Influence and Counter-influence in Networks

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Motivation

- A game of influence competition where agents in the network also play a role
- Their vote depends on how they have been influenced as well as what their neighbors believe

- Could be opinions, products…

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- Situations where two players compete to influence agents.
  - Political leader who wants to get a particular bill passed
  - Imagine the head of an organization who can shape how agents interact as well as their opinions

Voting

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- The incumbent wants members to vote in favor of the bill.
- The incumbent has the ability to influence the opinion of party members,
- But also shape the interaction pattern between party members by organizing appointments to committees, or work meetings

- The opposing leader may wish to sway the members in his favor or vote against the bill.

Robbers and cops

- Consider criminals (agents) who belong to an organization, with a Boss (Player 1) that can be captured by the Police (Player 2).

- The Boss wants the agents to be loyal while the Police wants them to be disloyal.

- The agents have two choices: be ***loyal*** to the organization or be ***disloyal***.

Robbers and cops

- The Boss can shape the pattern of interactions between the members because he can use physical coercion to enforce what they should do.

- The effectiveness of such measures depends on his ability to

  -use surveillance + punishment and

  - convince all that disloyal members will be punished.

- Alternatively, the Boss can exert surveillance and pressure only on specific agents and create a network among them to spread the opinion that his surveillance capacity is high and punishment will happen.

Model overview

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- *In* makes two decisions: she sets up the network by forming links between agents and influences a subset of them.
- Then, *Ci* choses the agents he will influence. *Forming links and exerting influence are costly activities*.

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- The influence activities of *In* and *Ci* determine the initial opinion of agents.
- The agent's *final opinion* is a convex combination of his initial opinion and the opinions of his neighbors.
- Finally, each agent votes on the basis of his final opinion – outcome of game is determined.

Related literature

- Closely related to several strands in the networks literature:
  - Centralized network protection is orchestrated by a designer, mirroring the structure of our model: Dziubinski and Goyal (GEB, 2013, TE, 2017), Goyal and Vigier (REStud, 2014), Hoyer and De Jaegher (JPET, 2016), Bravard et al. (JMaE, 2016).
  - Decentralized protection carried out by individual agents within the network: Cabrales, Gottardi and Vega-Redondo (RFS, 2017), Baccara and Bar-Isaac (REStud, 2008), Acemoglu et al. (JET, 2016), Haller and Hoyer (JEBO, 2019).

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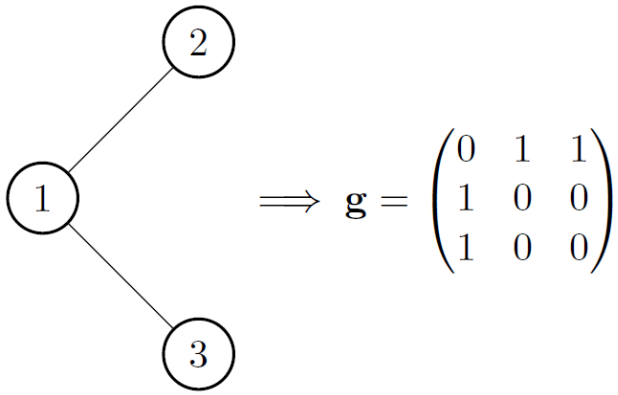
  - Spread of misinformation using social media to study diffusion: Bloch et al. (IER, 2018), Bravard et al. (GEB, 2023), Acemoglu et al. (REStud, 2024).
  - Strategic influence in networks studies similar questions with different objectives: Grabisch et al. (2018). Comola et al. (2024)

Model setup

- Two players: Influencer (*In*) and Counter-influencer (*Ci*) {strategic}
- Set of Agents: $\mathcal{N} = \{1, \dots, n\}$ $n \geq 4$ {non-strategic}
- Network: g is a pair $(\mathcal{N}, E(g))$
- g is undirected making $A(g)$ symmetric.



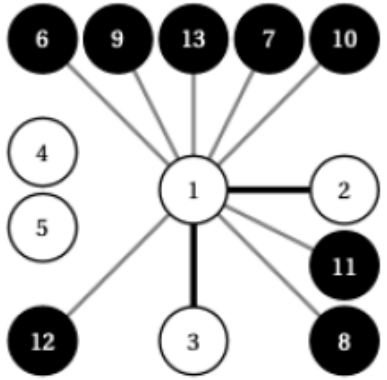
Model setup

- $\mathcal{N}_i(g)$: player i's neighbors
- Subnetwork: Network defined on a subset of the agents
- Path, Cycle, Path length, Geodesic
- Complete and Empty network

Model setup: Special networks

- Partial star: Partition $\mathcal{N}$ into sets $\mathcal{X}$ and $\mathcal{N}/\mathcal{X}$ where $g[\mathcal{X}]$ is a star and $\mathcal{N}/\mathcal{X}$ are isolated agents.

- Special type of partial star: $g(p, \mathcal{Y})$ where $\mathcal{Y}$ contains the central agents, the isolated agents and p peripheral agents. The agents in $\mathcal{N}/\mathcal{Y}$ are also peripheral agents.



(a) $(2, [\![1,5]\!])$-partial-star, g^1

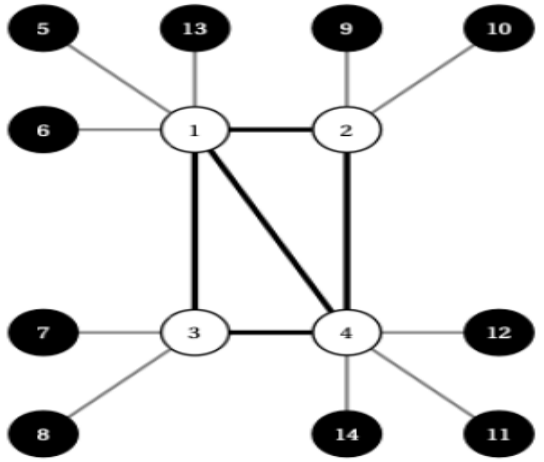
Model setup: Special networks

- g is a $(q, \mathcal{Y})$ – quasi core periphery network if it partitions $\mathcal{N}$ into two sets: $\mathcal{Y}$ (the core) and $\mathcal{N}/\mathcal{Y}$ (periphery)

(i)Every agent in the core has at has at least q times more neighbors that belong to $\mathcal{Y}$ than neighbors that belong to $\mathcal{N}/\mathcal{Y}$

(ii)Each agent in $\mathcal{N}/\mathcal{Y}$ is connected to only one agent that belongs to $\mathcal{Y}$

A minimal-QCP network is a quasi-core periphery network with fewest links.



(b) $(1, [\![1,4]\!])$-MQC network, g^2

Model setup: Strategies

- In's strategy: (g, I_{In}) : g is the network created by In and the set of agents influenced by In

- Ci's strategy: I_{Ci} the set of agents influenced by Ci.

- In wants to the agents to believe 1
- Ci wants the agents to believe 0.

Model setup: Opinions

- Initial opinion:

$$\theta_i = \begin{cases} 1 & if \quad i \in I_{In} \\ 0 & if \quad i \in I_{Ci} \backslash I_{In} \\ \emptyset & otherwise \end{cases}$$

- After agent i's neighbors are influenced by both In and Ci, he has the average opinion of his neighbors: Θ_i

Model setup: Opinions

- Final opinion:

$$\theta_i^F = \begin{cases} (1-\alpha)\theta_i + \alpha\boldsymbol{\Theta_i} & if\ i\ and\ her\ neighbors\ have\ been\ infleunced\ by\ both \\ \boldsymbol{\Theta_i} & if\ i\ has\ not\ not\ been\ influneced\ but\ neighbors\ have \\ \theta_i & if\ only\ i\ has\ been\ influnced\ but\ not\ the\ neighbors \end{cases}$$

Note: $\alpha \in \left(\frac{1}{2}, 1\right]$.

What if an agent has been influenced by both players?

Model setup: Voting

- Agent's vote:
  - They vote for 1 if $\theta_i^F \geq 1/2$
  - They vote for 0 if $\theta_i^F < 1/2$
  - They abstain if $\theta_i^F = \emptyset$

- In wins iff all agents vote 1: unanimous vote [benchmark model]

Model setup: Payoffs

- Influencer's payoffs:

$$u(In) = \begin{cases} 1 - C(\#E(g), \#I_{In}) & if\ In\ gets\ all\ the\ votes \\ -C(\#E(g), \#I_{In}) & otherwise \end{cases}$$

Note: $C(\#E(g), \#I_{In})$ is strictly increasing and concave.

Also, we allow I_{In} to use all her possible strtagies: $C\left(\frac{n(n-1)}{2}, n\right) < 1$

Model setup: Payoffs

- Counter-influencer's payoffs:

$$u(Ci) = \begin{cases} 1 - c_{Ci} \times \#I_{Ci} & if\ In\ does\ not\ get\ all\ the\ votes \\ -c_{Ci} \times \#I_{Ci} & otherwise \end{cases}$$

Note: c_{Ci} is linear and

$k_{Ci} = n$ (Ci's influence capacity)

- Equilibrium concept: *Subgame Perfect Nash Equilibrium (SPNE)*

Model setup: Timing of moves

- **Stage 1**: Influencer builds the network and influences agents
- **Stage 2**: Counter-influencer observes Influencer's strategy and influences a subset of agents
- **Stage 3**: Given their initial opinion, agents interact with neighbors, form their final opinion and vote; the two players receive their payoffs.

Solving the Model: Stage 2 intuition

- Suppose the network is fixed, only the opinion of neighbors' matters ($\alpha = 1$) AND In has the better influence technology. In an SPNE:
  - If g is the empty network, In influences all the agents and Ci influences no one.
  - If g is the complete network, In influences $(1 + n)/2$ agents and Ci influences no one.

Every agent has $n - 1$ neighbors. We need $\boldsymbol{\Theta_i} \geq 1/2$ Since $\boldsymbol{\Theta_i} = \frac{\#I_{In}-1}{n-1} \geq 1/2,$ it follows that In must influence $\frac{n+1}{2}$ agents.

Solving the Model: Stage 2 intuition

- Suppose the network is fixed, only the opinion of neighbors' matters ($\alpha = 1$) AND In has the better influence technology. In an SPNE:
  - If g is a star and i_c is the center of the star. Then In influences i_c and half of her neighbors. Ci influences no one.

If In does not influence i_c, then all the neighbors of i_c will vote 0.

Also, In must influence as many neighbors of i_c as Ci influences to obtain $\boldsymbol{\theta_i} \geq 1/2$. In other words of the maximal number of agents Ci can influence is $k_{Ci} = 2$, then In needs to influence 2 peripheral agents besides the center.

Stage 2 takeaways

- Every agent influenced by In has at least as many neighbors that are influenced by In as those who are not.

- Agents not influenced by In have to be linked with agents influenced by In. In fact, In would prefer that agents who are not influenced by her to not be connected with each other.

$\Rightarrow In$ has no incentive to create links between agents she does not influence. So, the complete network will never be an SPNE.

- Ci's capacity to influence, k_{Ci} plays an important role in In's decision

Key result

Theorem: Given n, α, and the Counter-influencer's influence capacity, the following strategies constitute an SPNE:

1. Form the empty network and influence all agents
2. Form a partial star with a minimum number of agents that need to be influenced, and the number of influenced peripheral agents must exceed Ci's influence capacity
3. Form a minimum quasi-core periphery networks with a sufficient number of agents in the core and influenced agents have to exceed a threshold
4. The Counter-influencer forms no links.

Allowing random links

- Suppose unlinked agents have a probability ω of getting linked.
- ω is iid and it is common knowledge.
- Costs are linear.
- **Propsn**: *Suppose* $\omega \leq \frac{c_L}{4n}$. *Then the optimal strategies are the same as those given by Theorem 1.*

Allowing random links

- **Implications**:
  - When ω is very low, Theorem 1 is preserved, and the outcome will depend on the relative magnitudes of c_L and c_{In}
  - When ω is very high, then In forms no links and influences all the agents. Whether In has lower costs relative to the benchmark model depends on the relative magnitudes of c_L and c_{In}
  - When ω is in the intermediate range, the outcome is one where some links are formed and some agents are influenced; this depends on the relative magnitudes of c_L and c_{In}

Securing a majority of the votes

- Suppose In only needs to obtain a majority of the votes.

- **Implications**:

  - In needs fewer resources than the benchmark model to win the game.

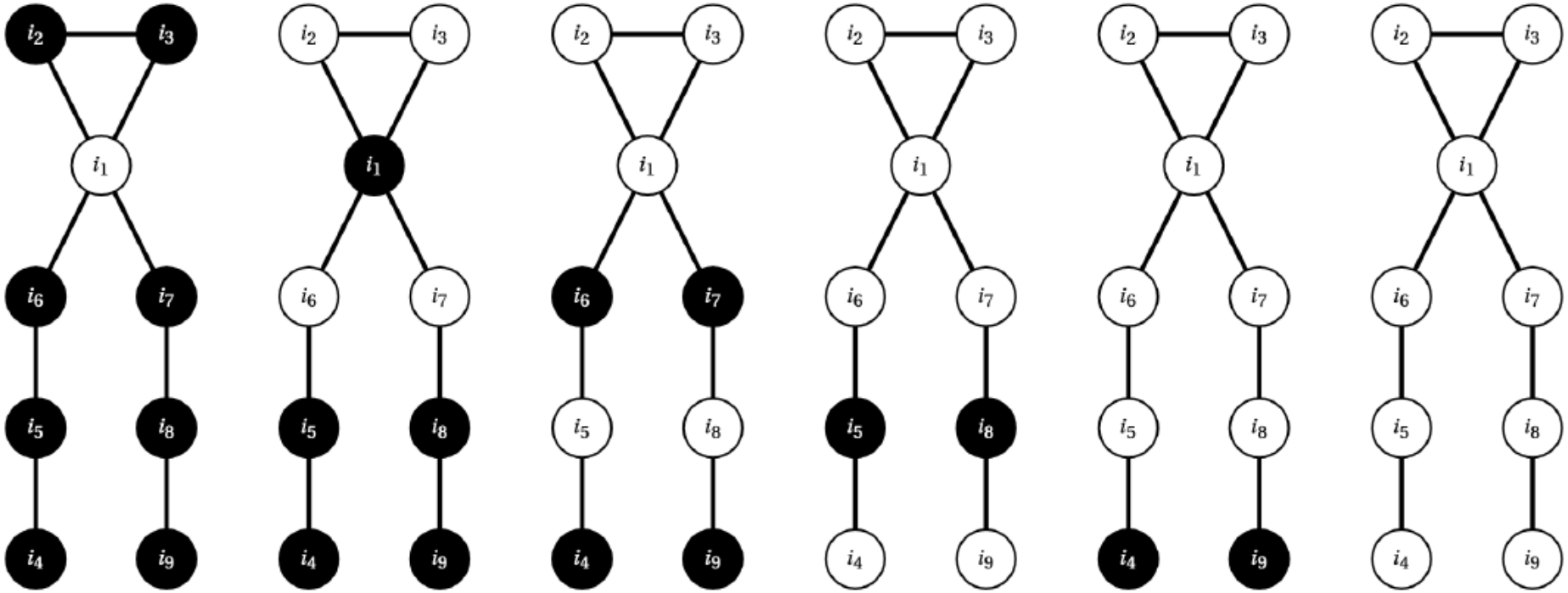
  - In can win the game without having to take into account (in some cases) how many agents Ci can influence.

Agents interact for multiple periods

- Agents they vote in each period, observe the outcome (straw vote). They cast the final vote (one that counts) in the last period.

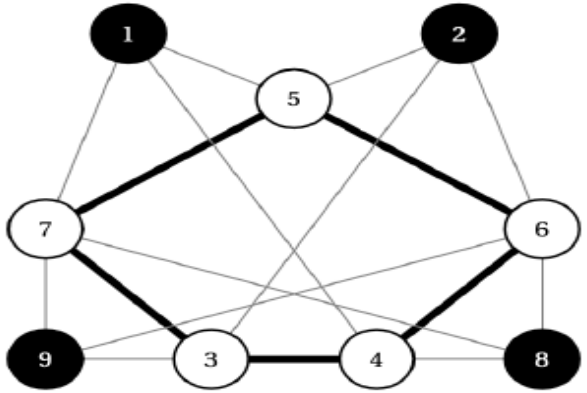
- In general, In needs fewer resources to win the game.

- In fact, In can win by influencing only 1 agent and building the right network.

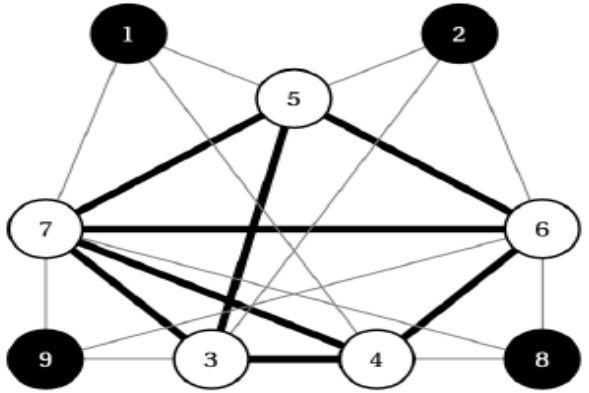

i_2 i_3 i_1 i_6 i_7 i_5 i_8 i_4 i_9
$t=0$
$t=1$
$t=2$
$t=3$
$t=4$
$t=5$

Adversary has better influence technology

- Equilibrium networks are quite different: they are either regular networks or contain regular networks



(b) a $(2,3,[\![3,7]\!])$-GR network



(c) a $(3,3,[\![3,7]\!])$-GR network

Note: If the Counter-influencer can also influence a large number of agents than the Influencer has no winning strategy

Key takeaways

- When the Influencer has the better technology: centralized architectures
- When the Counter-influencer has the better technology: symmetric architectures
- When the network can be altered, there is a range where the main results hold

Key takeaways

- When the Influencer has the better technology: centralized architectures
- When the Counter-influencer has the better technology: symmetric architectures
- When the network can be altered, there is a range where the main results hold
- When agents can interact repeatedly, the Influencer has to influence fewer agents (1 will suffice)
- Majority voting requires fewer resources, and the Influencer does not have to worry about how many agents the Counter-influencer can influence
- When the Counter-influencer has the better influence technology, the power to shape the interaction pattern between agents may not be sufficient

Thank You

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