Mobilization with polarization and campaign spending

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Abstract

This paper develops a formal model of electoral competition in which parties first choose their platforms and then allocate campaign resources that serve both persuasive and mobilization purposes. Voters, in turn, endogenously sort into ideological and impressionable types. We characterize a unique subgame perfect equilibrium and derive comparative statics that illustrate how the returns to mobilization and persuasion shape equilibrium platforms, campaign spending, and turnout. Among other results, we show that while campaign spending and polarization do not necessarily move in the same direction, turnout consistently increases with polarization.

Keywords: *electoral competition, campaign spending, polarization, mobilization*
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1 Introduction

In recent years, political campaigns have become increasingly sophisticated in targeting and persuading voters through advertising, social media, and grassroots mobilization efforts. Parties invest heavily in these activities not only to reinforce support among their base but also to sway and mobilize undecided voters. Despite the central role of campaign spending in modern democracies, the interaction between a party's ideological positioning and its strategic investments in persuasion and mobilization remains insufficiently understood. Theoretical models have often examined platform choice, campaign spending, and turnout in isolation, overlooking the feedback effects among these three key elements of electoral competition.

Our model features two parties competing in a two-stage game. In the first stage, parties simultaneously choose their policy platforms; in the second stage, after observing platforms, they simultaneously choose their campaign spending levels. The electorate comprises two distinct types of voters: ideological voters, who vote for the party whose platform is closest to their ideal point, and impressionable voters, who are influenced by campaign spending and may abstain if not sufficiently persuaded. The parties' campaign spending affects impressionable voters through two channels: it increases their likelihood of voting (mobilization) and, conditional on turnout, shifts their preferences toward the one party or the other (persuasion).

A key assumption in the model is that the division of voters into ideological and impressionable types is endogenous and depends on the level of polarization. Specifically, voters are ideologically committed or impressionable depending on the policy distance between the parties' platforms. When polarization is low and parties' platforms are close, a larger share of voters is impressionable, open to persuasion and mobilization through campaign spending. However, as polarization increases and the policy distance widens, more voters become ideologically committed to their preferred side, reducing the size of the impressionable electorate. This mechanism reflects the idea that greater ideological distance leads to stronger partisan identification, which in turn decreases voter responsiveness to campaign efforts. Thus, the model endogenizes voter type composition as a function of polarization, influencing the strategic choices of parties regarding policy and campaign spending.

The model delivers several important results concerning the interaction between polarization, campaign spending, and turnout. In the second stage, we establish the existence and uniqueness of a pure strategy Nash equilibrium in campaign spending, characterized by symmetric campaign spending from both parties. Spending increases with the persuasive power of campaigns and the returns to mobilization but decreases as electoral polarization intensifies, since higher polarization reduces the pool of impressionable voters.

In the first stage, parties strategically choose their platforms, balancing the incentive to appeal to the median voter with the cost implications of expanding the impressionable voter base. Our analysis demonstrates the existence of a unique symmetric interior equilibrium with diverging platforms. Although moderation tends to increase the share of impressionable voters—thereby elevating campaign costs—parties temper their platform choices to contain these expenditures, resulting in an equilibrium level of polarization.

The comparative statics reveal that campaign spending is concave in the degree of polarization, following an inverted U-shaped pattern. The platform position that maximizes spending is independent of campaign persuasiveness, mobilization, and uncertainty parameters, but increases with the baseline share of ideological voters in less polarized electorates. Moreover, as the persuasive effectiveness of campaigns rises, parties adopt more extreme platforms, which increases polarization and, depending on the equilibrium platform location, can lead to higher overall campaign spending. Thus, increased persuasive capacity intensifies costly electoral competition and drives parties toward polarization as a cost-containment strategy.

The study most closely related to ours is Herrera, Levine, and Martinelli (2008), where platforms determine voting preferences and campaign spending influences turnout. We build on this framework by introducing two interrelated channels through which turnout is determined. Specifically, in our model, the choice of policy platforms not only affects voting preferences but also directly shapes total turnout by influencing the share of ideological voters who are intrinsically motivated to participate. As in Herrera et al. (2008), campaign spending affects both turnout and voters' support. In our setting, however, this effect operates specifically through a group of impressionable voters, whose behavior is shaped by parties' mobilization and persuasion strategies.

Other recent papers, such as Grossman and Helpman (1996); Carrillo and Castanheira (2008), study the relationship between campaign spending and polarization but do so without modeling endogenous turnout. While empirical evidence suggests that campaign spending and polarization increase simultaneously, only Hirsch (2023) and Balart et al. (2022) provide theoretical accounts consistent with these empirical patterns. Balart et al. (2022) shows that greater polarization increases incentives for campaign spending even for purely office-motivated candidates, whereas Hirsch (2023) demonstrates that increased spending allows policy-motivated parties to move closer to their preferred, more extreme positions. However, neither framework allows for the possibility that campaigning itself shapes turnout. Our model addresses this gap by showing that simultaneous increases in polarization and campaign spending can arise in equilibrium when turnout is endogenous.

Our approach also connects to the literature on endogenous valence (Ashworth and Bueno de Mesquita, 2009; Zakharov, 2009). The main difference in our model is that campaign spending is non-additive (as in Balart et al. (2022)), affecting only impressionable voters and operating through two separate channels: mobilization and persuasion.

Finally, this paper contributes to the longstanding literature on voter turnout (Dhillon and Peralta, 2002; Herrera, Morelli, and Nunnari, 2016). Unlike the classical approach (Riker and Ordeshook, 1968) which focuses on individual incentives, models regarding the swing voter's curse (Feddersen and Pesendorfer, 1996; Herrera, Llorente-Saguer, and McMurray, 2019) or the group-based approach (Feddersen and Sandroni, 2006; Levine and Mattozzi, 2020) we abstract from the paradox of voting and instead concentrate on the aggregate effects of polarization and campaign spending on turnout. This perspective enables us to analyze how shifts in party competition dynamics can systematically alter overall participation rates.

The remainder of the paper is organized as follows: Section 1 presents the model, Section 2 characterizes equilibrium behavior, and Section 3 analyzes the comparative statics and policy implications. Section 4 concludes.

2 The model

There are two office-motivated parties D and R running for an election. Each party is characterized by two strategic decisions: In the first stage it chooses its policy platform and in the second stage its campaign spending. Policy platforms define parties' positions in the policy space represented by a point in the $[0,1]$ continuum, with d and $1-r$ denoting the platform of party D and R, respectively. Campaign spending is represented with $e_i \in \mathbb{R}_+$, $i \in \{D, R\}$. Parties' payoff for winning the election is normalized to 1; hence, campaign spending will always satisfy $e_i \leq 1 \ \forall i$.

Voters can be either ideological or impressionable (Baron, 1994; Grossman and Helpman, 1996). Ideological voters base their decisions on policy platforms, whereas impressionable voters are influenced by electoral spending. The division between these two voter types is endogenous and depends on the degree of political polarization, defined as the distance between the parties' platforms: $y = 1 - r - d$ (Balart et al., 2022). We assume that the share of ideological voters is given by $F(y) = z + (1-z)y$, where $0 < z < 1$ represents a mass of ideological voters who, even in the absence of polarization, vote based on policy considerations.

Ideological voters follow the framework of Herrera et al. (2008). Each has an ideal policy point v uniformly distributed over the policy space $[0,1]$. An ideological voter with ideal point v votes for party D (or R) whenever[1]

$$b_v + b - |v - d| \geq (\leq) \; - |v - (1 - r)|,$$

where b is an aggregate (party-level) shock and b_v is an individual (voter-level) shock. Both shocks are realized after parties have chosen their strategies. They are independently and uniformly distributed, with $b \sim \mathcal{U}[-\alpha, \alpha]$ and $b_v \sim \mathcal{U}[-\beta, \beta]$, where $\alpha \geq 1$ and $\beta \geq \alpha + 1$. These lower bounds on α and β ensure that, for any given pair of platforms $(d, 1 - r)$, neither the voting decision of any individual ideological voter nor the party with the majority of ideological votes can be predicted with certainty. Ideological voters never abstain and we let S_i denote the expected share of ideological voters who vote for party i.

The behavior of impressionable voters is influenced by electoral spending, which affects them through two distinct channels: mobilization and persuasion. First, electoral spending mobilizes impressionable voters regardless of its partisan source. That is, spending by both parties are perfect substitutes in generating turnout. We assume that the share of mobilized impressionable voters is given by $M(e_D, e_R) = (e_D^m + e_R^m)/2$ where $0 < m < 1$, captures the *returns to mobilization from spending.* The parameter m can reflect institutional, cultural, or technological factors that determine how easily voters are mobilized. Given that in our model campaign spending is lower than one, lower values of m imply higher returns to spending—relevant in settings where voters are more responsive to outreach, such as through organized campaigns or social media engagement. Conversely, higher values of m indicate lower returns, consistent with environments where voters are disengaged or turnout is structurally low.

Second, electoral spending also has a persuasive effect: parties can increase their support among impressionable voters by spending more. Following Meirowitz (2008), we model electoral competition for impressionable voters as a contest. In our case, the vote share of mobilized impressionable voters takes the Tullock form (Tullock, 1980):[2]

$$T_i(e_D, e_R) = \begin{cases} e_i^t/(e_D^t + e_R^t) & if \; e_D + e_R > 0 \\ 1/2 & if \; e_D + e_R = 0 \end{cases}$$

where $0 < t \leq 1$. The parameter t captures the effectiveness of spending in persuading impressionable voters—henceforth referred to as the *persuasiveness of spending.* A higher t reflects environments where even marginal increases in spending translate into a signifi-

[1] We assume ideological voters randomize with equal probability if indifferent between the two parties.

[2] The function is commonly used in the literature (see among others, Epstein and Nitzan, 2004; Crutzen et al., 2020; Balart et al., 2022), as it offers tractability and satisfies several axiomatic criteria Skaperdas (1996).

cant persuasive advantage (e.g., highly targeted digital advertising or a more charismatic candidate), whereas lower values of t reflect situations where persuasion is harder or more diffuse.

Parties are office-motivated: they seek to maximize their probability of winning the election net of electoral spending. Given the above notation, the total share of voters casting their ballot for party i is

$$v_i(d, r, e_D, e_R) = F(y)S_i(d, r) + (1 - F(y))M(e_D, e_R)T_i(e_D, e_R), \tag{1}$$

where the first term captures support from ideological voters and the second term reflects support from impressionable voters. Hence, the corresponding payoff for party i is

$$\pi_i(d, r, e_D, e_R) = \Pr\left(v_i(d, r, e_D, e_R) > v_{-i}(d, r, e_D, e_R)\right) - e_i, \tag{2}$$

where $-i$ denotes the opposing party.

The timing of the game is as follows. In the first stage, the two parties simultaneously choose the policy platforms that maximize their expected payoffs. In the second stage, after observing the platform choices and the resulting share of impressionable voters—determined by the level of polarization—the parties choose their levels of electoral spending. Consequently, the shocks are realized, and voters cast their votes. Given the sequential nature of the game, we focus on subgame perfect Nash equilibria (SPNE). Since the game is symmetric, if $(d, r, e_D^*(d, r), e_R^*(d, r))$ is an equilibrium of the game, then $(r, d, e_D^*(r, d), e_R^*(r, d))$ is also an equilibrium. Without loss of generality, we focus on the case $d \leq 1 - r$, or equivalently, $d + r \leq 1$.

The model captures key trade-offs in the strategic selection of policy platforms and electoral spending. First, by choosing a more centrist platform, parties increase their appeal among ideological voters but also expand the share of impressionable voters, for whom competition is costly. Second, greater electoral spending not only persuades more impressionable voters but also mobilizes them, amplifying the role of spending in determining electoral outcomes.

The framework above combines two key insights from recent studies that address the relationship between political polarization and electoral spending. The indirect effect of platform choice on spending is shared with Balart et al. (2022), but is absent in Herrera et al. (2008). In contrast, the mobilization effect resembles the targeting and mobilization mechanisms discussed in Herrera et al. (2008), but is not present in Balart et al. (2022). The combination of these two effects in the current model comes at the cost of focusing on a specific class of distribution $F(y)$.

3 Equilibrium

Following Herrera et al. (2008, Theorem 1), the share of ideological votes for each party in this model is given by:

$$
\begin{aligned}
S_D(d,r) &= \frac{1}{2} + \frac{b + d - d^2 - r + r^2}{2\beta}, \\
S_R(d,r) &= \frac{1}{2} - \frac{b + d - d^2 - r + r^2}{2\beta}.
\end{aligned}
$$

By replacing the expressions above in parties' vote shares (Eq. 1) and payoffs (Eq. 2) and isolating the party level shock b, we can write:

$$
\begin{aligned}
\pi_D(d,r,e_D,e_R) &= \Pr\left(b \leq \Delta(d,r,e_D,e_R)\right) - e_D, \\
\pi_R(d,r,e_D,e_R) &= \Pr\left(b > \Delta(d,r,e_D,e_R)\right) - e_R,
\end{aligned}
$$

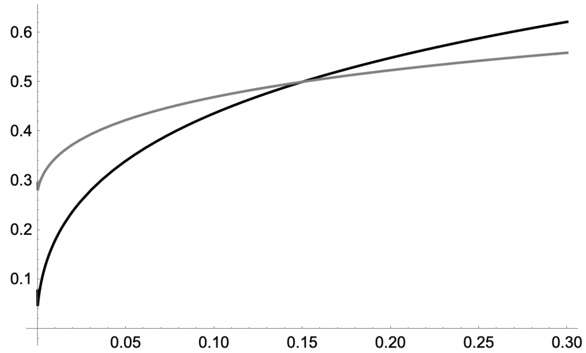
where $\Delta(d,r,e_D,e_R) = d - d^2 - r + r^2 + \beta H(y) M(e_D,e_R)\left(T_D(e_D,e_R) - T_R(e_D,e_R)\right)$ and $H(y) = (1 - F(y))/F(y)$.

As in Herrera et al. (2008), the first term of $\Delta(d,r,e_D,e_R)$ (i.e., $d - d^2 - r + r^2$) captures the effect of policy platforms on the share of ideological votes. The remaining term reflects the impact of campaign spending in our model, where spending has both mobilizing and persuading effects (through $M(e_D,e_R)$ and $T_i(e_D,e_R)$ respectively). Moreover, the influence of spending depends on the level of polarization, as it determines the composition of the electorate into ideological and impressionable voters (through $H(y)$). Given the above preliminary results we proceed by solving our game backwards.

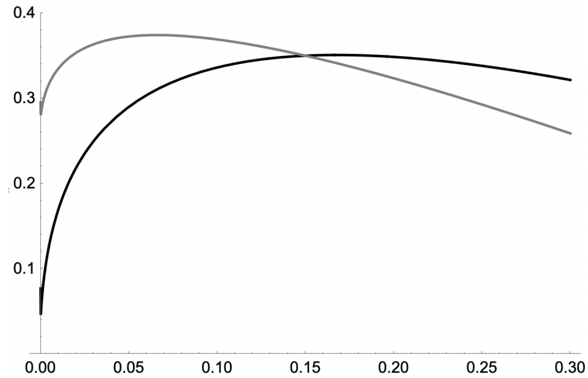
3.1 Second Stage: Campaign Spending

The platforms d and r chosen in the first stage determine not only the vote share for each party among ideological voters but also the share of impressionable voters, whose votes depend on the parties' costly campaign spending. Campaign spending operates in two ways: it mobilizes and persuades impressionable voters. This double role of spending makes the second stage interesting and different than standard contest models, as i) the size of the prize is endogenous through mobilization, and ii) the probability of a party winning the election is not necessarily increasing in its own spending.

Figure 1a illustrates how at low levels of campaign spending, an increase in a party's spending can actually reduce its probability of winning. For example, a party may be better off not campaigning at all than campaigning only marginally, as the voters it could mobilize might not be sufficiently persuaded to vote for it, thereby increasing the



(a) Probability of winning.



(b) Payoffs.

Figure 1: Probability of winning and party payoffs as functions of own spending. The black curve corresponds to $d = r = 0.25$, and the gray curve to $d = r = 0.15$. The opposing party's spending is fixed at 0.15. Parameters: $a = 1$, $b = 4$, $m = 0.2$, $t = 0.4$, $z = 0.25$.

opponent's probability of winning. An implication of the above non-monotonicity is that the party's payoff function is also non-monotonic. Figure 1b illustrates this feature, which clearly makes the problem non-standard. Figure 2, in turn, depicts the parties' first-order conditions, which hold twice: once at a local minimum and once at a local maximum. In our first result, we characterize the intersection of the first-order conditions corresponding to the maximum.

Proposition 1. *Let* $t < 1/2$, $m \leq \min\{t, 1-2t\}$ *and* $z \geq \frac{\beta t}{2^{m+1}\alpha^m(\alpha-1/4)^{1-m}+\beta t}$, *then*

$$e^*(d,r) = e^*_D(d,r) = e^*_R(d,r) = \left(\frac{\beta t H(1-d-r)}{4\alpha}\right)^{\frac{1}{1-m}}$$

is the unique pure strategy Nash equilibrium of the campaign spending stage of the game.

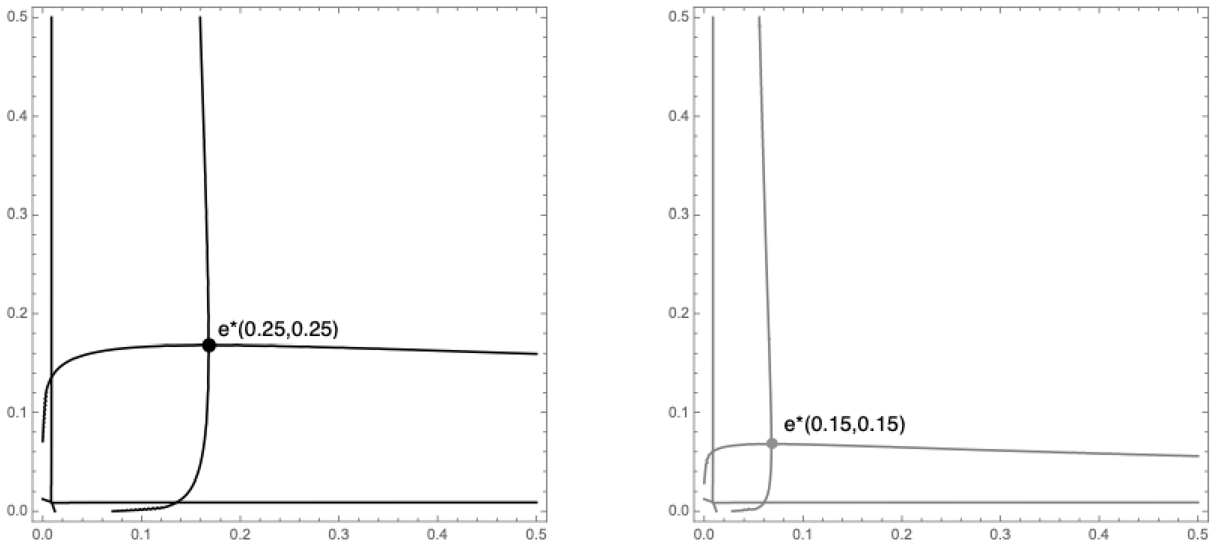
All proofs are in the Appendix.

Proposition 1 characterizes the unique pure strategy Nash equilibrium of the campaign stage. Spending is increasing in the persuasiveness parameter t. In the limit as $t \to 0$, impressionable voters are split equally between the parties, independently of the parties' spending, and thus there is no incentive to spend in campaign.[3] As persuasiveness increases, equilibrium campaign spending also increases, since a slightly greater effort than the opponent's can result in a significant gain in votes. As is standard in such models, an upper bound on the parameter t ensures the existence of a pure strategy equilibrium. In our setting, this condition is not limited to the standard requirement that vote shares

[3]Note that the equal division of impressionable voters renders the mobilization role of spending irrelevant as $t \to 0$.

be less than proportional to relative spending ($t \leq 1$); it is more stringent ($t \leq 1/2$) due to the non-concave nature of the objective function (see Figure 1).

Campaign spending is also increasing in the returns to mobilization. That is, as m decreases, the returns to mobilization increase, giving parties stronger incentives to engage in campaign spending and voter mobilization. The upper bound on the parameter m ensures that the payoffs reach a maximum in the proposed equilibrium spending given that the objective function is not concave.



(a) $d = r = 0.25 \implies y = 0.5$ **(b)** $d = r = 0.15 \implies y = 0.7$

Figure 2: First order conditions and equilibrium spending for different levels of polarization. Parameters: $a = 1$, $b = 4$, $m = 0.2$, $t = 0.4$, $z = 0.25$.

Last but not least, campaign spending decreases with polarization, defined as $y = 1 - d - r$. As polarization decreases, more voters are impressionable, and therefore parties have stronger incentives to engage in costly campaigning compared to situations where a larger share of voters is ideological. Indeed, the characterization of a pure strategy equilibrium requires imposing a lower bound on the parameter z, which restricts the amount of impressionable voters under minimal polarization. This requirement arises from the participation constraint in the spending stage. If the amount of impressionable voters is too large, competition for attracting their votes becomes excessively intense and expensive, precluding the existence of a pure strategy equilibrium.

The polarization effect on spending critically affects parties' incentives to optimally choose their platforms in the first stage, which we analyze next.[4]

[4]The two parameters governing the uncertainty of ideological voting also influence equilibrium spending. While α (aggregate uncertainty) reduces equilibrium spending, β (idiosyncratic uncertainty)increases it. This difference arises from the distinct nature of the two shocks. The aggregate shock b imbalances the competition between the two parties, thereby reducing the intensity of the spending stage. In contrast,

3.2 First stage: Platform Selection

Now we turn to the study of platform selection. Given the symmetric equilibrium in the second stage, parties equally split the impressionable voters. Hence, the probability of winning depends solely on platform selection. Specifically, we have:

$$\Delta(d, r, e_D^*, e_R^*) = d - d^2 - r + r^2,$$

and thus the probability of party D winning is:

$$\Pr\left(b < d - d^2 - r + r^2\right).$$

The above implies that a party's probability of winning increases as it moderates its platform, providing incentives to adopt a central platform. However, in our model, convergence to the center exacerbates electoral spending. As a result, parties optimally choose their platforms to balance this tradeoff as characterized in the following proposition:[5]

Proposition 2. *There is a unique subgame perfect Nash equilibrium where each party chooses platform* $0 < p^* < 1/2$*, where* p^* *is the solution to*

$$\frac{1}{2\alpha}(1 - 2p^*) = \left.\frac{\partial e^*(l, p^*)}{\partial l}\right|_{l=p^*} = \left.\frac{\partial e^*(p^*, r)}{\partial r}\right|_{r=p^*} \tag{3}$$

In equilibrium, parties propose symmetric interior platforms around the median. These platforms satisfy the parties' first-order conditions, as summarized in Equation (9). The left-hand side represents the benefit of moderation, which increases a party's probability of winning. The right-hand side captures the cost of moderation, stemming from higher campaigning in the second stage. As in Balart et al. (2022) and the broader literature on endogenous valence with office motives (Ashworth and Bueno de Mesquita, 2009; Zakharov, 2009), polarization arises as a means to reduce electoral spending.

3.3 Comparative Statics

Having characterized the equilibrium of the game, we can focus on its theoretical predictions. We start by analyzing how equilibrium spending changes depending on the equilibrium platforms.

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the independent idiosyncratic shock b_v adds noise —but no party bias— to the competition for ideological voters, thereby lowering its strategic relevance and shifting greater emphasis toward the contest for the impressionable electorate.

[5] A similar advantage of the central platform was also present in Herrera et al. (2008); however, in their setting, it was offset by the influence of party's policy motives and the cost of adopting a less preferred platform.

Proposition 3.

i) Equilibrium campaign spending $e^*(p^*, p^*)$ is concave in equilibrium platform p^*.

ii) The equilibrium platform p^{max} that maximizes equilibrium campaign spending is independent of the returns to mobilization and the persuasiveness of spending (t, m), as well as preference uncertainty (α, β).

iii) The equilibrium platform p^{max} that maximizes equilibrium campaign spending is strictly increasing in the mass of ideological voters when polarization is zero (i.e., z)

This result shows that equilibrium spending follows an inverted U-shape with respect to equilibrium platforms, and hence equilibrium polarization $y^* := 1 - 2p^*$. It hence highlights that any parameter change affecting equilibrium platforms and polarization may lead to spending and polarization moving either in the same or in opposite directions. The fact that equilibrium platform $p^{\max}$ does not depend on the two main parameters of interest t and m makes the comparative statics analysis possible despite not having a closed form solution for the equilibrium platforms.

Note that the result above describes the relationship between the equilibrium values of polarization and campaign spending, whereas the discussion following Proposition 1 captures only the direct effect of first-stage polarization on second-stage campaign spending. Specifically, while higher (lower) polarization tends to reduce (increase) campaign spending due to a smaller (larger) share of impressionable voters, it is still possible for equilibrium polarization and campaign spending to move in opposite directions. The following comparative statics results regarding changes in the returns to mobilization and the persuasiveness of campaigns further illustrate this.

Proposition 4. *An increase (decrease) in the persuasiveness of spending, t, leads to more (less) extreme equilibrium platforms (i.e., $\frac{\partial p^*}{\partial t} \leq 0$). Equilibrium electoral spending increases in t if and only if $p^* \geq p^{max}$ (i.e., $p^* \geq p^{max} \iff \frac{\partial e_i^*(p^*,p^*)}{\partial t} \geq 0$).*

Polarization in equilibrium increases monotonically with the persuasiveness of spending. As t increases, the competition for impressionable voters intensifies, triggering a rat race between the two parties: Both simultaneously increase their spending without necessarily improving their chances of winning. To curb the escalating cost of electoral spending, parties polarize their platforms, thereby reducing the share of impressionable voters. This reduction should, in turn, lead to a decrease in campaign spending. Whether the direct effect of increased persuasiveness or the indirect effect through polarization dominates ultimately determines whether campaign spending increases or decreases with t. According to Proposition 4, electoral spending in equilibrium decreases in t when polarization is sufficiently high due to sufficiently extreme equilibrium platforms (i.e., $p^* < p^{\max}$).

Considering changes in the returns to mobilization, m, we observe a similar pattern. Recall that given that electoral spending is smaller than 1, returns to mobilization are decreasing in m. Hence, a reduction in m is interpreted as an increase in the returns to mobilization. Proposition 5 presents this result.

Proposition 5. *An increase in the returns of mobilization (i.e., a decrease in m), leads to more extreme equilibrium platforms (i.e., $\frac{\partial p^*}{\partial m} \geq 0$). Equilibrium electoral spending increases in the returns to mobilization if $p^* \geq p^{max}$ (i.e., $p^* \geq p^{max} \Rightarrow \frac{\partial e_i^*(p^*,p^*)}{\partial m} \leq 0$).*

Similar to an increase in the persuasiveness of spending, an increase in the returns to mobilization (i.e., a decrease in m) raises the stakes in the competition for impressionable voters. As a result, the direct effect of a decrease in m by Proposition 1 suggests an increase in their campaign spending. Yet in equilibrium such increase in spending does not affect their probability of winning the election. Consequently, in the first they are incentivized to polarize their platforms in order to reduce the share of impressionable voters and offset the rising cost of electoral spending of the second stage. Hence, a change in m also has an indirect effect of a decrease in spending through the increase in polarization. As in the previous case, the indirect effect on spending might more than offset the direct effect of a decrease in m resulting in an overall reduction in spending. However, given the direct effect of the returns to mobilization on the equilibrium spending level, the condition for observing such a reduction is more demanding than in the case of the persuasiveness of spending. This is captured by the fact that the condition $p^* \geq p^{max}$ was sufficient and necessary for an increase in spending after an increase in t but it is only sufficient after a decrease in m.

Next, we analyze the effects of changes in m or t on turnout. In equilibrium, turnout is given by:

$$Turnout = F(y^*) + (1 - F(y^*))M(e_D^*, e_R^*). \tag{4}$$

Clearly, if a parameter change induces both an increase in campaign spending and an increase in polarization, then turnout rises. This is because, in such a scenario, both $F(y^*)$ and $M(e_D^*, e_R^*)$ increase. This occurs whenever the equilibrium platforms satisfy $p^* > p^{\max}$ and the parameter change increases polarization—that is, either through an increase in t or a decrease in m.

However, when equilibrium platforms involve $p^* \leq p^{max}$, the effect on turnout is not straightforward. In this case, any change that induces greater polarization results in lower campaign spending, leading to two opposing effects on turnout. On the one hand, polarization increases the share of ideological voters who always participate. On the other hand, it reduces the mobilization of impressionable voters. The following demonstrates how the indirect effect operating through polarization dominates the direct effect on campaign spending, thereby determining the comparative statics with respect to t and m:

Proposition 6. *An increase (decrease) in the persuasiveness of spending, t, always leads to higher (lower) turnout (i.e., $\frac{\partial Turnout}{\partial t} \geq 0$). Similarly, an increase (decrease) in the returns to mobilization (i.e., a decrease (increase) in m) also leads to higher (lower) turnout (i.e., $\frac{\partial Turnout}{\partial m} \leq 0$).*

Proposition 6 shows that turnout increases with improvements in either dimension of campaign technology—greater persuasiveness (t) or greater returns to mobilization (i.e., lower m). Crucially, this holds even in the high-polarization regime (i.e., when $p^* \leq p^{\max}$), where technological improvements induce an increase in polarization but a decrease in campaign spending. Despite these opposing effects on turnout—greater participation from ideological voters versus reduced mobilization of impressionable voters—the net effect is positive. That is, the turnout-enhancing effect of increased polarization dominates the turnout-reducing effect of lower campaign effort, ensuring that polarization and turnout move in the same direction in response to changes in t or m.

3.4 Robustness

One of the main assumptions in our model regarding the division of impressionable and ideological voters via $F(y)$ can be relaxed. Specifically, we can instead assume that

$$F(y) = z + (1-z)y^w, \quad \text{where } 0 < w < 1.$$

The exponent w captures the concavity in the distribution of voters across the two types. When $w < 1$, the function $F(y)$ is concave, implying that as parties become more polarized, the marginal increase in the share of ideological voters diminishes. This functional form provides a more flexible, non-linear specification of how polarization maps into voter types, compared to the baseline linear case where $w = 1$.

Proposition 7. *Let $F(y) = z + (1-z)y^w$, where $0 < w < 1$. Then Propositions 1, 2,3–i) and ii)–, 4 and 5 continue to hold.*

The concavity introduced affects the distribution of voter types but does not alter the qualitative logic of the strategic interactions underlying Propositions 1–5. Regarding Proposition 6, we conjecture that it also holds.

4 Conclusion

We developed a two-stage electoral competition model in which parties strategically select policy platforms and campaign spending. Our analysis highlights the dual role of campaign spending in both mobilizing and persuading impressionable voters, and how the endogenous composition of the electorate depends on political polarization. We show that equilibrium platforms balance the benefits of moderation against the escalating costs of campaign spending, leading parties to sometimes adopt more polarized positions to reduce costly competition over impressionable voters. The model generates rich comparative statics predictions, revealing that increases in the persuasiveness of spending or the returns to mobilization can simultaneously increase polarization and produce non-monotonic effects on campaign spending. Nevertheless, any such change in the persuasiveness of spending or the returns to mobilization leads also to an increase in turnout. By integrating these mechanisms, the paper provides a novel theoretical framework that connects turnout policy platforms, and campaigns.

While our model advances the understanding of the interplay between policy platforms, campaign spending, and turnout, it also relies on certain assumptions that future work may seek to relax. In particular, we model abstention as arising from indifference, consistent with the idea that "rather than turning off the public and depressing turnout, polarization energizes the electorate and stimulates political participation" (Abramowitz and Saunders, 2008). However, one may be interested in incorporating forms of abstention due to alienation—where extreme polarization discourages participation among moderate or disengaged voters—thereby exploring the potential of a more nuanced relationship between polarization and turnout.

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5 Appendix

We prove Propositions 1–5 for the case where $F(y) = z + (1-z)y^w$, with $0 < w \leq 1$ so that our proofs also contain the proof of Proposition 7.

Proof of Proposition 1

Taking as given first stage strategies such that $d+r \leq 1$ and assuming $Prob[v_i(d,r,e_D,e_R) > v_{-i}(d,r,e_D,e_R)] \in (0,1)$, the first order conditions for the second stage are:

$$\frac{\beta}{2\alpha}H(y)\left(\frac{me_D^{m-1}}{2}\left(\frac{e_D^t - e_R^t}{e_D^t + e_R^t}\right) + \frac{e_D^m + e_R^m}{2}\left(\frac{2te_D^{t-1}e_R^t}{(e_D^t + e_R^t)^2}\right)\right) = 1 \quad \text{for party D,}$$
$$\frac{\beta}{2\alpha}H(y)\left(\frac{me_R^{m-1}}{2}\left(\frac{e_R^t - e_D^t}{e_D^t + e_R^t}\right) + \frac{e_D^m + e_R^m}{2}\left(\frac{2te_R^{t-1}e_D^t}{(e_D^t + e_R^t)^2}\right)\right) = 1 \quad \text{for party R.}$$

By subtracting the two expressions, it follows that a necessary condition for an interior solution ($e_L^* > 0$, $e_R^* > 0$) is:

$$m\left(e_D^{m-1} + e_R^{m-1}\right)\left(e_D^{2t} - e_R^{2t}\right) = \left(e_D^m + e_R^m\right)\left(2te_D^{t-1}e_R^{t-1}(e_D - e_R)\right)$$

A symmetric profile $e_D = e_R$ guarantees that the condition above holds with

$$e_D^*(d,r) = e_R^*(d,r) = \left(\frac{\beta t H(1-d-r)}{4\alpha}\right)^{\frac{1}{1-m}} \equiv e^*(d,r)$$

We cannot rely on the second-order conditions to confirm that the proposed equilibrium is indeed a maximum, as the concavity of the objective functions is not guaranteed. Instead, we will proceed by identifying all local extrema and providing the conditions under which the proposed equilibrium constitutes the global maximum.

The FOC of party D can be rewritten as:

$$\frac{\beta(1-z)\left((-d-r+1)^w - 1\right)\left(me_D^m\left(-e_R^{2t}\right) + 2te_D^t e_R^t\left(e_D^m + e_R^m\right) + me_D^{m+2t}\right)}{4\alpha e_D\left(e_D^t + e_R^t\right)^2\left((z-1)(-d-r+1)^w - z\right)} - 1 = 0$$

Grouping the two terms and simplifying the resulting numerator, it follows that the sign of the expression is the opposite sign of:

$$\begin{aligned}&4\alpha e_D\left(e_D^t + e_R^t\right)^2\left(z - (z-1)(-d-r+1)^w\right)\\ &- \beta(z-1)\left((-d-r+1)^w - 1\right)\left(me_D^m\left(-e_R^{2t}\right) + 2te_D^t e_R^t\left(e_D^m + e_R^m\right) + me_D^{m+2t}\right).\end{aligned}$$

Writing it as a generalized polynomial on e_D:

$$
\begin{aligned}
&4\alpha\left(z-(z-1)(-d-r+1)^{w}\right)e_D^{2t+1}\\
&+8\alpha\left(z-(z-1)(-d-r+1)^{w}\right)e_R^{t}e_D^{t+1}\\
&+4\alpha\left(z-(z-1)(-d-r+1)^{w}\right)e_R^{2t}e_D\\
&-m\beta(z-1)\left((-d-r+1)^{w}-1\right)e_D^{2t+m}\\
&-2t\beta(z-1)\left((-d-r+1)^{w}-1\right)e_R^{t}e_D^{t+m}\\
&-2t\beta(z-1)\left((-d-r+1)^{w}-1\right)e_R^{t+m}e_D^{t}\\
&+\beta(z-1)\left((-d-r+1)^{w}-1\right)me_R^{2t}e_D^{m}.
\end{aligned}
$$

Notice that if $m \leq t$ and $2t+m \leq 1$, then the terms are ordered decreasingly in terms of the exponent on e_D. We know from the generalized Descartes rule of signs, that the generalized polynomial can have at most as many positive roots as the number of times that the sign of subsequent terms' coefficients change when the terms of the polynomial ordered decreasingly in their exponents (Jameson (2006). Hence, if $m \leq t$ and $2t+m \leq 1$ the (generalized) polynomial has at most two positive roots, and hence, it can only have one interior maximum. Moreover, Descartes' rule also implies that the difference between the number of times that the sign of the coefficient changes (two) and the actual number of roots has to be an even number. Hence, the generalized polynomial has two positive roots, since having zero roots is not possible as we have found already one above. Then the objective function can have one interior maximum and one interior minimum. Note that the objective function is non-positive when $e_i \to 1$ and that its second derivative is negative when evaluated at $e^*_D(d,r) = e^*_R(d,r) = e^*(d,r)$, which guarantees that it is the unique interior maximum and that it exists and interior minium at a lower spending level.

To conclude the proof, we have to show that $e^*_D(d,r) = e^*_R(d,r) = e^*(d,r)$ is the global maximum. Given the existence of an interior minimum at the left of the equilibrium candidate, there exist only one alternative candidate to the global maximum at zero spending.

We verify that parties do not want to deviate to zero spending. Considering party D, a deviation to zero spending can result in either: i) a positive probability of winning (this happens if $-\Delta(d,r,0,e^*) \geq -\alpha$), or ii) a zero probability of winning (this happens if $-\Delta(d,r,0,e^*) < -\alpha$).

For case i), party D has no incentives to deviate to zero expenditure if and only if:

$$
\begin{aligned}
&\pi_D\left(d, r, 0, e^*(d, r)\right) \leq \pi_D\left(d, r, e^*(d, r), e^*(d, r)\right)\\
&Prob\left(b < -\Delta(d, r, 0, e^*(d, r))\right) \leq Prob\left(b < -\Delta(d, r, e^*(d, r), e^*(d, r))\right) - e^*(d, r)\\
&-\frac{\beta}{4\alpha} H(1-d-r) e^*(d, r)^m \leq -e^*(d, r)\\
&\frac{\beta}{4\alpha} H(1-d-r) \geq e^*(d, r)^{1-m}\\
&\frac{\beta}{4\alpha} H(1-d-r) \geq \frac{\beta t H(1-d-r)}{4\alpha}\\
&1 \geq t.
\end{aligned}
$$

Case ii) imposes a participation constraint demanding that the expected utility is non-negative in equilibrium:

$$0 \leq Prob\left(b < -\Delta(d, r, e^*(d, r), e^*(d, r))\right) - e^*(d, r)$$

$$\left(\frac{\beta t H(1-d-r)}{4\alpha}\right)^{\frac{1}{1-m}} \leq \frac{1}{2\alpha}\left(d - d^2 - r + r^2 + \alpha\right). \tag{5}$$

Replacing $H(1-d-r)$ by its upper bound $\frac{1-z}{z}$ and $(d - d^2 - r + r^2)$ by its lower bound $-1/4$, we obtain a sufficient condition.

$$\left(\frac{\beta t(1-z)}{4\alpha z}\right)^{\frac{1}{1-m}} \leq \frac{1}{2\alpha}\left(\alpha - \frac{1}{4}\right). \tag{6}$$

From (6), we see that a sufficient condition for the participation constraint (5) is that z is sufficiently close to 1, which guarantees the existence of a non empty interval $z \in [\bar{z}, 1]$, where $\bar{z} = \frac{\beta t}{2^{m+1}\alpha^m(\alpha-1/4)^{1-m}+\beta t}$ for which the participation constraint holds for any pair $\{d, r\}$.

Finally, the existence of a global maximum and the symmetry of equilibrium spending, guarantee that the initial assumption $Pr[v_i(d, r, e_D^*, e_R^*) > v_{-i}(d, r, e_D^*, e_R^*)] \in (0, 1)$ holds.

Proof of Proposition 2

We begin the proof by remarking the positive effect of adopting a central platform on the probability of winning. This observation will serve for Lemma 1 and Lemma 2.

Remark 1. *The probability of winning monotonically increases by approaching to* $1/2$.

As explained in the main text, this observation follows directly from substituting the equilibrium spending expressions into the first-stage objective functions.

Lemmas 1 and 2 establish properties of the best responses that will be instrumental in characterizing the equilibrium of the platform subgame.

Lemma 1. *If party R chooses a policy $r \in [0, 1/2]$, the best response of party D is in the interval $d \in [0, 1/2]$. Similarly, if party D chooses a policy $d \in [0, 1/2]$, the best response of party R is in the interval $r \in [0, 1/2]$.*

Proof. Consider party D. If party R selects a policy $r \in [0, 1/2]$, then $d = 1/2$ strictly dominates any $d \in (1/2, 1-r]$ (the probability of winning is greater –Remark 1– and the electoral spending lower). Moreover, any choice $d > 1 - r$ is dominated by its symmetric counterpart $d' = 1 - r - (d - 1 + r)$ (greater probability of winning –Remark 1– and less electoral spending). We can proceed analogously to complete the proof for party R.

□

Lemma 2. *If party R chooses a policy $r > 1/2$, the best response of party D is in the interval $(1 - r, 1]$. Similarly, if party L chooses a policy $d > 1/2$, the best response of party R is in the interval $(0, d]$.*

Proof. Any $d \in [0, 1 - r)$ is dominated by $d' = 1 - r + (1 - r - d)$ (greater probability of winning –Remark 1–, same spending). Finally, choosing $d = 1 - r$ is strictly dominated by $d = 1/2$ (greater probability of winning –Remark 1–, less spending).

□

The above two lemma imply that if we want to characterize an equilibria such that $d \leq 1 - r$, it must hold that $d \in [0, 1/2]$ and $r \in [0, 1/2]$. From now on we restrict ourselves to these intervals. The FOCs of the first stage subject to these conditions are:

$$f(d, r) \equiv \frac{1}{2\alpha}(1 - 2d) - \frac{\partial e_D^*(d, r)}{\partial d} = 0 \tag{7}$$

$$g(d, r) \equiv \frac{1}{2\alpha}(1 - 2r) - \frac{\partial e_R^*(d, r)}{\partial r} = 0 \tag{8}$$

In the next lemma, we show that the second-order conditions hold, hence the two conditions above are necessary and sufficient for an interior equilibrium $d \in (0, 1/2)$ and $r \in (0, 1/2)$.

Lemma 3. *The function $f(d, r)$ is decreasing in d, and $g(d, r)$ is decreasing in r. Hence, the SOCs of the problem are negative.*

Proof. We provide the proof for $f(d, r)$. The one for $g(d, r)$ is analogous. Since the first term of $f(d, r)$ is decreasing in d, we can complete the proof by showing that $\frac{\partial^2 e^*(d,r)}{\partial d^2} > 0$. Although the expression is long, a sufficient condition guaranteeing that it is positive is

$$(1-d-r)^w(2(m-1)z-m+w+1)-(m-1)(w+1)(z-1)(1-d-r)^{2w}+(m-1)(w-1)z > 0$$

The next four steps show that the above inequality holds, completing the proof:

$$(1-d-r)^w(2(m-1)z-m+w+1)-(m-1)(w+1)(z-1)(1-d-r)^{2w}+(m-1)(w-1)z >$$
$$(1-d-r)^w\left((2(m-1)z-m+w+1)-(m-1)(w+1)(z-1)(1-d-r)^w+(m-1)(w-1)z\right) >$$
$$(1-d-r)^w\left((2(m-1)z-m+w+1)-(m-1)(w+1)(z-1)+(m-1)(w-1)z\right)$$
$$= mz(1-d-r)^w > 0.$$

□

From the equilibrium expressions at the spending subgame, it follows that the two first-order conditions are symmetric. Hence, in an interior equilibrium it must hold that $d = r = p^*$ with $p^* \in [0, 1/2]$. Consequently, the two first-order conditions above collapse to the same one, with the only relevant equation for interior equilibria being $f(p^*, p^*) = g(p^*, p^*) = 0$. Hence the equilibrium condition for the two parties is defined by the single equation:

$$\frac{1}{2\alpha}(1-2p^*) = \left.\frac{\partial e^*(d,p^*)}{\partial d}\right|_{d=p^*} = \left.\frac{\partial e^*(p^*,r)}{\partial r}\right|_{r=p^*} \tag{9}$$

Such an interior equilibrium exists if $f(0,0) = g(0,0) \geq 0$ and $f(1/2,1/2) = g(1/2,1/2) \leq 0$. The former inequality strictly holds –guaranteeing that in equilibrium $p^* > 0$– from replacing $d = r = 0$ in equations (7 and 8) and noting that $\frac{\partial e^*(d,r)}{\partial i}|_{r=d=0} = 0$ for $i \in \{d, r\}$.[6] The latter inequality strictly holds –guaranteeing that in equilibrium $p^* < 1/2$– by replacing $d = r = 1/2$ in equations (7) and (8) and observing that spending is strictly decreasing in polarization (i.e., increasing in d and r).

Finally, by strict concavity, the interior equilibrium is unique.

Proof of Proposition 3

Let us rewrite equation (9) as

$$e_D^*(p^*,p^*) = \frac{1}{2\alpha}(1-2p^*)\left(-\frac{(m-1)\left((1-2p^*)^w-1\right)(1-2p^*)^{1-w}\left((z-1)(1-2p^*)^w-z\right)}{w}\right) \tag{10}$$

$$\frac{\partial e_D^*(p^*,p^*)}{\partial p^*} = -\frac{(m-1)(1-2p^*)^{1-w}\left(2(2z-1)(1-2p^*)^w-(w+2)(z-1)(1-2p^*)^{2w}+(w-2)z\right)}{aw}$$

[6]Note that $\frac{\partial e^*(d,r)}{\partial i}|_{r=d=0} = 0$ for $i \in \{d, r\}$ is a direct consequence of $H(1) = 0$.

Thus, the sign of the $\frac{\partial e_D^*(p^*,p^*)}{\partial p^*}$ is precisely the sign of

$$h(p^*) \equiv \left(2(2z-1)(1-2p^*)^w - (w+2)(z-1)(1-2p^*)^{2w} + (w-2)z\right).$$

It immediately follows that $h(0) > 0$ and $h(1/2) < 0$. Moreover,

$$h'(p^*) = 4(1-2p^*)^{-1+w}w(1+(1-2p^*)^w(2+w)(-1+z)-2z),$$
$$h''(p^*) = 8w(1-2p^*)^{w-2}\left((w-1)(2z-1)-(w+2)(2w-1)(z-1)(1-2p^*)^w\right) > 0.$$

So the second derivative has at most one zero in $p^* \in (0,1/2)$, $h'(0) < 0$, $h'(1/2) > 0$ which proves the inverted U-shape of equilibrium effort when varying policy platform.

Let's see that p^{max} providing the maximum level of expenditure only depends on z and w. In fact, we can find it explicitly by solving $h(p^{max}) = 0$; notice that it results in a quadratic equation in $(1-2p^*)^w$ with solution:

$$p^{max} = \frac{1}{2}\left(1 - \left(\frac{\sqrt{w^2(z-1)z+1} - 2z + 1}{(w+2)(1-z)}\right)^{1/w}\right).$$

The expression does not depend on t, m, α nor β and it is increasing in z.

For the case $w = 1$, the expression is

$$p^{max}|_{w=1} = \frac{1}{2}\left(1 - \left(\frac{\sqrt{(z-1)z+1} - 2z + 1}{(1-z)}\right)\right)$$

which is strictly increasing in z.

Proof of Proposition 4

By concavity of the objective function and the Implicit Function Theorem, we can move all terms of equation (9) to the left to see that $\frac{\partial p^*}{\partial t} \leq 0$ if and only if:

$$\frac{4^{\frac{1}{m-1}} w(1-2p^*)^{w-1} \left(\frac{a\left(\frac{1}{(z-1)((1-2p^*)^w-1)}-1\right)}{bt}\right)^{\frac{1}{m-1}}}{(m-1)^2 t\left((1-2p^*)^w - 1\right)\left(z-(z-1)(1-2p^*)^w\right)} \leq 0$$

Given the constraints in the parameters of the model $z \in (0,1)$, $m \in (0,1)$, $w \in (0,1]$ and in equilibrium platforms $p^* \in (0,1/2)$; the above inequality always holds.

To analyze changes in equilibrium spending, we should note that from equation (10), in equilibrium electoral spending only depends on t through changes in policy platforms. Consequently, the change in spending follows from the comparison of p^* and p^{max} as indicated in Proposition 2.

Proof of Proposition 5

By concavity of the objective function and the Implicit Function Theorem, we can move all terms of equation (9) to the left to see that $\frac{\partial p^*}{\partial m} \geq 0$ if and only if:

$$\frac{4^{\frac{1}{m-1}} w(1-2p^*)^{w-1} \left(\frac{a\left(\frac{1}{(z-1)((1-2p^*)^w-1)}-1\right)}{bt}\right)^{\frac{1}{m-1}} \left(\ln\left(\frac{4a\left(\frac{1}{(z-1)((1-2p^*)^w-1)}-1\right)}{bt}\right)+m-1\right)}{(m-1)^3\left((1-2p^*)^w-1\right)\left(z-(z-1)(1-2p^*)^w\right)} \geq 0 \tag{11}$$

Given the constraints in the parameters of the model $z \in (0,1)$, $m \in (0,1)$, $w \in (0,1]$ and in the equilibrium platforms $p^* \in (0,1/2)$; the sign of the left-hand side of equation (11) is determined by the sign of the term $\ln\left(\frac{4a\left(\frac{1}{(z-1)((1-2p^*)^w-1)}-1\right)}{bt}+m-1\right)$.

This expression is clearly decreasing in t, so let us consider $0 < t^* < 1/2$ that solves

$$\ln\left(\frac{4a\left(\frac{1}{(z-1)((1-2p^*)^w-1)}-1\right)}{bt^*}\right) = 1-m.$$

If such t^* does not exist, the proof is over, since (11) would be positive for all t (note that (11) is continuous and positive at $t=0$).

Solving for t^*:

$$t^* = \frac{4ae^{m-1}\left(\frac{1}{(z-1)((1-2p^*)^w-1)}-1\right)}{b}.$$

Moreover, we can see that the resulting expression from moving all terms of equation (9) to the left-hand side is decreasing in t. Thus, if after replacing $t=t^*$ in the resulting expression from moving all terms of equation (9) to the left-hand side, it is negative, then it would be negative for any t making the condition on t^* incompatible with equilibrium platforms p^* and concluding the proof. By doing so, we want to check if:

$$-2a\left(e^{1-m}\right)^{\frac{1}{m-1}} w(1-2p^*)^w - (1-2p^*)^2(m-1)\left((1-2p^*)^w-1\right)\left((z-1)(1-2p^*)^w-z\right) \leq 0$$

Since given any values of p^* and w, the left-hand side of the above expression is is maximized for $m=0$, $z=1$ and $a=1$. We can substitute for these values to obtain the sufficient condition:

$$\frac{2w(1-2p^*)^w}{e} \geq (1-2p^*)^2\left(1-(1-2p^*)^w\right) \iff \frac{2w}{e} \geq (1-2p^*)^{2-w}\left(1-(1-2p^*)^w\right)$$

Since the left-hand side of the latter expression is constant, a sufficient condition can be obtained by considering the maximum of the right-hand side and compare it. By derivating we find that the right-hand side has a maximum when

$$1 - 2p^* = \left(\frac{2-w}{2}\right)^{1/w}$$

Substituting this expression into the latter inequality immediately confirms that t^* is incompatible with equilibrium platforms confirming that equation (11) holds.

To analyze changes in equilibrium spending, we differentiate Equation (10) with respect to m, yielding

$$\frac{\partial e_D^*(p^*, p^*)}{\partial m} = -\frac{1}{2\alpha w}(1-2p^*)\left[(1-(1-2p^*)^w)(1-2p^*)^{1-w}((1-z)(1-2p^*)^w + z)\right] + \frac{\partial e_D^*(p^*, p^*)}{\partial p^*}\frac{\partial p^*}{\partial m} \tag{12}$$

We know from Proposition 2 that the second term is negative if and only if $p^* \geq p^{max}$. Moreover, given the constraints imposed on the parameters of the model, the first term of expression (12) is always negative. Consequently, the condition $p^* \geq p^{max}$ is sufficient but not necessary for $\frac{\partial e_D^*(p^*,p^*)}{\partial m} < 0$

6 Proof of Proposition 6

In equilibrium, turnout is characterized by the following expression

$$1 - 2p^*\left(-1 + \left(\frac{p^*(-1+2p^*)(-1+m)(1+2p^*(-1+z))}{a}\right)^m\right)(-1+z) \tag{13}$$

6.1 Comparative statics of turnout with respect to t

Note that t does not appear explicitly in the expression, indicating that changes in t influence turnout only indirectly, through their effect on the equilibrium platforms p^*. Since $\frac{p^*}{t} < 0$, to prove the result we have to show that the derivative of 13 with respect to p^* is negative.

The derivative of (13) with respect to p^* is the opposite sign of the expression below:

$$\begin{aligned}&\left(4p^{*2}(3m+1)(z-1) - 2p^*(2m+1)(z-2) - m - 1\right)\left(\frac{p^*(2p^*-1)(m-1)(2p^*(z-1)+1)}{a}\right)^m \\ &- (2p^*-1)(2p^*(z-1)+1).\end{aligned} \tag{14}$$

To establish the result, we must demonstrate that the expression above is positive. We start by dividing the expression by the strictly positive term $-(2p^*-1)(2p^*(z-1)+1)$, obtaining:

$$\frac{p^*(m-1)\left(-4p^{*2}(3m+1)(z-1)+2p^*(2m+1)(z-2)+m+1\right)\left(\frac{p^*(2p^*-1)(m-1)(2p^*(z-1)+1)}{a}\right)^{m-1}}{a}+1 \tag{15}$$

Note that the expression above is strictly positive for $p^*=0$; thus, it suffices to show that for $p^* \in (0, 1/2]$:

$$\frac{p^*(m-1)\left(-4p^{*2}(3m+1)(z-1)+2p^*(2m+1)(z-2)+m+1\right)}{a}+\left(\frac{p^*(2p^*-1)(m-1)(2p^*(z-1)+1)}{a}\right)^{1-m} \geq 0 \tag{16}$$

Note that the first and second terms of the equation are equal in absolute value but with opposite sign when $m=0$. Furthermore, since the values of m, z and p^* lie within the interval $[0,1]$, the second term is strictly positive. Therefore, it suffices to show that: i) the second term is increasing in m; and ii) the first term is increasing in m whenever it is negative.

Part i) is guaranteed by noting that the two terms of the derivative are positive: $-\log\left(\frac{p^*(2p^*-1)(m-1)(2p^*(z-1)+1)}{a}\right)+1$ (as m, z and p^* are bounded within $[0,1]$ and $\alpha>1$, then the argument of the logarithmic expression lies in the interval $[0,1]$).

For part ii), it will be useful to define the first term of expression (16) and its derivative with respect to m:

$$f(p^*)=\frac{p^*(m-1)\left(-4p^{*2}(3m+1)(z-1)+2p^*(2m+1)(z-2)+m+1\right)}{a}$$
$$\frac{\partial f(p^*)}{\partial m}=g(p^*)=\frac{2p^*\left(-4p^{*2}(3m-1)(z-1)+p^*(4m-1)(z-2)+m\right)}{a}$$

It is immediate to verify that $f(0)=g(0)=0$ and that $f'(0)<0$ and $g'(0)>0$. Hence, it is enough to see that: a) both $f(p^*)$ and $g(p^*)$ have at most one zero in $(0,1/2)$, and $f(p^*)$ has exactly one; b) if $f(p^*)=0$ with $p^*>0$, then $g(p^*)\geq 0$.

Let's start with part a). Given that $f(0)=0$ and $f'(0)<0$, we just have to show that $f(1/2)>0$, which is immediate as, after simplifying, we see that $f(1/2)=\frac{(1-m)mz}{2a}$. Furthermore, given the explicit solution, it is immediate that the zero must be:

$$\tau(m,z)=\frac{2mz-4m+z-2+\sqrt{4m^2z^2-4m^2z+4m^2+4mz^2+z^2}}{4(3mz-3m+z-1)},$$

which is well-defined.

Next we see that $g(p^*)$ has at most one zero in the interval of interest. Note that, given that $g(p^*)$ is a polynomial of third degree in p^*, it can be solved explicitly. One solution is obviously $p^* = 0$, and another one is given by:

$$s(m,z) = \frac{4mz - 8m - z + 2 - \sqrt{(-4mz + 8m + z - 2)^2 + 4m(12mz - 12m - 4z + 4)}}{2(12mz - 12m - 4z + 4)}.$$

Since the values of m and z lie within the interval $[0, 1]$, the expression inside the square root is always positive and the solution is real and well defined whenever the denominator is not zero.

Next, we will see that $s(m, z) \notin (0, 1/2)$ for any $0 < m < 1$ and $0 < z < 1$. First of all, note that the denominator is positive if $m \leq 1/3$ and negative otherwise. Hence, if $m \leq 1/3$, clearly $s(m, z) < 0$, since the numerator is negative. Next we see that, if $m > 1/3$, $s(m, z) \geq 1/2$. Given that the denominator is negative, we have to see that:

$$-\sqrt{(-4mz + 8m + z - 2)^2 + 4m(12mz - 12m - 4z + 4)} \leq m(8z - 4) - 3z + 2.$$

The above inequality necessarily holds whenever the right-hand side of the expression above is positive. So, we focus on checking whether the following two conditions do not hold simultaneously:

$$\begin{aligned}
&m(8z - 4) - 3z + 2 < 0,\\
&(-4mz + 8m + z - 2)^2 + 4m(12mz - 12m - 4z + 4) < (m(8z - 4) - 3z + 2)^2.
\end{aligned}$$

The last condition reduces to $0 < 8(1 + m(-5 + 6m))(-1 + z)z$, which holds if, and only if, $1 + m(6m - 5) < 0$, that is, if and only if, $1/3 < m < 1/2$. However, when m lies in this interval, the condition $m(8z - 4) - 3z + 2 < 0$ is violated for any $z \in [0, 1]$

We finally move on to prove part b), which states that if $f(p^*) = 0$ and $p^* > 0$, then it follows that $g(p^*) \geq 0$. We know that there is one, and only one, $p^* > 0$ such that $f(p^*) = 0$ for which we have an explicit value $\tau(m, z)$. We plug it in function $g(p^*)$ and we obtain that $g(\tau(m, z)) \geq 0$:

$$\begin{aligned}
&-\frac{(m-1)\left(\sqrt{-4m^2z + 4m^2 + (2mz + z)^2} + 2m(z-2) + z - 2\right)}{8a(3m+1)^3(z-1)^2}\\
&\cdot ((z-2)\left(\sqrt{-4m^2z + 4m^2 + (2mz + z)^2}\right) + 2m(z(z+2) - 2)) + z^2.
\end{aligned}$$

If we see that:

$$\begin{aligned}
&\sqrt{-4m^2z + 4m^2 + (2mz + z)^2} + 2m(z-2) + z - 2 \leq 0 \text{ and}\\
&(z-2)\left(\sqrt{-4m^2z + 4m^2 + (2mz + z)^2}\right) + 2m(z(z+2) - 2) + z^2 \leq 0,
\end{aligned}$$

it implies that $g(\tau(m,z)) \geq 0$, concluding the proof. The first is easy to verify, given that $2m(z-2)+z-2 < 0$ in general. For proving the second it suffices to show that the following two conditions cannot hold at the same time:

$$
\begin{aligned}
&2m(z(z+2)-2)+z^2 > 0,\\
&(2m(z(z+2)-2)+z^2)^2 > (z-2)^2(-4m^2z+4m^2+(2mz+z)^2).
\end{aligned}
$$

Which follows immediately by noting that

$$
\begin{aligned}
&(2m(z(z+2)-2)+z^2)^2 - ((z-2)^2(-4m^2z+4m^2+(2mz+z)^2))\\
&= 4(z-1)(3mz+z)^2 \leq 0.
\end{aligned}
$$

6.2 Comparative statics of turnout with respect to m

Since we have shown that $\frac{\partial Turnout^*}{\partial p^*} \leq 0$ and, by Proposition 4, $\frac{\partial p^*}{\partial m} \geq 0$ we know that the indirect effect of m on turnout through polarization is negative; a sufficient condition for $\frac{\partial Turnout^*}{\partial m} \leq 0$ is that the partial derivative of equation (13) with respect to m is negative:

$$
2p^*(1-z)\left(\frac{p^*(-1+2p^*)(-1+m)(1+2p^*(-1+z))}{a}\right)^m
\left(\frac{m}{m-1} + \log\left(\frac{p^*(-1+2p^*)(-1+m)(1+2p^*(-1+z))}{a}\right)\right) < 0
$$

The inequality necessarily holds for all m, z and p^* lying within the interval $[0,1]$.