Asymmetric Passive Cross Ownership and Tacit Collusion

Rupayan Pal[1] and Emmanuel Petrakis[2]

[1]Indira Gandhi Institute of Development Research (IGIDR), India,

rupayan@igidr.ac.in

[2]Departamento de Economía, Universidad Carlos III de Madrid, Spain and

Department of Economics, University of Crete, Rethymnon, Greece,

empetrak@eco.uc3m.es, petrakis@uoc.gr

Abstract: In a Cournot duopoly, we analyze the implications of asymmetric passive cross ownership (PCO) for the stability of tacit collusion. In contrast to the symmetric PCO that always stabilizes collusion, a higher level of average PCO need not necessarily be anticompetitive. When the distribution of bargaining power over sharing collusive profits is exogenously given and the firm that owns less shares of its rival is neither too powerful nor too weak, an increase in the average level of cross-holdings makes collusion less likely to be stable. This also occurs under broader conditions when the PCO becomes more asymmetric. Yet, if firms' bargaining powers are determined via their levels of PCO, a 'mean-increasing spread-preserving' shift in the distribution of PCO stabilizes collusion, while a 'mean-preserving spread-increasing' shift does not affect collusion stability. Our findings have important policy implications for antitrust authorities.

Keywords: Passive cross ownership; Tacit collusion; Collusion stability; Asymmetric cross-holdings.

JEL Classification: L41, L13, K21

Corresponding Author: Emmanuel Petrakis

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1. Introduction

Passive cross ownership (PCO) among rival firms is a widely observed phenomenon in many industries, including banking and finance, information technology, natural resources, automobile, consumer goods, and other manufacturing.[1] Moreover, the pattern of PCO is often found to be asymmetric, i.e. the extent of passive cross-holdings vary across firms in the same industry. For example, in the automobile industry, Toyota bought a 4.94% stake in Suzuki, while Suzuki bought 48-billion-yen worth of shares in Toyota in 2019. In 2018, Renault owned 43% of Nissan, while Nissan had a 15% share in Renault. From 2011 to 2015, VW and Suzuki held shares in each other 19.89% and 2.5%, respectively (Hariskos et al., 2022). While a firm does not gain control rights over its rivals by holding their shares, it is well argued that passive cross ownership softens product market competition. The reason is that ownership of a rival's share by a firm induces it to consider the implications of its market strategy, not only on its own operating profit, but also on that of its rival. The same is true, regardless of whether the pattern of PCO is symmetric or asymmetric.

This paper analyzes the implications of asymmetric PCO patterns for the stability of tacit collusion. We consider a linear demand homogeneous good Cournot duopoly in which firms have the same marginal cost but differ both in terms of their passive cross-holdings and their bargaining powers over sharing collusive profits. We consider both a scenario with firms' bargaining powers exogenously given, and another in which they are determined in accordance with their levels of cross-holdings.

We show that if the pattern of PCO is symmetric, a higher level of cross-holdings makes tacit collusion more likely to be stable, not only in case firms equally share collusive profit à la Malueg (1992), but also under any given distribution of firms' bargaining powers.

Notably, we show that under asymmetric PCO and exogenously given bargaining powers, a higher average level of cross-holdings may destabilize tacit collusion. This holds whenever the bargaining power of the firm holding fewer shares of its rival is neither too high nor too low and the spread of firms' cross-holdings is high enough. Moreover, greater heterogeneity in cross-holdings makes tacit collusion less likely to be stable if the bargaining power of the firm that owns fewer rival's shares is relatively high. We, thus, identify conditions under which asymmetries in PCO lead to procompetitive outcomes.

[1] See Nain and Wang (2018), Heim et al. (2022), Azar et al. (2022), Barth et al (2022), Hariskos et al. (2022), Dai et al. (2022), among many others.

We also consider a scenario in which firms' bargaining powers are determined via their levels of cross-holdings. In line with Fershtman and Gandal (1994)'s market division rule, a firm owing relatively more shares of its rival has higher bargaining power, such that the ratio of firms' owners' payoffs under collusion is the same as that under Cournot competition. We show that a 'mean-increasing spread-preserving' shift in the distribution of cross-holdings stabilizes tacit collusion, but a 'mean-preserving spread-increasing' shift has no impact on the stability of collusion.

Interestingly, we find that if the exogenously given bargaining power of a firm is more than the degree of the bargaining power consistent with the Fershtman and Gandal (1994)'s market division rule, it is optimal for that firm to forfeit some of its bargaining power, and that in turn makes collusion more likely to be stable.

Our paper contributes to the strand of the literature that studies the impact of passive cross ownership on collusion stability. In a seminal paper, Reynolds and Snapp (1986) explore the pricing effects of asymmetric cross ownership, yet they do not explicitly address collusion stability. Malueg (1992) shows that under Cournot competition, symmetric cross ownership with equal sharing of collusive profits facilitates collusion whenever the demand is (weakly) concave. Under similar assumptions, Gilo et al. (2006) confirms the latter result under Bertrand competition; it also argues that asymmetric PCO facilitates collusion by aligning firms' interests. Gilo et al. (2013) examine symmetric PCO with asymmetric firms' costs, emphasizing how the latter shape collusion incentives. In a similar setting, Yang and Zheng (2021) show that an increase in the larger firm's stakes on its rival destabilizes collusion, while the smaller firm's level of stakes has ambiguous effects. De Haas and Paha (2021), considering a duopoly with asymmetric PCO in which firms equally divide monopoly output, show that collusion stability increases. In contrast, our paper challenges the assumption of equal profit sharing under collusion and identifies conditions under which asymmetric PCO destabilizes collusion.

2. The Model

Two firms, 1 and 2, produce a homogeneous good using identical constant returns to scale technologies. Let $p = \alpha - q_1 - q_2$ be the inverse demand function and $c \in [0, \alpha)$ be the marginal production cost. Firm i owns $s_i \in [0, \frac{1}{2})$ passive (i.e. non-controlling) shares of firm j, $i, j = 1, 2$, $i \neq j$. The objective function of firm i's owner is:

$$\Gamma_i(q_i, q_j) = (1 - s_j)\pi_i(q_i, q_j) + s_i\pi_j(q_j, q_i) = (\alpha - q_i - q_j - c)\big[(1 - s_j)q_i + s_i q_j\big],$$

with q_i and π_i denoting firm i's quantity and operating profits, respectively. That is, each firm's owner cares about the sum of its firm's retained operating profit - after dividend payout to its rival - and the dividend received by its rival. This objective is in line with Reynolds and Snapp (1986) and Reitman (1994). We consider an infinitely repeated game between the firms that set quantities in each stage. The pattern of cross ownership is assumed to be exogenously given. Let $\delta \in (0,1)$ be the firms' common discount factor. The equilibrium analysis under alternative single-period scenarios follows, with proofs relegated to the Appendix.

2.1 Cournot Competition

From the first order condition of firm i's owner maximization problem,

$$\max_{q_i} \Gamma_i(q_i, q_j) = \max_{q_i}(\alpha - q_i - q_j - c)\big[(1 - s_j)q_i + s_i q_j\big],$$

we get firm i's reaction function, $R_i(q_j) = \frac{1}{2}[\alpha - c - q_j(1 + \frac{s_i}{1-s_j})]$. Note that $\frac{\partial^2 \Gamma_i}{\partial q_j \partial q_i} < 0$ and $\frac{\partial R_i}{\partial s_i} = -\frac{q_j}{2(1-s_j)} < \frac{\partial R_i}{\partial s_j} = -\frac{q_j s_i}{2(1-s_j)^2} \leq 0$ for all $s_i, s_j \in \left[0, \frac{1}{2}\right)$. That is, firms' strategies are strategic substitutes; also, the ownership of rival's passive shares induces a firm to compete less aggressively in the market.

In equilibrium, quantities, firms' operating profits, owners' payoffs, consumer surplus (CS) and social welfare (SW) are, respectively,

$$q_{iN}(s_i, s_j) = \frac{(\alpha-c)(1-s_i)}{3-s_i-s_j}, \quad \pi_{iN}(s_i, s_j) = \frac{(\alpha-c)^2(1-s_i)}{(3-s_i-s_j)^2}, \quad \Gamma_{iN}(s_i, s_j) = \frac{(\alpha-c)^2(1-s_j)}{(3-s_i-s_j)^2}, \tag{1}$$

$$CS_N(s_i, s_j) = \frac{(\alpha-c)^2(2-s_i-s_j)^2}{2(3-s_i-s_j)^2}, SW_N(s_i, s_j) = \frac{(\alpha-c)^2(4-s_i-s_j)(2-s_i-s_j)}{2(3-s_i-s_j)^2},$$

where subscript N stands for the Cournot competition outcome.

Clearly, $\frac{\partial q_{iN}}{\partial s_i} < 0 < \frac{\partial q_{iN}}{\partial s_j}$, $\frac{\partial \pi_{iN}}{\partial s_i} < 0 < \frac{\partial \pi_{iN}}{\partial s_j}$, $\frac{\partial \Gamma_{iN}}{\partial s_j} < 0 < \frac{\partial \Gamma_{iN}}{\partial s_i}$, $\frac{\partial CS_N}{\partial s_i} = \frac{\partial CS_N}{\partial s_j} < 0$ and $\frac{\partial W_N}{\partial s_i} = \frac{\partial W_N}{\partial s_j} < 0$, for all $i, j = 1, 2; i \neq j$.

2.2 Collusion: Sharing the surplus via bargaining

When firms collude, the industry profit is $\Pi_C = \frac{(\alpha-c)^2}{4}$, where the subscript C stands for collusion. These profits are larger than those under Cournot competition, $\Pi_N = \pi_{1N} + \pi_{2N} =$

$\Gamma_{1N} + \Gamma_{2N} = \frac{(\alpha-c)^2(2-s_1-s_2)}{(3-s_1-s_2)^2}$, by an amount $\Delta\Pi = \Pi_C - \Pi_N = \frac{(\alpha-c)^2(1-s_1-s_2)^2}{4(3-s_1-s_2)^2} > 0, \forall\, s_1, s_2 \in \left[0, \frac{1}{2}\right)$.

Firms do better by engaging in tacit collusion than competing. The question is how firms determine their respective shares of collusive profits. To address this issue, we adopt the generalized Nash bargaining solution, which encompasses (i) Nash bargaining à la Harrington (1989) and (ii) Schmalensee (1987)' market division collusive technology à la Fershtman and Gandal (1994), as special cases.

Let y_i be firm i's operating profit under collusion: $y_1 + y_2 = \Pi_C$. Owners' payoffs are then,

$$\Gamma_i(y_i, y_j) = (1 - s_j)y_i + s_i y_j,\ i, j = 1, 2,\ i \neq j, \tag{2}$$

with $\Gamma_1 + \Gamma_2 = \Pi_C > \Pi_N$, and $\frac{\partial \Gamma_i}{\partial y_i} = 1 - s_1 - s_2 = -\frac{\partial \Gamma_j}{\partial y_i}$ (since $y_i = \Pi_C - y_j$). Let γ and $1-\gamma$, $\gamma \in [0, 1]$ be the bargaining power of firm 1 and 2, respectively. The collusive profit-sharing generalized Nash Bargaining problem is:

$$\max_{y_1, y_2} Z = (\Gamma_1(y_1, y_2) - \Gamma_{1N})^{\gamma} (\Gamma_2(y_1, y_2) - \Gamma_{2N})^{1-\gamma}$$
$$\text{s.t. } y_1 + y_2 \leq \Pi_C, y_1 \geq 0 \text{ and } y_2 \geq 0. \tag{3}$$

For expositional easiness, let for the moment set $\gamma_1 = \gamma$ and $\gamma_2 = 1 - \gamma$. From the first order conditions of the logarithm of (3) and using $\frac{\partial \Gamma_i}{\partial y_i} = 1 - s_1 - s_2 = -\frac{\partial \Gamma_j}{\partial y_i}$, it is easy to verify that $\Gamma_{iC} = \Gamma_{iN} + \gamma_i(\Pi_C - \Gamma_{1N} - \Gamma_{2N}) \geq \Gamma_{iN}$, $i = 1,2$. Also, solving the system of equations (2), we get $y_{iC} = \frac{\Gamma_{iC} - s_i(\Gamma_{iC} + \Gamma_{jC})}{1 - s_1 - s_2}$. Hence firms' operating profits and owners' payoffs are, $i, j = 1, 2,\ i \neq j$,

$$y_{iC}(s_i, s_j, \gamma_i) = \frac{(\alpha - c)^2 \left(4 + \gamma_i + s_i^2 - \gamma_i s_j - s_i(5 + \gamma_i - s_j)\right)}{4(3 - s_1 - s_2)^2} > 0$$

$$\Gamma_{iC}(s_i, s_j, \gamma_i) = \frac{(\alpha-c)^2\left(4+\gamma_i - 4s_j - \gamma_i(2-s_1-s_2)(s_1+s_2)\right)}{4(3-s_1-s_2)^2} > 0. \tag{4}$$

Finally, firms can get their operating profits without resorting to side payments, by allocating production as follows:

$$q_{iC}(s_i, s_j, \gamma_i) = \frac{(\alpha-c)\left((1-s_i)(4+\gamma_i-s_i) - (\gamma_i - s_i)s_j\right)}{2(3-s_1-s_2)^2}. \tag{5}$$

Clearly, $q_{1C}(s_1, s_2, \gamma_1) + q_{2C}(s_1, s_2, \gamma_2) = \frac{\alpha - c}{2}$; hence, consumers surplus and social welfare are: $CS_C = \frac{(\alpha-c)^2}{8}$ and $W_C = \frac{3(\alpha-c)^2}{8}$. Then $\frac{\partial q_{iC}}{\partial s_i} < 0 < \frac{\partial q_{iC}}{\partial s_j}$, $\frac{\partial y_{iC}}{\partial s_i} < 0 < \frac{\partial y_{iC}}{\partial s_j}$, $\frac{\partial \Gamma_{iC}}{\partial s_j} < 0 < \frac{\partial \Gamma_{iC}}{\partial s_i}$, $\frac{\partial CS_C}{\partial s_i} = \frac{\partial CS_C}{\partial s_j} = 0$ and $\frac{\partial W_C}{\partial s_i} = \frac{\partial W_C}{\partial s_j} = 0$, for all $i, j = 1, 2, i \neq j$. As expected, $\frac{\partial q_{iC}}{\partial \gamma_i} > 0$, $\frac{\partial y_{iC}}{\partial \gamma_i} > 0$ and $\frac{\partial \Gamma_{iC}}{\partial \gamma_i} > 0$.

Note that

(a) When $\gamma_i = \frac{1}{2}$, $\Gamma_{iC}(s_i, s_j, \gamma_i) = \Gamma_{iN} + \frac{1}{2}\Delta\Pi, i = 1,2$, which corresponds to the solution under Nash bargaining à la Harrington (1989).

(b) When $\gamma_i = \frac{1-s_j}{2-s_1-s_2}$, the above solution corresponds to that under the market division rule à la Fershtman and Gandal (1994) – see below.

2.3 Firms' Owners' payoffs under deviation

Let firm *i* deviates, while firm *j* sticks to its collusive output q_{jC}. Firm *i*'s best response is given by its reaction function, $q_{iD} = R_i(q_{jC})$, where the subscript *D* stands for deviation. Thus, firm *i*'s output and its owner's payoff are:

$$q_{iD}(s_i, s_j, \gamma_i) = \frac{(\alpha-c)\left(s_i^2(3-\gamma_i-3s_j)+(1-s_j)\left(13+\gamma_i-(6+\gamma_i)s_j+s_j^2\right)-4s_i(4-s_j)(1-s_j)\right)}{4(1-s_j)(3-s_1-s_2)^2}$$

$$\Gamma_{iD}(s_i, s_j, \gamma_i) = \frac{(\alpha-c)^2\left(s_i^2(1+\gamma_i-s_j)+(1-s_j)\left(13+\gamma_i-(6+\gamma_i)s_j+s_j^2\right)-2s_i(3+\gamma_i-s_j)(1-s_j)\right)^2}{16(1-s_j)(3-s_1-s_2)^4} > \Gamma_{iC}(s_i, s_j, \gamma_i).$$

Note that $\frac{\partial \Gamma_{iD}}{\partial s_j} < 0 < \frac{\partial \Gamma_{iD}}{\partial s_i}$, $\frac{\partial \Gamma_{iD}}{\partial \gamma_j} < 0 < \frac{\partial \Gamma_{iD}}{\partial \gamma_i}$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iC})}{\partial s_i} < 0$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iC})}{\partial s_j} < 0$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iC})}{\partial \gamma_i} < 0$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iC})}{\partial \gamma_j} > 0$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iN})}{\partial s_i} < 0$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iN})}{\partial s_j} < 0$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iN})}{\partial \gamma_i} > 0$, $\frac{\partial(\Gamma_{iD}-\Gamma_{iN})}{\partial \gamma_j} < 0$, for all $i, j = 1, 2; i \neq j$.

3. Stability of Tacit Collusion

Suppose that if in any period *t* firm *i* deviates, its rival chooses the Cournot competition output in all subsequent periods. Firm *i* has no incentive to deviate from the collusive output if

$$\delta > \frac{\Gamma_{iD}(s_i,s_j,\gamma_i)-\Gamma_{iC}(s_i,s_j,\gamma_i)}{\Gamma_{iD}(s_i,s_j,\gamma_i)-\Gamma_{iN}(s_i,s_j)} \equiv \hat{\delta}_i(s_i, s_j, \gamma_i), \text{ with}$$

$$\hat{\delta}_i(s_i, s_j, \gamma_i) = \frac{\left((1-s_j)(5-\gamma_i-s_j)-s_i(1-\gamma_i-s_j)\right)^2}{(1+\gamma_i-s_j)\left(s_i^2(1+\gamma_i-s_j)-2s_i(5+\gamma_i-s_j)(1-s_j)+(1-s_j)\left(25+\gamma_i-(10+\gamma_i)s_j+s_j^2\right)\right)}, \quad (6)$$

the firm *i*'s critical discount factor. Note that $\frac{\partial \hat{\delta}_i}{\partial s_i} > 0$, $\frac{\partial \hat{\delta}_i}{\partial s_j} < 0$, $\frac{\partial \hat{\delta}_i}{\partial \gamma_i} < 0$ and $\frac{\partial \hat{\delta}_i}{\partial \gamma_j} > 0$ for all $i, j = 1, 2, i \neq j$.

Tacit collusion is then stable if no firm has incentive to deviate, i.e., if $\delta > \hat{\delta}(s_1, s_2, \gamma) = max\ [\hat{\delta}_1(s_1, s_2, \gamma), \hat{\delta}_2(s_1, s_2, 1-\gamma)]$ holds true (Remember that $\gamma_1 = \gamma$ and $\gamma_2 = 1-\gamma$). A higher (lower) value of $\hat{\delta}(s_1, s_2, \gamma)$ implies that collusion is less (more) likely to be stable.

Without loss of generality, suppose that $0 < s_1 \leq s_2 < \frac{1}{2}$. Let $\bar{s} = \frac{s_1+s_2}{2}$ and $2\Delta = s_2 - s_1$, denote, respectively, the mean and the spread of the PCO distribution. Then $s_1 = \bar{s} - \Delta$ and $s_2 = \bar{s} + \Delta$.

Proposition 1: If passive cross ownership is symmetric, i.e., $s_1 = s_2 = \bar{s}$, a higher level of cross-holdings makes tacit collusion more likely to be stable: $\frac{\partial\, \hat{\delta}(\bar{s}, \bar{s}, \gamma)}{\partial\, \bar{s}} < 0$ for all $\gamma \in [0,1]$.

Proposition 1 confirms Malueg (1992)'s finding that when firms share equally collusive profits ($\gamma_1 = \gamma_2 = \frac{1}{2}$), an increase in symmetric cross-holdings makes collusion more likely to be stable. Importantly, Proposition 1 extends this result for any possible sharing of collusive profits between firms. Intuitively, when $s_1 = s_2$ and firm *i*'s bargaining power reduces below that of firm *j* ($\gamma_i < \frac{1}{2} < \gamma_j$), firm *i*'s critical discount factor becomes higher than that of firm *j*: $\hat{\delta}_j(\cdot) < \hat{\delta}_i(\cdot) = \hat{\delta}(\cdot)$ This is because (i) firm *i*'s deviation payoff becomes lower ($\frac{\partial \Gamma_{iD}}{\partial \gamma_i} > 0$), while its punishment payoff remains unaltered ($\frac{\partial \Gamma_{iN}}{\partial \gamma_i} = 0$), implying that $\Gamma_{iD}(.) - \Gamma_{iN}(.)$ decreases, and (ii) firm *i*'s deviation payoff in excess of its collusive payoff $\Gamma_{iD}(.) - \Gamma_{iC}(.)$ increases ($\frac{\partial (\Gamma_{iD} - \Gamma_{iC})}{\partial \gamma_i} > 0$). Now, due to a symmetric increase in firms' cross-holdings, the reduction in firm *i*'s gain from deviation, $\Gamma_{iD}(.) - \Gamma_{iC}(.)$, is larger than the reduction in its deviation payoff in excess of that under punishment, $\Gamma_{iD}(.) - \Gamma_{iN}(.)$. Thus, firm *i*'s critical discount factor $\hat{\delta}_i(\cdot)$ decreases and firm *i* is less likely to

deviate whenever it owns a larger share of its rival.[2] Therefore, a higher level of cross-holdings makes collusion more likely to be stable under symmetric PCO.

Let $\gamma_l(\bar{s},\Delta) \equiv \frac{1}{2} - \frac{\Delta}{2(1-\bar{s})}$ and $\gamma_u(\bar{s},\Delta) \equiv \frac{1}{2} + \Delta - \frac{(3-2\bar{s})^2(1-2\bar{s})}{2(5-4\bar{s}^2+8\Delta)}$. It can be checked that for all $s_1, s_2 \in \left[0,\frac{1}{2}\right)$, $0 < \gamma_l(\bar{s},\Delta) < \frac{1}{2}$ and $\gamma_u(\bar{s},\Delta) < 1$. Moreover, $\gamma_l(\bar{s},\Delta) < \gamma_u(\bar{s},\Delta)$ only if $\Delta \in \left(\underline{\Delta}(\bar{s}),\ \frac{1}{4}\right)$, where $\underline{\Delta}(\bar{s}) = \frac{3-4\bar{s}(2-\bar{s})}{8}$, i.e., only if the spread Δ of the PCO distribution is high enough (for given average level $\bar{s}$).[3] Note that since $\frac{d\underline{\Delta}}{d\bar{s}} < 0$ for all $\bar{s} \in \left(0,\frac{1}{2}\right), \Delta > \underline{\Delta}(\bar{s})$ is more likely to hold when $\bar{s}$ is larger.

Lemma 1: (i) $\hat{\delta}_1(\bar{s}-\Delta,\bar{s}+\Delta,\gamma) \gtreqless \hat{\delta}_2(\bar{s}-\Delta,\bar{s}+\Delta,1-\gamma)$ if and only if $\gamma \gtreqless \gamma_l(\bar{s},\Delta)$.

(ii) $\frac{\partial\hat{\delta}_1}{\partial\bar{s}} < 0$ if $\hat{\delta}_1(.) > \hat{\delta}_2(.)$; also, $\frac{\partial\hat{\delta}_2}{\partial\bar{s}} > 0$ if $\hat{\delta}_2(.) > \hat{\delta}_1(.)$ and $\gamma_l(\bar{s},\Delta) < \gamma < \gamma_u(\bar{s},\Delta)$, otherwise, $\frac{\partial\hat{\delta}_2}{\partial\bar{s}} < 0$.

(iii) $\frac{\partial\hat{\delta}_1}{\partial\Delta} < 0$ and $\frac{\partial\hat{\delta}_2}{\partial\Delta} > 0$ for all $\gamma \in [0,1]$.

Lemma 1(i) confirms that the firm with relatively low bargaining power has a higher critical discount factor under asymmetric PCO too. Yet, the turning point is now lower, i.e., $\gamma_l(.) < \frac{1}{2}$. This is because for given s_2 (s_1), a lower s_1 (higher s_2) leads to smaller $\hat{\delta}_1$ ($\frac{\partial\hat{\delta}_1}{\partial s_1} > 0, \frac{\partial\hat{\delta}_1}{\partial s_2} < 0$). An immediate consequence of the latter is that $\frac{\partial\hat{\delta}_1}{\partial\Delta} < 0$; moreover, since $\frac{\partial\hat{\delta}_2}{\partial s_1} < 0$ and $\frac{\partial\hat{\delta}_2}{\partial s_2} > 0$, we get $\frac{\partial\hat{\delta}_2}{\partial\Delta} > 0$ (Lemma 1(iii)). Finally, Lemma 1(ii) states that whenever firm i's bargaining power is low enough, given the asymmetry in PCO Δ, an increase in the average level of cross-holdings $\bar{s}$ – i.e., an equal increase in s_1 and s_2 - makes collusion more likely to be stable. Nevertheless, in case that the bargaining power of the firm that has relatively lower shares in its rival lies in an *intermediate* region, $\gamma_l(.) < \gamma < \gamma_u(.)$, an increase in $\bar{s}$ leads to a higher critical discount factor for its rival ($\frac{\partial\hat{\delta}_2}{\partial\bar{s}} > 0$). Since $\hat{\delta}_2(.) > \hat{\delta}_1(.)$ in this region, an increase in $\bar{s}$, makes collusion less

[2] This argument is along the lines of Malueg (1992), but it applies to the firm with the lower bargaining power that has the higher critical discount factor.

[3] Note that in terms of (s_1, s_2), the inequality $\gamma_l(.) < \gamma_u(.)$ is satisfied as long as $s_2 > 4 - s_1 - \sqrt{13-8s_1} \equiv s_{2c}(s_1)$, with $\frac{ds_{2c}}{ds_1} > 0$, $s_{2c}(0) = 0.394$ and $s_{2c}\left(\frac{1}{2}\right) = \frac{1}{2}$.

likely to be stable. Note, however, that such a region exists only if, given $\bar{s}$, the spread of the distribution of PCO is high enough.

From Lemma 1, the following Proposition is immediate.

Proposition 2: If passive cross ownership is asymmetric, $0 < s_1 = \bar{s} - \Delta < s_2 = \bar{s} + \Delta < \frac{1}{2}$, then

(i) As the average cross-holdings $\bar{s}$ increase, tacit collusion becomes less likely to be stable when asymmetry in cross ownership is high enough, $\Delta > \underline{\Delta}(\bar{s})$, and γ is neither too high nor too low, $\gamma_l(\bar{s}, \Delta) < \gamma < \gamma_u(\bar{s}, \Delta)$; otherwise, tacit collusion is more likely to be stable.

(ii) As cross ownership becomes more asymmetric (higher Δ), tacit collusion becomes less likely to be stable if and only if the bargaining power of the firm that owns less shares of its rival is high enough, $\gamma > \gamma_l(\bar{s}, \Delta)$.

Example: Let $(s_1, s_2) = (0.1, 0.42)$. Then $(\bar{s}, \Delta) = (0.26, 0.16)$, $\underline{\Delta}(\bar{s}) \approx 0.149$, and $(\gamma_l, \gamma_u) \approx (0.392, 0.414)$. Suppose that $\gamma = 0.4$. Then $\hat{\delta}_1 \approx 0.50362 < \hat{\delta}_2 \approx 0.51342$. Now, consider two alternative scenarios: (1) A spread-preserving mean-increasing change in PCO to $(s_1', s_2') = (0.11, 0.43)$ that leads to $(\bar{s}', \Delta) = (0.27, 0.16)$, $\underline{\Delta}(\bar{s}') \approx 0.141$, $(\gamma_l', \gamma_u') \approx (0.390, 0.423)$, $\hat{\delta}_1' \approx 0.50190$ and $\hat{\delta}_2' \approx 0.51346 > \hat{\delta}_2$, making collusion less likely to be stable. (2) A mean-preserving spread-increasing change in PCO to $(s_1'', s_2'') = (0.09, 0.43)$ that leads to $(\bar{s}, \Delta'') = (0.26, 0.17)$, $\underline{\Delta}(\bar{s}) \approx 0.149$, $(\gamma_l', \gamma_u') \approx (0.385, 0.428)$, $\hat{\delta}_1' \approx 0.49857$ and $\hat{\delta}_2' \approx 0.51660 > \hat{\delta}_2$, making again collusion less likely to be stable.

From Proposition 2(ii) it follows that when firms have equal bargaining powers ($\gamma = \frac{1}{2}$) over collusive profit sharing (Harrington, 1989), a more asymmetric PCO makes tacit collusion less likely to be stable. Importantly, there are parameter constellations under which an increase in the average level of PCO destabilizes collusion.[4]

[4] Clearly, $\gamma = \frac{1}{2} > \gamma_l(\bar{s}, \Delta)$. Now for $\gamma = \frac{1}{2} < \gamma_u(\bar{s}, \Delta)$, we need $\Delta > \frac{(3-2\bar{s})^2(1-2\bar{s})}{2(5-4\bar{s}^2+8\Delta)}$, which is true when Δ is large enough for given $\bar{s}$.

3.1 "Endogenous" Bargaining Power

Let firms' bargaining powers be determined according to their levels of cross-holdings. In line with Fershtman and Gandal (1994)'s market division rule, a firm owing relatively more shares of its rival has higher bargaining power, such that the ratio of firms' owners' payoffs under collusion is the same as that under Cournot competition.

From (1), we get $\frac{\Gamma_{1N}}{\Gamma_{2N}} = \frac{1-s_2}{1-s_1}$. For the latter to be equal to $\frac{\Gamma_{1C}}{\Gamma_{2C}}$, it should be that $\gamma_i = \frac{1-s_j}{2-s_1-s_2}$. This is because the solution to (3) implies that $\Gamma_{iC} = \Gamma_{iN} + \gamma_i(\Pi_C - \Gamma_{1N} - \Gamma_{2N})$. Therefore, $\frac{\Gamma_{1C}-\Gamma_{1N}}{\Gamma_{2C}-\Gamma_{2N}} = \frac{\gamma_1}{\gamma_2}$. And since $\gamma_1 + \gamma_2 = 1$, for $\frac{\Gamma_{1C}}{\Gamma_{2C}} = \frac{\Gamma_{1N}}{\Gamma_{2N}} = \frac{\Gamma_{1C}-\Gamma_{1N}}{\Gamma_{2C}-\Gamma_{2N}} = \frac{1-s_2}{1-s_1}$, it must be that $\gamma = \gamma_1 = \frac{1-s_2}{2-s_1-s_2}$ and $1-\gamma = \gamma_2 = \frac{1-s_1}{2-s_1-s_2}$.

Note that $\gamma_l\left(\frac{s_1+s_2}{2}, \frac{s_2-s_1}{2}\right) = \frac{1-s_2}{2-s_1-s_2} = \gamma_1 = \gamma$. From Lemma 1(i) we know that in this case, $\hat{\delta}_1(.) = \hat{\delta}_2(.) \equiv \hat{\delta}(s_1, s_2) = \frac{(3-s_1-s_2)^2}{17+(s_1+s_2)^2-10(s_1+s_2)}$ (by (6)), with $\frac{\partial\hat{\delta}}{\partial s_i} = -\frac{4(3-s_1-s_2)(1-s_1-s_2)}{(17+(s_1+s_2)^2-10(s_1+s_2))^2} < 0$. Hence, an increase in s_i makes collusion more likely to be stable.

Now if $s_1 < s_2$, an increase in s_1 results (i) in an increase in the average level of cross-holdings and (ii) in lower asymmetry in the cross ownership pattern. In particular, $\hat{\delta}(\bar{s}, \Delta) = \frac{(3-2\,\bar{s})^2}{17-4(5-\bar{s})\,\bar{s}}$, with $\frac{\partial\,\hat{\delta}}{\partial\bar{s}} < 0$ and $\frac{\partial\,\hat{\delta}}{\partial\,\Delta} = 0$. Proposition 3 summarizes.

Proposition 3: (i) A mean-increasing spread-preserving shift in the distribution of cross ownership makes tacit collusion more likely to be stable.

(ii) The stability of tacit collusion remains unchanged due to mean-preserving spread-increasing shift in the distribution of cross ownership.

3.3 Enhancing Stability of Collusion by Forfeiting Bargaining Power

Let firm i can choose to exercise bargaining power $\hat{\gamma}_i \in [0, \gamma_i]$, where γ_i is its actual power, i.e., firm i can forfeit part of its bargaining power at zero cost.[5] From above, we know that $\frac{\partial \hat{\delta}_1}{\partial \gamma} < 0$ and $\frac{\partial \hat{\delta}_2}{\partial \gamma} > 0$. Also, from Lemma 1(i), we have $\hat{\delta}_1(.) \gtreqless \hat{\delta}_2(.)$ if and only if $\gamma \lesseqgtr \gamma_l(\bar{s}, \Delta) = \frac{1-s_2}{2-s_1-s_2}$. An immediate consequence is that collusion is the most likely to be stable when firm 1's bargaining power is $\gamma = \gamma_1 = \frac{1-s_2}{2-s_1-s_2}$, i.e., when the distribution of bargaining power is determined via the market division rule à la Fershtman and Gandal (1994).

Suppose firm 1's bargaining power γ is larger than $\frac{1-s_2}{2-s_1-s_2}$ such that its discount factor $\delta < \hat{\delta}_2(\bar{s} - \Delta, \bar{s} + \Delta, 1 - \gamma)$, rendering thus collusion non-sustainable. Then firm 1 has incentive to forfeit the least possible part of its bargaining power such that collusion can be sustained. This is because firm 1's owners' collusive payoffs are increasing with its bargaining power. Thus, by forfeiting $\gamma - \hat{\gamma}_1 > 0$ such that $\delta = \hat{\delta}_2(\bar{s} - \Delta, \bar{s} + \Delta, 1 - \hat{\gamma}_1)$, firm 1 makes tacit collusion sustainable with the least possible losses over the collusion surplus. A similar argument applies to firm 2 when its bargaining power $1 - \gamma$ is larger than $\frac{1-s_1}{2-s_1-s_2}$.

4. Conclusions

We challenge the maintained in the existing literature assumption that collusive profits are equally divided even if firms own asymmetric shares in their rivals and identify conditions under which an increase in the average level of rivals' shares destabilizes collusion.

Our findings have important policy implications for antitrust authorities. To judge whether asymmetric PCOs lead to anti-competitive outcomes or not, they should first investigate how the involved firms divide the ensuing from collusion profits. Our analysis suggests that negotiations over the surplus division under collusion - as reflected by the firms' bargaining power distribution - may lead to pro-competitive (instead of anti-competitive) outcomes under certain market conditions. Future research should delve into the investigation of those market conditions.

[5] In a different context, Marx and Shaffer (2010) identify conditions under which an agent has incentives to forfeit part of his bargaining power. Forfeiting bargaining power can be materialized by delegating negotiations to a weaker in the negotiation table agent.

Appendix

Proof of Proposition 1: If $s_1 = s_2 = \bar{s}$, then $\Delta = 0$, $\hat{\delta}_1(\bar{s}, 0) = \frac{[(5-2\bar{s})(1-\bar{s})-\gamma(1-2\bar{s})]^2}{(1+\gamma-\bar{s})[(5-2\bar{s})^2(1-\bar{s})+(1-2\bar{s})^2\gamma]}$, and $\hat{\delta}_2(\bar{s}, 0) = \frac{[(5-2\bar{s})(1-\bar{s})-(1-\gamma)(1-2\bar{s})]^2}{(2-\gamma-\bar{s})[(5-2\bar{s})^2(1-\bar{s})+(1-2\bar{s})^2(1-\gamma)]}$. It follows that, $\hat{\delta}_1(\bar{s}, 0) \gtreqless \hat{\delta}_2(\bar{s}, 0)$ as long as $\gamma \lesseqgtr \frac{1}{2}$, $\gamma \in [0, 1]$. Further, it can be checked that if $0 \leq \gamma \leq \frac{1}{2}$, $\frac{\partial \hat{\delta}_1(.)}{\partial \bar{s}} < 0$; and if $\frac{1}{2} \leq \gamma \leq 1$, $\frac{\partial \hat{\delta}_2(.)}{\partial \bar{s}} < 0$. Therefore, $\frac{\partial \hat{\delta}(\cdot)}{\partial \bar{s}} < 0$ for all $\gamma \in [0, 1]$.

Proof of Lemma 1: Lemma 1(i) follows from straight comparison of the expressions for $\hat{\delta}_1(\bar{s}, \Delta, \gamma)$ and $\hat{\delta}_2(\bar{s}, \Delta, 1-\gamma)$. Differentiating then $\hat{\delta}_1(\cdot)$ and $\hat{\delta}_2(\cdot)$ with respect to $\bar{s}$ and Δ, we obtain Lemma 1(ii) and 1(iii).

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