mK.$$

Solving this equation and using the formulae for n^*_{CE} derived above we get

$$n_{ME} = \left(\frac{\varepsilon}{1+\varepsilon}\right)^{\varepsilon} n^*_{CE} < \left(\eta + \frac{\varepsilon}{1+\varepsilon}\right)^{\frac{1}{\eta(\alpha+\beta)}} \left(\frac{\varepsilon}{1+\varepsilon}\right)^{\varepsilon} n^*_{CE} = n^*_{ME},$$

where the inequality holds since $\eta + \varepsilon/(1+\varepsilon) > 1$. Thus, there is *insufficient entry* in a monopolistic economy, which is an additional source of inefficiency. This result arises because the marginal social benefit of installing a new plant per sector is above the monopolist's marginal profit, as can be seen by noticing that equations (47) and (49) imply

$$W'_{ME}(n) = \left(\eta + \frac{\varepsilon}{1+\varepsilon}\right)\Pi'_{ME}(n) > \Pi'_{ME}(n).$$

Even though the monopolist internalizes consumers preference for variety, which deactivates the issue of product differentiation, this misalignment between the monopolist's interests and those of society leads to insufficient entry.

We now consider an oligopolistic economy. By Proposition 5

$$W_{OE}(n) = \frac{1+(1-(\alpha+\beta)\gamma(n))\varepsilon}{1+\varepsilon}\gamma(n)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1)n^{\eta(1+\varepsilon)(1-\alpha-\beta)}.$$

It is unclear whether W_{OE} is a concave function of n, and even if it is, obtaining an analytical solution to the equation $W'_{OE}(n) = mK$ is not feasible. Likewise, calculating the number of firms per sector in the *free entry oligopolistic equilibrium* requires solving the equation

$$\Pi_{OE}(n) = (1-(\alpha+\beta)\gamma(n))\gamma(n)^{(\alpha+\beta)\eta\varepsilon} Y_{CE}(1)n^{\eta(1+\varepsilon)(1-\alpha-\beta)} = mnK. \tag{50}$$

However, the existence of an analytical solution to this equation is unclear.

We inquire into the conditions for insufficient or excessive entry to arise, which depends on the sing of

$$W'_{OE}(n_{OE}) - \frac{\Pi_{OE}(n_{OE})}{n_{OE}}$$

where n_{OE} is the number of firms per sector in the free entry oligopolistic equilibrium, i.e., the solution to equation (50). We show that the sign of this inequality coincides with the sign of

$$\phi(m, n_{OE}, \varepsilon) - \theta \gtreqless 0, \tag{51}$$

where

$$\phi(m, n, \varepsilon) := \frac{(m-1)(1 + mn\varepsilon(1+\varepsilon))}{mn(1 + \varepsilon(mn-1))}.$$

The function ϕ increases with ε and m, and decreases with n. Obviously, n_{OE} decreases with K. Hence, insufficient entry tends to arise when the elasticity of labor supply ε and the cost of entry K are large and when θ is small, whereas excessive entry tends to arise under the reverse conditions.

It is easy to produce examples of oligopolistic economies in which the free entry equilibrium involves either insufficient or excessive entry. Consider the economy studied in Section 7.4, in which

$$(m, A, \alpha, \beta, \theta, \varepsilon, \lambda) = (2, 1, 3/5, 1/5, 6/5, 1/4, 1/4).$$

When the entry cost is $K = 0.005$ the number of firms per sector in the free entry oligopolistic equilibrium is $n_{OE} \simeq 24.997$, while the socially optimal number of firms is $n^*_{OE} \simeq 17.414$. Hence, there is excessive entry into the economy. Increasing the elasticity of labor supply to $\varepsilon = 5$ and the cost of entry to $K = 0.0175$, generates a new economy in which $n_{OE} \simeq 1.887 < 2.357 \simeq n^*_{OE}$; hence, there is insufficient entry. Moreover, if we consider the integer constraint, we see that for $K = 0.005$, $[n_{OE}] = 24 > 17 = [n^*_{OE}]$, i.e., excessive entry arises, whereas for $K = 0.0175$, $[n_{OE}] = 1 < 2 = [n^*_{OE}]$, i.e., insufficient entry arises.

Firms' complex interactions in an oligopolistic economy, which involve competition among firms that produce the same good as well as among firms producing differentiated goods, may lead to either insufficient or excessive entry. Mankiw and Whinston (1986) study entry in imperfectly competitive markets with product diversity and show that entry has two effects that may drive a wedge between the social and private (i.e., firms') benefits of marginal entry: One relates to the business-stealing effect, and the other to the impact of product variety. Because these effects have opposite signs, the sign of the entry bias is ambiguous. Consistent with this assessment, the parameters identifying the economy determine the strength of these two effects and, ultimately, the sign of the net effect.

It is interesting to consider whether the phenomenon uncovered in Proposition 7 and in Proposition 4 of Azar and Vives (2021), vanishes in the long run as the economy reaches its free-entry equilibrium. We show that this is not the case. Indeed, the economy identified by the parameter values above provides an example when $K = 0.0175$. In the free-entry monopolistic equilibrium of this economy the number of firms per sector is $n_{ME} = 0.575$. Thus, considering the integer constraint requires verifying whether the monopolistic profit-maximizing number of firms per sector is $n = 0$ or $n = 1$. It turns out that monopoly profits net of entry cost for $n = 1$ are positive, and therefore in the free-entry monopolistic equilibrium, $n_{ME} = 1$. Moreover, as shown above, in the free-entry oligopolistic equilibrium, $n_{OE} = 1$. Furthermore, for this economy the bound of Proposition 7 is $\bar{n} = 2$. Therefore, in this economy the free-entry monopolistic equilibrium allocation is Pareto superior to the free-entry oligopolistic equilibrium allocation.

Proposition 11 summarizes these results.

Proposition 11. *Under free and costly entry, an optimal number of firms enter a competitive economy (i.e., $n_{CE} = n^*_{CE}$). In a monopolistic economy, there is insufficient entry (i.e., $n_{ME} < n^*_{ME}$) even though the socially optimal number is below that in a competitive economy (i.e., $n^*_{ME} < n^*_{CE}$). In an oligopolistic economy, excessive or insufficient entry may arise. Moreover, the monopolistic equilibrium allocation may be Pareto superior to the oligopolistic equilibrium allocation.*

Alleviating excessive/insufficient entry may require specific entry taxes/subsidies or the design of legal procedures to facilitate or obstruct entry. A corporate tax and/or a minimum wage discourage entry, thus having a positive (negative) efficiency effect when the equilibrium involves excessive (insufficient) entry. Public spending programs, such as those considered in Section 7.5, cannot correct entry biases.

9 Conclusions

Market power reduces the surplus generated by the economic activity and shifts its distribution in favor of consumers whose main source of income is the returns of their shares of firms' property. Free entry does not resolve the inefficiency and distribution bias caused by market power. Antitrust and competition policies are appropriate instruments to improve efficiency and distribution in the long run. Moreover, policy interventions may help effectively reduce the negative effects of market power. For instance, an appropriate minimum wage improves efficiency (i.e., increases surplus) and reduces the distributional bias caused by market power. However, high minimum wages create unemployment and increase inequality in the absence of palliative measures such as unemployment subsidies. Government spending financed via non-distortionary taxes, such as investments in human capital and/or technological innovation, has the potential to improve efficiency and income distribution. However, the cost-benefit analysis of such programs surely involve dynamic factors of the economic activity that are not present in our static setting.

Even though our framework is simple, it captures the basic elements that determine the extent to which market power affects the results of the economic activity: The strength of consumers preference for variety, which relates to the number of goods and the elasticity of substitution among goods; the elasticity of labor supply, which relates to workers' value of leisure; and the degree of concentration as identified by the number of firms. Our assumptions about consumers and workers preferences lead to a constant elasticity of substitution among goods and a constant elasticity of labor supply, allowing us to analytically identify the equilibrium in different settings. These formulae, which involve a small number of parameters, facilitate the derivation of precise results for policy intervention. Thus, the model has the potential to allow for a variety of applications such as those related to the political economy of alternative policies.

Naturally, the specificity of the assumptions regarding firms' technology and consumers' and workers' preferences limits the scope of our conclusions. An essential feature facilitating the analysis is symmetry, which is present in several elements but, most importantly, on the assumption that all goods are produced using the same technology. While assuming a Cob-Douglas production function is non-essential, departing from symmetry seems to be a difficult task. This is an important restriction, as the empirical literature on the macroeconomic effects of market power emphasizes the role of firms' heterogeneity (arising from differences in technology, size, or market structure) in explaining recent phenomena such as the rise in wage inequality – see Deb *et al.* (2024) and Jarosch, Nimczik and Sorkin (2024). (In our symmetric setting, in contrast, market power does not affect the skill premium.) These studies use numerical methods to address firm heterogeneity.

Considering general preferences for consumption goods and leisure presents significant challenges. Perhaps a viable strategy involves working directly with the aggregate demand for goods and the aggregate supply of the different types of labor. Assuming properties of these functions amenable to the analysis, and implicit differentiation of equilibrium conditions may allow the derivation of qualitative similar results on the inefficiency and distributional biases of the allocations generated in economies in which firms have market power. Such task is relegated to future work.

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