Should R&D Subsidies be Financed with Public Debt?

Nikolaos Motsios

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European University Institute

Motivation

- **Innovation** $\Rightarrow$ **Positive Externalities** (Arrow 1972) $\Rightarrow$ **Government Intervention** (grants, subsidies, tax credits)
- Literature shows:

$$\text{Public R\&D} \uparrow \Rightarrow \text{Innovation} \uparrow \Rightarrow g \uparrow$$

However, no answer on how to finance: deficits or taxes?

Why it matters:

- Policy Implications (Draghi, ReArm Europe, Maastricht)
- Few Theoretical Models on public debt's effect on innovation **(and almost all ignore capital!)**
- Taxation can harm innovation (Akcigit and Stantcheva 2020)
- Debt is also harmful for innovation (Today's Model)

Motivation and Main Reasoning

Main Idea

- Labor/Consumption taxes $\Rightarrow$ Welfare loss/ Disliked
- Firm taxes $\uparrow \Rightarrow$ Innovation $\downarrow$
- So instead, maybe finance subsidies with debt?

Mechanism

$$\text{Debt } (b) \uparrow \Leftrightarrow \text{ Subsidies} \uparrow \Rightarrow \underbrace{(r-g)}_{\text{Cost of Debt}} \downarrow \Rightarrow \underbrace{r < g}_{\text{Sustainability with 0 taxes}}$$

▸ ODE solution

However, this logic ignores

- $b \uparrow \Rightarrow \ k \downarrow$ (crowding-out)
- $g = g(k)$, $g'(k) > 0$ (Howitt and Aghion 1998; Aghion and Howitt 2017)

Research Question & Progress

Research Question: **Should R&D subsidies be financed with debt instead of taxes?**

1. Free lunch? $(r < g)$
2. Welfare Implications

Contributions

1. **Today**: Theoretical Model (Debt $\rightarrow$ Innovation)
   - Perpetual Youth + Schumpeterian Growth
   - Capital Accumulation
   - $b^* \uparrow \Rightarrow \ (r^* - g^*) \uparrow \Leftrightarrow$ Cost of Debt $\uparrow$
2. **Future:** Use debt-financed subsidies to answer debt or tax financing of subsidies.
3. **Future:** Quantitative Exercise

Literature

Empirical ($\Leftarrow$ Model replicates results qualitatively)

- Public R&D on Innovation Inputs (private R&D expenditures) and Outputs (patents etc.) (Almus and Czarnitzki 2003; Rao 2016; Howell 2017; Acconcia and Cantabene 2018)
- Debt on growth (Reinhart and Rogoff 2010; Checherita-Westphal and Rother 2012; Woo and Kumar 2015; Chudik, Mohaddes, Pesaran, and Raissi 2017)

Theoretical ($\Leftarrow$ Model: Debt Sustainability + Capital + Creative Destruction)

- Creative Destruction (Howitt and Aghion 1998; Huetsch, Krueger, and Ludwig 2023)
- Debt & Innovation (Ferraro and Peretto 2020; Cozzi 2023; Minami and Horii 2025)
- Debt Sustainability (Blanchard 2019; Angeletos, Lian, and Wolf 2024; Minami and Horii 2025)

In the Future ($\Leftarrow$ Answer: Debt/Tax Financing of R&D programs)

- Public Finance (Krueger and Ludwig 2016)

Model

Perpetual Youth[1] + Schumpeterian Growth (with capital)[2]

- t continuous
- 1 Final Good (PC)
- $i \in [0, 1]$ Intermediate goods (MC)
- Innovation (improves quality of $i \in [0, 1]$)
- Perpetual Youth Households (OLG)
- Government R&D subsidies

Key Assumptions

1. Perpetual Youth (breaks Ricardian Equivalence)
2. Labor Productivity decreases with Age (allows $r < g$)

[1] Blanchard 1985

[2] Howitt and Aghion 1998; Aghion and Howitt 2017

Production - Final Good

- Competitive Final Good Firm

$$Y(t) = Z \int_0^1 (q_i(t)L)^{1-\alpha} x_i(t)^{\alpha} di \tag{1}$$

q_i : quality of i good, $x_i(t)$: quantity of i good, Z : scaling parameter, $L = 1$ labor

- This implies
  1. Labor Demand:

$$w(t) = (1-\alpha) Z \int_0^1 q_i(t)^{1-\alpha} x_i(t)^{\alpha} di \tag{2}$$

  2. Demand for i good

$$p_i(t) = \alpha Z q_i(t)^{1-\alpha} x_i(t)^{\alpha-1} \tag{3}$$

Production - Intermediate Good Sector

- Monopolistic Competition
- $i \in [0,1]$ firm solves[3]

$$\max_{x_i(t)} \left(p_i(t)x_i(t) - r(t)x_i(t)\right), \quad \text{s.t. (3)}$$

- i production $x_i(t) = K_i(t)$ & Capital MC $\Rightarrow$

$$x_i(t) = q_i(t)k(t) \tag{4}$$

$$r(t) = \alpha^2 Z k(t)^{\alpha-1} \tag{5}$$

$$\pi_i(t) = \alpha(1-\alpha)Z q_i(t)k(t)^{\alpha} \tag{6}$$

where:

$k(t) := K(t)/Q(t)$,

$Q(t) := \int_0^1 q_i(t)di$ is Aggregate Quality (TFP)

[3] No capital depreciation

Production - Aggregates

- Output

$$Y(t) = ZQ(t)k(t)^{\alpha} \tag{7}$$

- Profits

$$\pi(t) = \int_0^1 \pi_i(t)di = \alpha(1-\alpha)ZQ(t)k(t)^{\alpha} \tag{8}$$

- Wage

$$w(t) = (1-\alpha)ZQ(t)k(t)^{\alpha} \tag{9}$$

Note

$$Y(t) = \pi(t) + w(t)L + r(t)K(t)$$

Innovation Sector

- **Successful Innovation:**
  1. $q_i \to \lambda q_i$ $(\lambda > 1)$
  2. **Innovator replaces i monopolist (Creative Destruction)**
- Flow rate of Innovation (Acemoglu 2008)

$$\phi_i(t|q_i(t)) = \eta \frac{R_i(t)}{q_i(t)} \tag{10}$$

$R_i(t)$: Private R&D by innovator i

- Value of Firm i

$$V(t|q_i(t)) = \frac{\pi_i(t)}{r(t) + \phi_i(t|q_i(t))} \tag{11}$$

Innovation Sector

- Government Subsidy: $\sigma R_i(t)$, $\sigma \in (0,1)$
- Free Entry implies

$$\phi_i(t|\lambda^{-1}q_i(t))V(t|q_i(t) = (1-\sigma)R_i(t)$$

- Equilibrium Flow Rate (using (8),(12),(13))

$$\phi(k(t);\sigma) := \phi_i(t) = Z\Big[\frac{\alpha(1-\alpha)\eta\lambda}{1-\sigma}k(t)^{\alpha} - \alpha^2 k(t)^{\alpha-1}\Big] \tag{12}$$

- Using (12)

$$R_i(t) = \frac{1}{\eta}q_i(t)\phi(k(t);\sigma) \Rightarrow \tag{13}$$

$$R(t) = \frac{1}{\eta}Q(t)\phi(k(t);\sigma) \tag{14}$$

Aggregate Growth

- Growth Rate for Aggregate Quality $Q(t)$

$$g(k(t);\sigma) = \frac{\dot{Q}(t)}{Q(t)} = (\lambda - 1)\phi(k(t);\sigma) \tag{15}$$

- BGP Growth Rate

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{d\ lnY(t)}{dt} = g(k^*;\sigma) \tag{16}$$

- Partial Equilibrium:

1. $\partial g/\partial \sigma > 0$
2. $\partial g/\partial k > 0$

Households

- s Time of birth
- μ Poisson death rate
- Population:

$$N = \int_{-\infty}^{t} \mu e^{-\mu(t-s)} ds = 1$$

- Labor Supply = Inelastic
- $\xi e^{-\beta(t-s)}$ Labor Productivity (decreasing over lifetime)[4]
- $\xi := \frac{\beta+\mu}{\mu}$ ($\Rightarrow$ Aggregate Productivity = 1)
- Households participate in an innovation fund according to their productivity (exogenously spend for R&D and earn profits[5])

[4] This is assumed only to allow for dynamic inefficiency $r < g$ (see Blanchard 1985; Saint-Paul 1992; Minami and Horii 2025)

[5] See Nuno 2011

Household Optimization

Household born at s solves

$$\max_{c(s,t),\, a(s,t)} \int_t^{\infty} e^{-(\rho+\mu)(\tau-t)} \ln\big(c(s,t)\big)\, d\tau \tag{17}$$

s.t.

$$\dot{a}(s,t) = (r(t)+\mu)a(s,t) + \xi e^{-\beta(t-s)} f(t) - c(s,t) \tag{18}$$

$$f(t) := (1-\tau(t))w(t) + \pi(t) - (1-\sigma)R(t)$$

$$lim_{T\to\infty} e^{-\int_t^T (r(v)+\mu)dv} a(s,T) = 0$$

Euler Equation:

$$\dot{c}(s,t) = (r(t)-\rho)c(s,t) \tag{19}$$

Household Aggregation

- Consumption Function

$$c(s,t) = (\rho+\mu)\Big[a(s,t) + \xi e^{-\beta(t-s)} \int_t^T e^{-\int_t^T (r(v)+\rho+\beta)dv} f(\tau)d\tau\Big] \tag{20}$$

- Aggregating[6] yields

1. Aggregate Euler Equation:

$$\dot{C}(t) = (r(t)+\beta-\rho)C(t) - (\mu+\beta)(\mu+\rho)A(t) \tag{21}$$

2. Aggregate Budget Constraint:

$$\dot{A}(t) = r(t)A(t) + f(t) - C(t) \tag{22}$$

[6] $X(t) := \int_{-\infty}^{t} x(s,t)\mu e^{-\mu(t-s)}ds$

Government

Government:

- $B(t)$, Debt (1-period bonds)
- $G(t) = \sigma R(t)$, Innovation Subsidy
- $T(t) = \tau(t)w(t)$, Proportional labor taxes[7]

Government Budget Constraint:

$$\dot{B}(t) = r(t)B(t) + \sigma R(t) - \tau(t)w(t) \tag{23}$$

[7] Because of the inelastic labor, they are essentially Lump-sum taxes

Competitive Equilibrium

Market Clearing[8]:

$$A(t) = K(t) + B(t) \tag{24}$$

Use (23)–(26) and de-trend[9] by dividing by $Q(t)$ to get:

Model Equations

$$\dot{c}(t) = [r(t) + \beta - \rho - g(k(t);\sigma)]c(t) - (\mu + \beta)(\mu + \rho)[k(t) + b(t)] \tag{25}$$

$$\dot{k}(t) = y(t) - \frac{R(t)}{Q(t)} - c(t) - g(k(t);\sigma)k(t) \tag{26}$$

$$\dot{b}(t) = [r(t) - g(k(t);\sigma)]b(t) + \sigma\frac{R(t)}{Q(t)} - \tau(t)\frac{w(t)}{Q(t)} \tag{27}$$

where $c := C/Q$ and $b := B/Q$

▸ Jump to Appendix

Equilibrium Properties - Existence

Proposition 1 - Existence

Let the economy be characterized by equations (25) and (26). Assume that $k(0) > \underline{k}$[10] and that the government stabilizes debt at $b^* < \bar{b}$[11]. Then $\exists \bar{k} \in (\underline{k}, +\infty)$ such that the economy has at least one steady state $(k^*, c^*) >> \mathbf{0}$ with $k^* \in (\underline{k}, \bar{k})$

[10] $\underline{k} := \frac{\alpha(1-\sigma)}{\eta\lambda(1-\alpha)}$

[11] $\bar{b} := \frac{\alpha^2 Z^2}{(\mu+\beta)(\mu+\rho)\underline{k}^{3-2\alpha}} - 1$

Phase Diagram

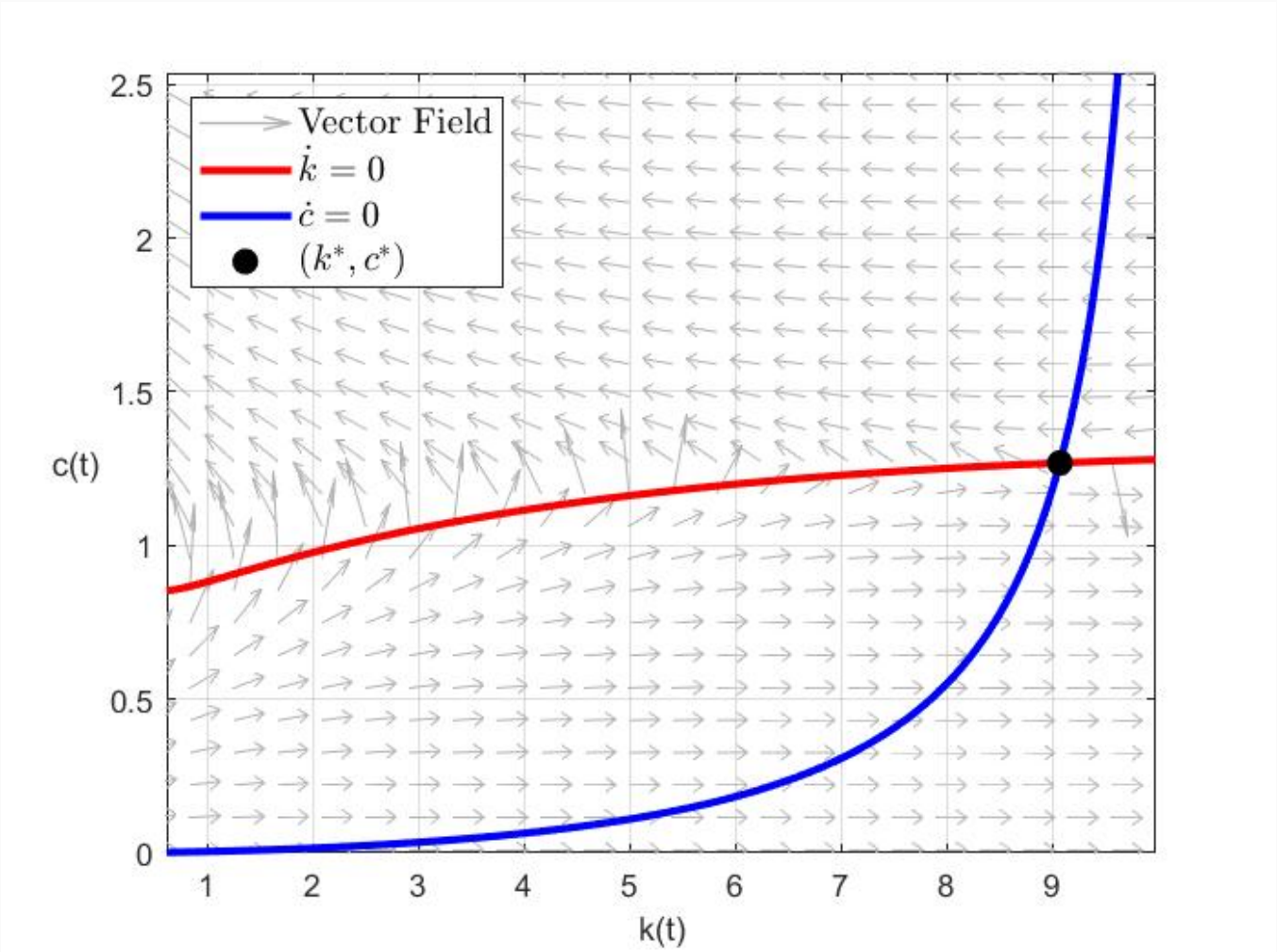

Figure 1: Phase Diagram and Directional vectors.

Equilibrium Properties - Comparative Statics

Proposition

Let (k^*, c^*) be a steady state for equations (27),(28) that is characterized by saddle-path stability. Then,

$$\frac{\partial}{\partial b^*}(r^* - g^*) > 0$$

Heuristic Proof: An increase in b^* crowds out capital. This increases the steady state interest rate $r^* = \alpha^2 Z k^{*\alpha-1}$ and decreases the steady state growth rate $g^*(k^*; \sigma) = (\lambda - 1)Z[\frac{\alpha(1-\alpha)\eta\lambda}{1-\sigma} k^{*\alpha} - \alpha^2 k^{*\alpha-1}]$

Comparative Statics

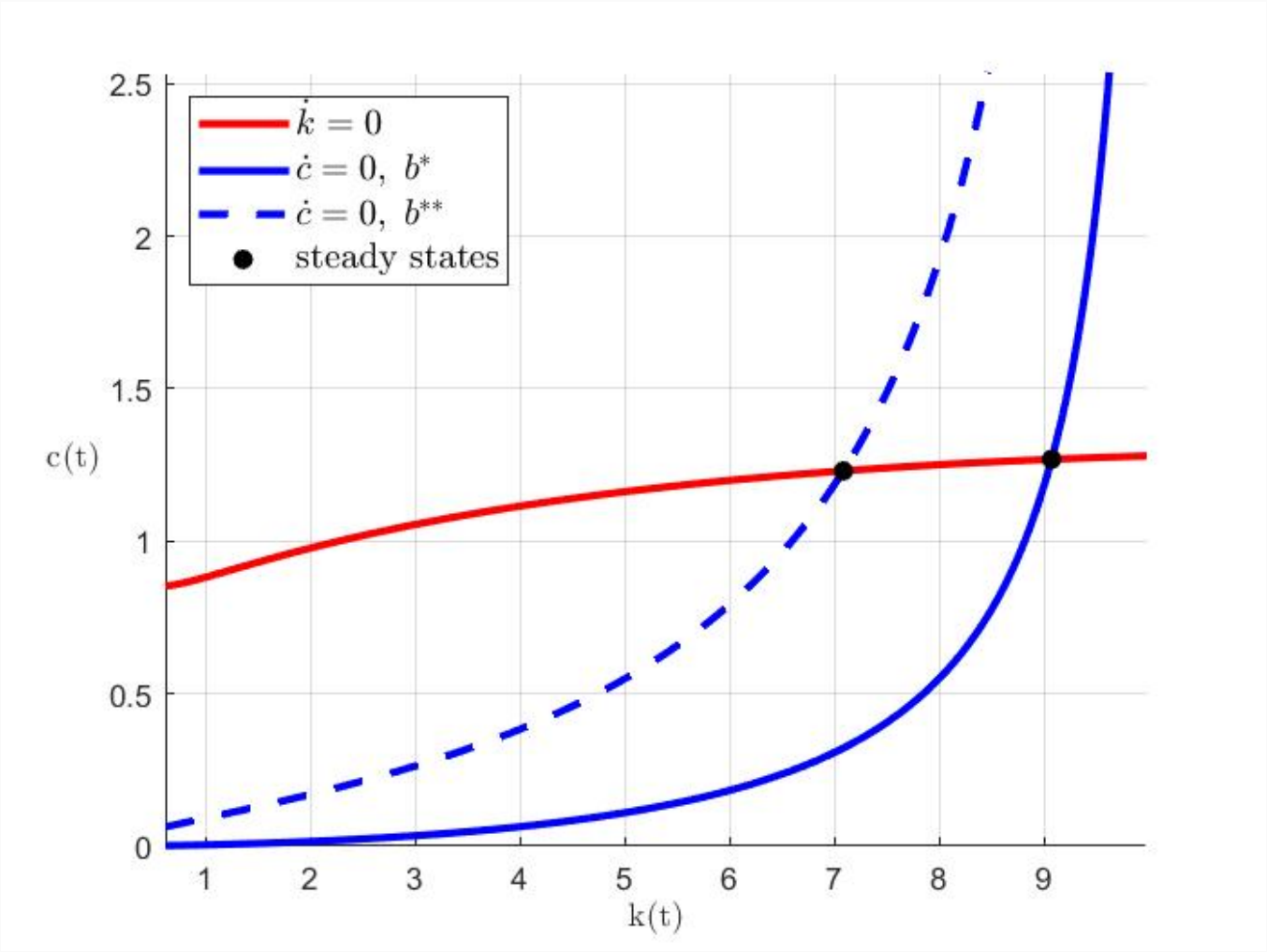

Figure 2: Effect of an increase of b^* to b^{**}

$$b^* \uparrow \Rightarrow c \uparrow \Rightarrow k \downarrow \Rightarrow (r^* - g^*) \uparrow$$

Steady-state results for Various B^*/Y^*

B^*/Y^*	k^*	C^*/Y^*	r^*	ϕ^*	g^*	r^*-g^*	K^*/Y^*	Stability
0%	2.1646	0.5631	0.0332	**0.2611**	0.0444	−0.0112	3.3467	Yes
25%	2.0327	0.5757	0.0346	**0.2536**	0.0431	−0.0085	3.2093	Yes
50%	1.9085	0.5879	0.0361	**0.2461**	0.0418	−0.0057	3.0772	Yes
100%	1.6715	0.6121	0.0394	**0.2305**	0.0392	0.0003	2.8169	Yes
150%	1.4588	0.6355	0.0432	**0.2148**	0.0365	0.0067	2.5725	Yes
200%	1.2674	0.6585	0.0474	**0.1988**	0.0338	0.0136	2.3423	Yes

Table 1: Steady-state results for different values of B^*/Y^*

Parameter	Description	Value	Source
α	Capital share	1/3	Standard
λ	Innovation step size	1.17	Ad hoc
η	R&D productivity per quality	1.4	Ad hoc
σ	Subsidy rate	0.2	Ad hoc
ρ	Pure rate of time preference	0.01	Implied $1/(1+\rho) = 0.99$
μ	Poisson death rate	0.1	Blanchard (1985)
β	Labor productivity depreciation rate	0.25	Ad hoc
Z	Output scaling factor	0.5	Ad hoc

Table 2: Model parameter values

Conclusion

- This project aims to answer:

  Should public R&D subsidies aimed at startups be financed with Public Debt?

- Built a model that:
  1. Studies effects of Debt on Innovation
  2. Takes into account capital crowding out effects
  3. Allows $r < g$ ($\Leftrightarrow$ Debt Sustainability with T=0)

- Model: Debt $\downarrow$ $\Rightarrow$ Innovation $\uparrow$ $\Rightarrow$ $(r - g)\downarrow$

Next Steps

Theoretical:

1. Debt-Financed Subsidies (Policy Counterfactuals)
2. Welfare analysis (Krueger and Ludwig 2016)
3. Stochastic Environment (Blanchard 2019) (separate r^f and r and remove β)

Empirical:

- Calibration (η, λ) to estimate elasticities of $\phi, (r - g)$ on debt-financed subsidies (Using aggregates of SMEs - Orbis, EPO)

Solve Issues:

- Not as analytical as I hoped
- Dependent on β (will be solved with stochastic environment)

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Appendix

The dynamic equations that describe the economy are

$$\dot{k}(t) = -Z(\lambda-1)\frac{\alpha(1-\alpha)\eta\lambda}{1-\sigma}k(t)^{\alpha+1} + Z\left(1-\frac{\alpha(1-\alpha)\lambda}{1-\sigma}+(\lambda-1)\alpha^2\right)k(t)^{\alpha} + \frac{Z\alpha^2}{\eta}k(t)^{\alpha-1} - c(t) \quad (28)$$

$$\dot{c}(t) = \left[-Z(\lambda-1)\frac{\alpha(1-\alpha)\eta\lambda}{1-\sigma}k(t)^{\alpha} + Z\alpha^2\lambda k(t)^{\alpha-1}\right]c(t) + (\beta-\rho)c(t) - (\mu+\beta)(\mu+\rho)[k(t)+b(t)] \quad (29)$$

$$\dot{b}(t) = \left[r(k(t)) - (\lambda-1)Z\left[\frac{\alpha(1-\alpha)\eta\lambda}{1-\sigma}k(t)^{\alpha} - \alpha^2 k(t)^{\alpha-1}\right]\right]b(t) + \frac{\sigma}{\eta}Z\left[\frac{\alpha(1-\alpha)\eta\lambda}{1-\sigma}k(t)^{\alpha} - \alpha^2 k(t)^{\alpha-1}\right] - \tau(t)(1-\alpha)Z\,k(t)^{\alpha} \quad (30)$$

▸ Back to Talk

Appendix - Sustainability

The solution to Governments ODE is

$$b(T) = e^{\int_t^T \left[r(u) - g(k(u);\sigma)\right] du} \times \left[b(t) - \underbrace{\int_t^T e^{-\int_t^u \left[r(v) - g(k(v);\sigma)\right] dv} \left(\tau(u)\,(1-\alpha)\,Z\,k(u)^{\alpha} - \frac{\sigma}{\eta}\,\phi\Big(k(u);\sigma\Big) \right) du}_{\text{PDV of Future Surpluses}} \right]$$

ODE converges if:

- $r < g$ (in this case $\tau \to 0$)
- $b(t) \leq$ PDV of Future Surpluses

▸ Back to Talk

Appendix - Steady State Taxes

At the Steady state $k^* = k(b^*, \sigma)$ with $k_{b^*} < 0$ and $k_\sigma < 0$.

Steady State GBC:

$$0 = \Big[r(k(b^*,\sigma)) - g(k(b^*,\sigma);\sigma))\Big] b^* + \frac{\sigma}{\eta}\phi(k(b^*,\sigma);\sigma) - \tau^* w(k(b^*,\sigma)) \Rightarrow$$

$$\tau^* = \frac{\Big[r(k(b^*,\sigma)) - g(k(b^*,\sigma);\sigma))\Big] b^* + \frac{\sigma}{\eta}\phi(k(b^*,\sigma);\sigma)}{w(k(b^*,\sigma))}$$

At SS, if b^* and σ are too low, then $r^* - g^* < 0$, then there is room for $\tau*$ to decrease at the steady state with an increase in b^*, σ. (**could be also set to $\tau^* = 0$ for very low debts, as in that case r<g**)

Appendix - Complete Phase Diagram

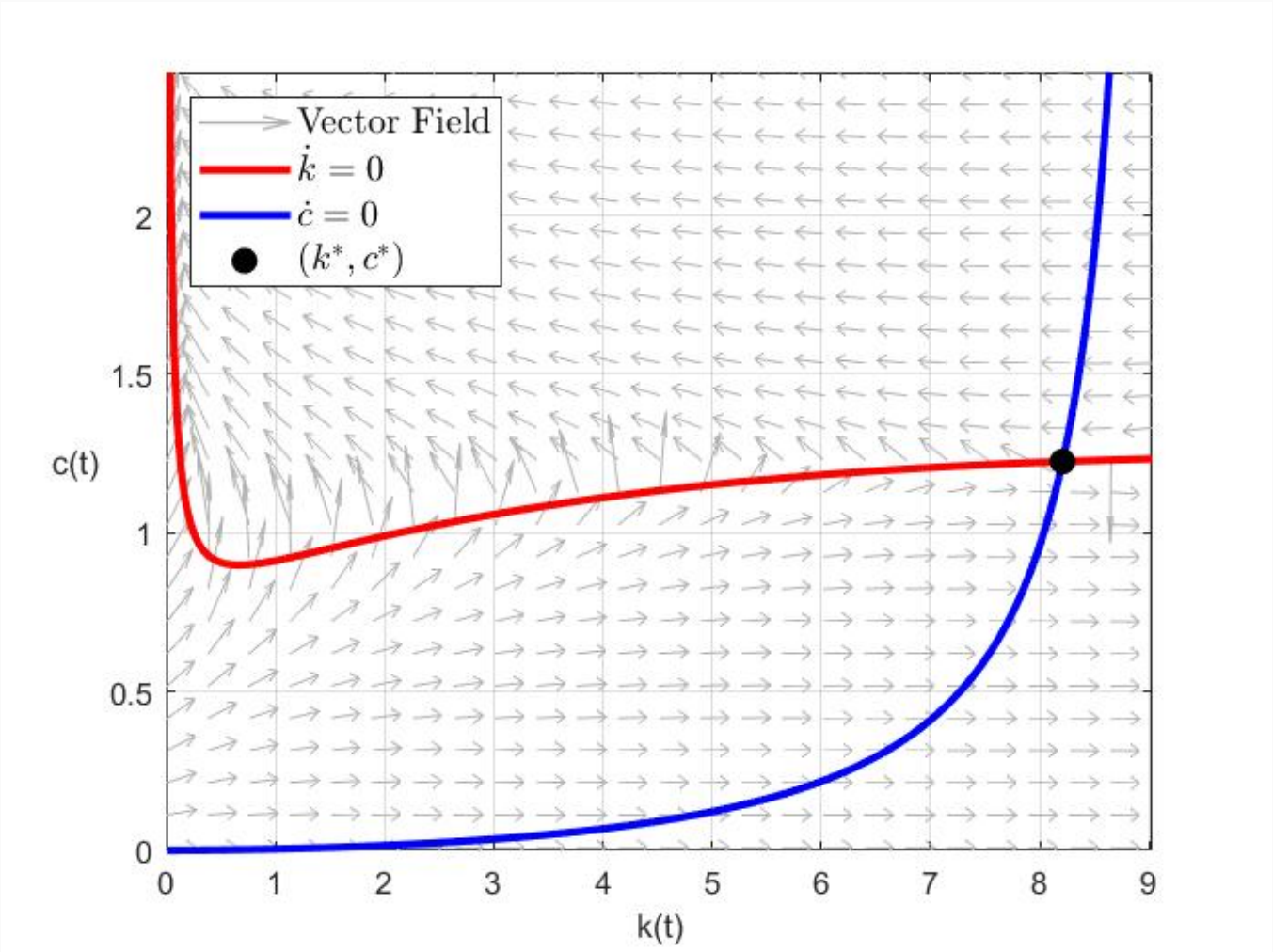

Figure A.1: Untrimmed Phase Diagram

Appendix - $r - g$ for different σ

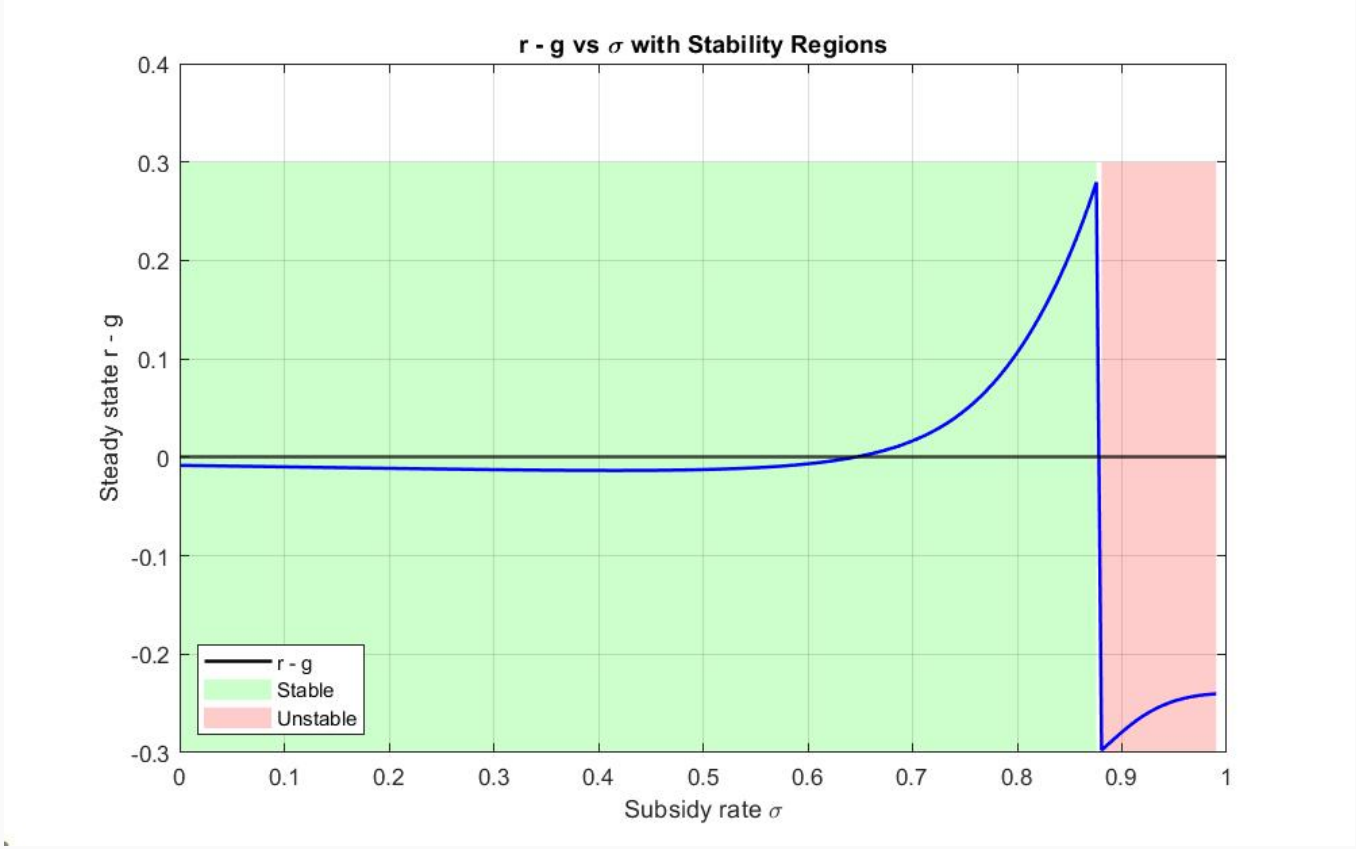

Figure A.2: Effect of different σ to $r - g$ (under fixed b^*)