Time Flies: Cooperation in Repeated Finite Time-Horizon Games

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Cooperation in dynamic setups

- Price wars in IO vs tacit collusion
- Cooperation in the absence of formal contracts and court systems
- Managers-employees relationships when not everything is contractible
- Trade wars
- Informal (and to some extent formal) finance
- Social contract in political economy
- Fighting global warming
- ...

Cooperation in Repeated Games - What do we know?

		Player B	
		C	D
Player A	C	$(4,4)$	$(0,5)$
	D	$(5,0)$	$(1,1)$

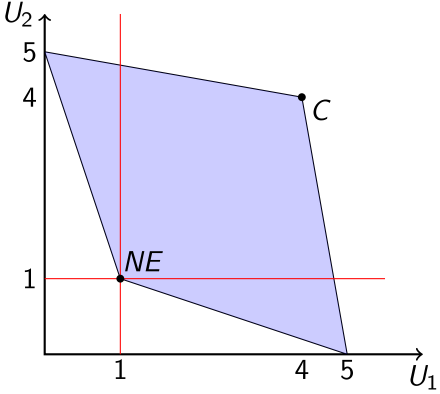
- PD with arbitrary payoffs. Only NE $\Rightarrow (D,D)$
- Yet cooperation can be sustained with (infinite) repetition: **Folk theorem**
- δ the discount rate; cooperation sustainable with Grim trigger:

$$4+\delta 4+\delta^2 4+\ldots\delta^{\infty}4>5+\delta+\delta^2+\ldots+\delta^{\infty}$$

$$\Leftrightarrow \frac{4}{1-\delta}>5+\frac{\delta}{1-\delta}\Leftrightarrow \delta>\frac{1}{4}$$

Cooperation in Repeated Games

The **folk theorem** is much more general: *Any vector of feasible and individually rational (minimax) payoffs can be attained as the average discounted payoff vector of a SPNE if δ is high enough*



Cooperation in Repeated Games

- But the Folk theorem collapses if we lift that $t \to \infty$ assumption!
- Backward induction: you deviate in T, so you deviate in $T-1$, $\ldots$ you deviate in $t = 1$.
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- Result can be “saved” if the last period of the game is unknown: there can always be an $t+1$ period meaning the game is never actually terminated (conceptually equivalent to see δ as termination probability; the game though could <u>never</u> end)

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- But what about <u>really finite games</u>?

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- In real life we do observe cooperation, even when we know the game will end at some point
- Has been rationalized with behavioural models; altruism, reciprocity, revenge, etc... (can even support cooperation in one-shot setups)
- We propose an alternative explanation based on how fast time passes

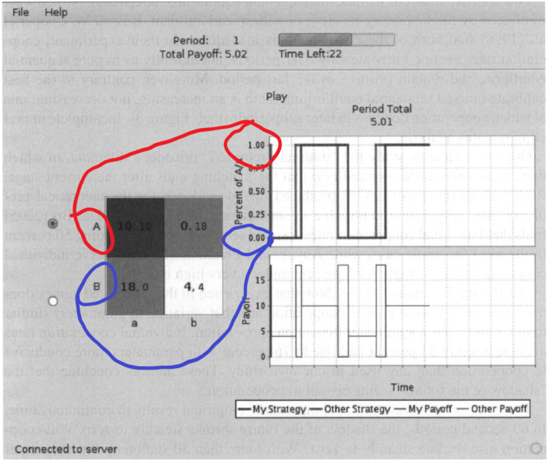
Cooperation in Finite Time Repeated Games - a Hint

Friedman and Oprea (2012)

- Lab experiment on the standard Prisoner's dilemma
- 3 treatments all lasting 60 secs:
  - One shot treatment (1 decision made at 60 secs)
  - Grid of 60 decisions (1 sec $=$ 1 decision)
  - Continuous treatment with decisions implemented in $<$ 50 milliseconds

Cooperation in Finite Time Repeated Games - a Hint

What participants see



Cooperation in Finite Time Repeated Games - a Hint

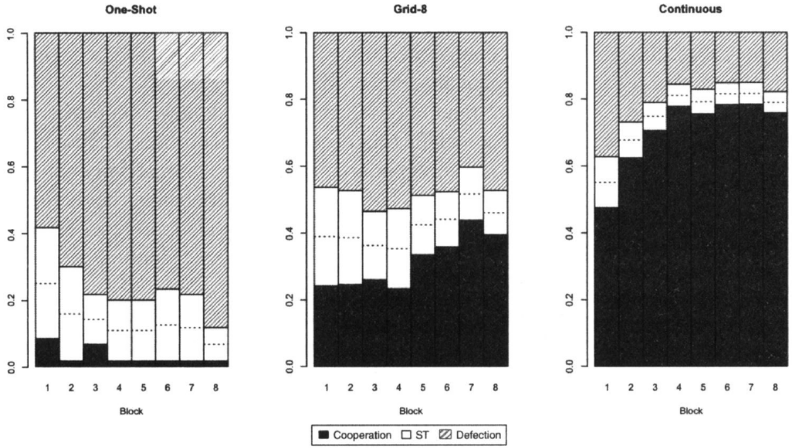


FIGURE 2. OUTCOMES OVER BLOCKS BY TREATMENT

Friedman and Oprea (2012)

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  - If you can *instantaneously* punish the deviator, that will always discipline him!
  - Indeed, as long as the game is "not finished", no matter how close you are from the end, if you deviate, you can slice the remaining time, and punish!

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    - Indeed, as long as the game is “not finished”, no matter how close you are from the end, if you deviate, you can slice the remaining time, and punish!
    - Technical problem: what does instantaneous means in continuous time?

Simon, Stichcombe (1989) & Bergin, MacLeod (1993)

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- Bergin and MacLeod (1993) introduce "inertia equilibria": if time is discrete, modeled with "inertia" periods, and if you can (almost) instantaneously react to a deviation, then you do not deviate
- This makes time "discrete", but as $\epsilon \to 0$, you approximate the continuous reasoning

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Theory predicting asymptotic change in equilibrium; not what their experiment shows (gradual change)

Bigoni et al. (2015)

- Improve on Oprea and Friedman (2012) by allowing for both deterministic and stochastic termination of the game ($\approx$ making it infinitely played)
- Also look at different time horizons (60sec, 20sec).
  - Do confirm Oprea and Friedman (2012)
  - in 20 sec treatments cooperation is higher with deterministic termination (!)
  - "End game" effect
  - Longer games $\Rightarrow$ more cooperation

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- **Theory?** Kreps et al. (1982): uncertainty on strategy played by rivals

Our explanation: **time flies**

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- "textbook" discrete version: you stop the time, observe options and make a decision.
- But if time "flows" continuously rather than experiencing discrete jumps (as in reality and in "continuous time" experiments):
  - even if players know what they should play at each "point in time"
  - may fail to *implement the right decision at the right time* because *they may not know for sure where they are in the game!*

Time flies

- Scan the QR code to convince yourself
- 6 seconds, try hitting the black bar (i.e. $T - 3$)
- Attempt with 6 secs divided in 10 (probably no mistake)
- Refresh the page and try with 50, 100, 200...

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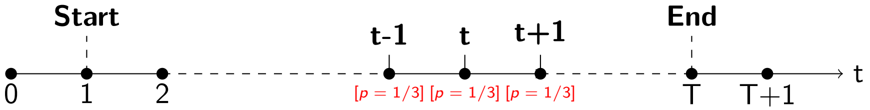
Time flies - a simple illustration

- If we are imprecise, that implies that *we do not know exactly what period we are in* when making a decision
- We could be playing "too late", "on time", or "too early"
- Keep things simple: 1/3 probability of being too early and 1/3 too late
- Perception of where we are when **in period t**



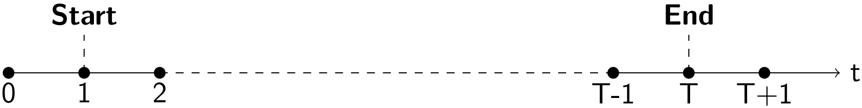
Time flies - a simple illustration

- The game starts with a "**default**" pair of actions (can be any)
- Players can either play the default again and again, or pick a different action
- If either player does not play default in t, in $t+1$ time uncertainty vanishes completely [you now know where you are]
    - Observe we need not make such a strong assumption; could partly vanish, not vanish, or even re-appear

A simple illustration

- **Default:** (C, C) [one possibility]
- We are in T
- No one played D before T

Player A \ Player B	C	D
C	$(4, 4)$	$(0, 5)$
D	$(5, 0)$	$(1, 1)$



Observation If either player chooses D in $T-1$, then in T both players choose D (Non-cooperative equilibrium "survives")

A simple illustration

- **Default:** (C, C)
- We are in T
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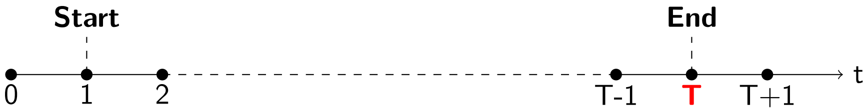

'Deviate' D: $\frac{1}{3}[5 + 1\frac{2}{3} + 5\frac{1}{3}] + \frac{1}{3}[4+5] + \frac{1}{3}[4+4]$

'Cooperate' C: $4 + 4$

'Deviate' D if: $29/3 > 8$ **False**

A simple illustration

- **Default:** (C, C)
- We are in $T - 1$
- No one played D before $T - 1$
- Expect C in terminal node T

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		C	D
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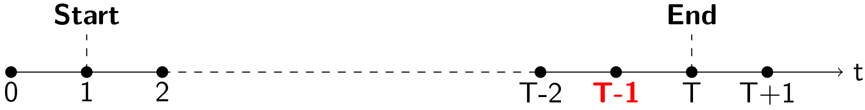


'Deviate' D: $\frac{1}{3}[5+1+1] + \frac{1}{3}[4+5+\frac{2}{3}1+\frac{1}{3}5] + \frac{1}{3}[4+4+5]$

'Cooperate' C: $4+4+4$

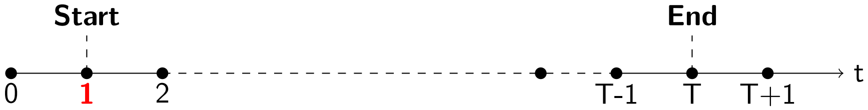
'Deviate' D if: $94/9 > 3 \times 4$ **False**

A simple illustration

- **Default:** (C, C)
- We are in $t = 1$
- Expect C in all $t > 1$

Player B

		C	D
Player A	C	$(4, 4)$	$(0, 5)$
	D	$(5, 0)$	$(1, 1)$



'Deviate' D:

$\frac{1}{3}[5+1+\ldots+1]+\frac{1}{3}[4+5+1+\ldots+1]+\frac{1}{3}[4+4+5+1+\ldots+1]$

'Cooperate' C: $4+4+\ldots+4$

'Deviate' D if: $5+(T-2)1 > (T-1)\times 4$ **False** (the earliest, the lower the incentives to deviate)

Time flies - Generalization and question marks

How to formalize “not knowing where you are”?

- Both players know that they are uncertain as to where they are and that the same holds true for the other player [playing against a perfectly informed player would cancel the result]
- “nature” draws where you are. There are 3 “states” of the world in our earlier example:
  1. your first move is too early; and you don’t come to choose the “last move”
  2. you are just on time
  3. your last move is too late, and you don’t come to choose the “first move”

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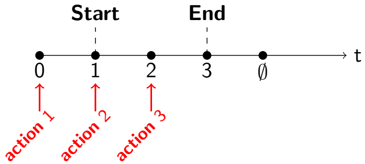
  3. your last move is too late, and you don't come to choose the "first move"
- We assumed immediate reaction of rival; result survives with delayed reaction (again cardinality matters)

Time flies - Generalization and question marks

Probability $1/3$



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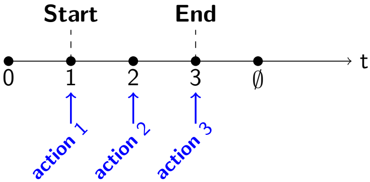

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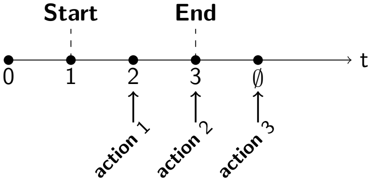

Time flies - The Setup

- 2 players: A and B
- Finite horizon, discrete time game: $\tilde{T} = \{0, \underbrace{1, \ldots T}_{\text{payoffs}}, T+1\}$ periods
  - Periods $K = \{0, T+1\}$ yield no payoff
  - Periods $T = \{1, \ldots T\}$ yield payoffs (below)
- Discount factor: $\delta \leq 1$
- The static game:
  - Player i's action: $x_i \in X_i$, where X_i is the action set
  - $X := \times_{i \in I} x_i$ is the set of action profiles
  - $u_i(x_A, x_B)$ the stage-payoff of player i

Time flies - The Setup

The payoffs - for payoff-relevant time periods

- The simultaneous move static game admits a Nash equilibrium described by (x_{AN}, x_{BN})
- $\exists$ at least 1 pair of strategies $(\dot{x}_A, \dot{x}_B)$ yielding Pareto-superior payoffs:

$$(\dot{U}_A, \dot{U}_B) = (U_A(\dot{x}_A, \dot{x}_B), U_B(\dot{x}_A, \dot{x}_B))$$
$$> (U_{AN}, U_{BN}) = (U_A(x_{AN}, x_{BN}), U_B(x_{AN}, x_{BN}))$$

- Deviation from $\dot{x}_i \Rightarrow \check{x}_j(\dot{x}_i)$, with $U_i(\dot{x}_i, \check{x}_j(\dot{x}_i)) = U_{iD} < U_{iN}$
- Deviation from $\dot{x}_j \Rightarrow \check{x}_i(\dot{x}_j)$, with $U_i(\check{x}_i(\dot{x}_j); \dot{x}_j) = \check{U}_i > \dot{U}_i$

The payoffs - for $t = \{0, T+1\}$

- $U_i^0 = U_i^{T+1} = 0$

Time flies - The Setup

The repeated game - with mistakes

- Game starts with an inherited history: $h^0 := (\check{x}_A, \check{x}_B) \in X$
- Each player plays T periods
- Denote by T^I, the time set of player I
- Three types of players (symmetric incomplete information)
  - Early types (with probability p^E): $T^E = \{0, \dots T-1\}$
  - On-time types (with probability p^O): $T^O = \{1, \dots T\}$
  - Late types (with probability $p^L = (1 - p^E - p^O)$): $T^L = \{2, \dots T+1\}$
- **NB:** w\o information to update belief on actual time period, in $t = k$, a player attributes $\check{p}^E$ of being in $k-1$, $\check{p}^O$ in k and $\check{p}^L$ in $k+1$
- **NB2:** For early type, $x^T = x^{T-1}$.

Time flies - The Setup

The repeated game - the history

- Each player plays T times, and accordingly observes T histories
- Yet, players know they can be mistaken in their perception of timing
- History of the game thus also includes probabilities
- Denote by h^t the game's *true* history where we omit probabilities since $p^O = 1$: $h^t = ((\check{x}_A, \check{x}_B), (x_A^1, x_B^1), \ldots (x_A^{t-1}, x_B^{t-1}))$

Time flies - The Setup

Early types (E). Assume player A is E:

- Unless some update takes place, $(p^E, p^O, p^L) = (\check{p}^E, \check{p}^O, \check{p}^L)$
- In $\tilde{t} = 1$ $(t = 0)$, $\tilde{h}^{1E} = h^0 = (\check{x}_A, \check{x}_B; \check{p}^E, \check{p}^O, \check{p}^L)$
- In $\tilde{t} = 2$ $(t = 1)$
$$\tilde{h}^{2E} = \begin{cases} ((\check{x}_A, \check{x}_B), (\check{x}_A, \check{x}_B); \check{p}^E, \check{p}^O, \check{p}^L) & \text{if } x_A^0 = \check{x}_A \\ ((\check{x}_A, \check{x}_B), (\check{x}_A, \check{x}_B); p^E = 1, p^O = 0, p^L = 0) & \text{otherwise} \end{cases}$$
- In $\tilde{t} = 3$
$$\tilde{h}^{3E} = \begin{cases} ((\check{x}_A, \check{x}_B), (\check{x}_A, \check{x}_B)), (x_A^1, x_B^1); \check{p}^E, \check{p}^O, \check{p}^L) & \text{if } x_A^0 = \check{x}_A \\ ((\check{x}_A, \check{x}_B), (\check{x}_A, \check{x}_B)), (x_A^1, x_B^1); 1, 0, 0) & \text{otherwise} \end{cases}$$
- . . .

Time flies - The Setup

Late types (L). Assume player A is L:

- In $\tilde{t} = 1$ ($t = 2$), $\tilde{h}^{1E} = h^0 = (\check{x}_A, x_B^1; \check{p}^E, \check{p}^O, \check{p}^L)$
- In $\tilde{t} = 2$ ($t = 3$) $\tilde{h}^{2E} = ((\check{x}_A, x_B^1), (x_A^2, x_B^2); \check{p}^E, \check{p}^O, \check{p}^L)$
- In $\tilde{t} = 3$ ($t = 4$) $\tilde{h}^{3E} = ((\check{x}_A, x_B^1), (x_A^2, x_B^2), (x_A^3, x_B^3); \check{p}^E, \check{p}^O, \check{p}^L)$
- ...

Time flies - Equilibrium

Benchmark: $p^O = 1$

- Unique SPNE of this game such that $X^t = (x_{AN}, x_{BN})$, $\forall t \in T$
- Equilibrium payoff of player i: $U_{AN} \sum_{t \in T} \delta^t$

Time flies - Equilibrium

Uncertainty: $(\check{p}^E, \check{p}^O, \check{p}^L)$

- Assume $\{\check{x}_A, \check{x}_B\} = \{\dot{x}_A, \dot{x}_B\}$
- The strategy profile $(x_A^t, x_B^t) = (x_{AN}, x_{BN})$, $\forall t$, is an equilibrium strategy profile
- If rival plays that strategy profile, no profitable deviation

Time flies - Equilibrium

Uncertainty: $(\check{p}^E, \check{p}^O, \check{p}^L)$

- Assume $\{\check{x}_A, \check{x}_B\} = \{\dot{x}_A, \dot{x}_B\}$
- The game admits a SPNE where $(x_A^t, x_B^t) = (\dot{x}_A, \dot{x}_B)$, $\forall t$ if no deviation in last perceived time period
- Consider player A and apply OSDP:
  - No deviation yields $\dot{U}_A \sum_{t \in T} \delta^T$
  - Deviation in *perceived* last period yields:

$$\sum_{t=1}^{T-2} \delta^t \dot{U}_A + p^E(\delta^{T-1} \check{U}_A + \delta^T U_{AN})$$
$$+p^O(\delta^{T-1} \dot{U}_A + \delta^T \check{U}_A) + p^L(\delta^{T-1} + \delta^T) \dot{U}_A$$

Time flies - Equilibrium. **OSDP:**

$$\left(p^E\delta^{T-1} + p^O\delta^T\right)\check{U}_A + p^E\delta^T U_{AN} > \left(p^E(\delta^{T-1} + \delta^T) + \delta^T p^O\right)\dot{U}_A$$

where $\check{U}_A > \dot{U}_A > U_{AN}$

More likely all else equal if:

- the lower $\dot{U}_A$
- the larger $\dot{U}_{AN}, \check{U}_A$
- For fixed p^E the larger p^O
- For fixed p^O the larger p^E *if* $\delta^{T-1}\check{U}_A + \delta^T U_{AN} > (\delta^{T-1} + \delta^T)\dot{U}_A$
- the larger p^E if $\frac{\check{U}_A + U_{AN}}{2} > \dot{U}_A$
- If $\check{U}_A >> \dot{U}_A > U_{AN}$, increasing in δ (otherwise decreasing)

Time flies - Equilibrium

- With uncertainty, and for given payoffs and probabilities (cardinality matters!), two pure strategy equilibria:
  - $(x_A^t, x_B^t) = (x_{AN}, x_{BN}), \forall t \Rightarrow (U_{AN}, U_{BN})$
  - $(x_A^t, x_B^t) = (\dot{x}_A, \dot{x}_B), \forall t \Rightarrow (\dot{U}_A, \dot{U}_B)$
  - There is obviously scope for a mixed strategy yielding an intermediate payoff too (but an ex-ante one!)

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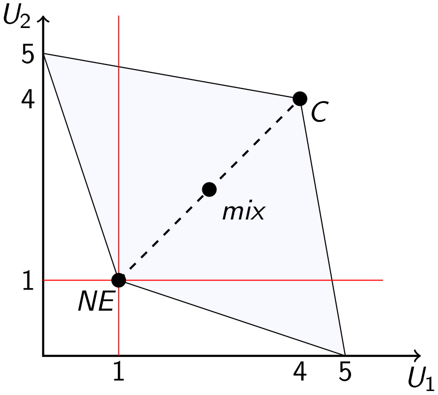
  - There is obviously scope for a mixed strategy yielding an intermediate payoff too (but an ex-ante one!)
- A **Folk Theorem?**
  - No: it can be shown that any strategy deviating from the Pareto-superior strategy implies a total unravelling into the Nash equilibrium play

Cooperation in Repeated Games



Time flies - designing a lab experiment

- Subjects called to play a discrete PD game with finite time horizon.
- The time length of a stage game is fixed
- We overlap (probabilistically) at what time period each player is
  - with prob $1/9$: both players take actions from $t = 0$ until $t = T$
  - with prob $1/9$: player 1 takes actions from $t = 0$ until $t = T$ and 2 from $t = 1$ to $t = T + 1$
  - ...
- If true t is revealed, then total recall may be implemented (cleaner)

Thank you!