Collective Effects of Limited Energy in Voting

Zoi Terzopoulou

GATE, St-Etienne School of Economics
University of Lyon/St-Etienne

Collective Decisions

Classical Social Choice Theory

- Agents $N = \{1, ..., n\}$.
- Alternatives $A = \{a, b...\}$.

- Every agent i holds a preference $\succ_i$ over all alternatives in A. This is a linear order (a transitive and complete binary relation).
- A profile $\succ = (\succ_1, ..., \succ_n)$ collects the preferences of all agents.

- A voting rule takes as input a profile of preferences and outputs a winning alternative in A. $F : \succ \mapsto x$.

Two Famous Voting Rules: Plurality

1	a	a	a	d	d
0	b	b	b	b	b
0	c	c	c	c	c
0	d	d	d	a	a

Winner is a!

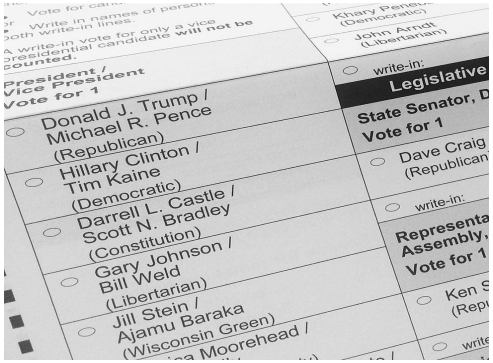
Two Famous Voting Rules: Borda

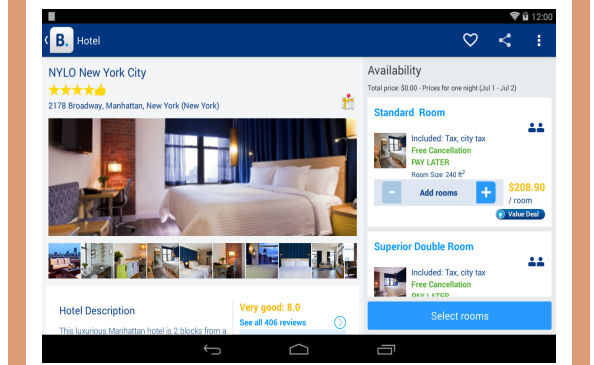
3	a	a	a	d	d
1	b	b	b	b	b
−1	c	c	c	c	c
−3	d	d	d	a	a

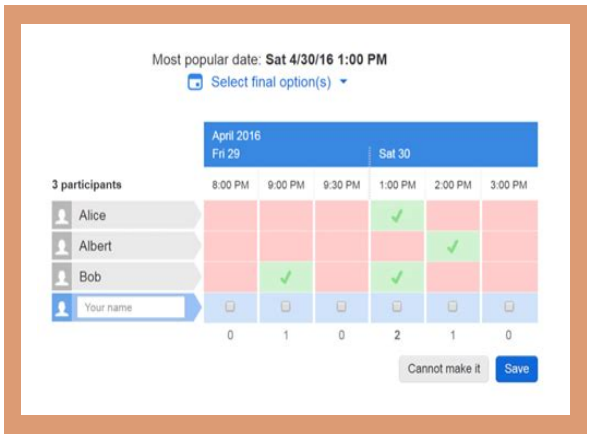
Winner is b!

A Problem of Classical Social Choice

Agents may not examine the whole set of alternatives due to limited time and energy, or other cognitive constraints.

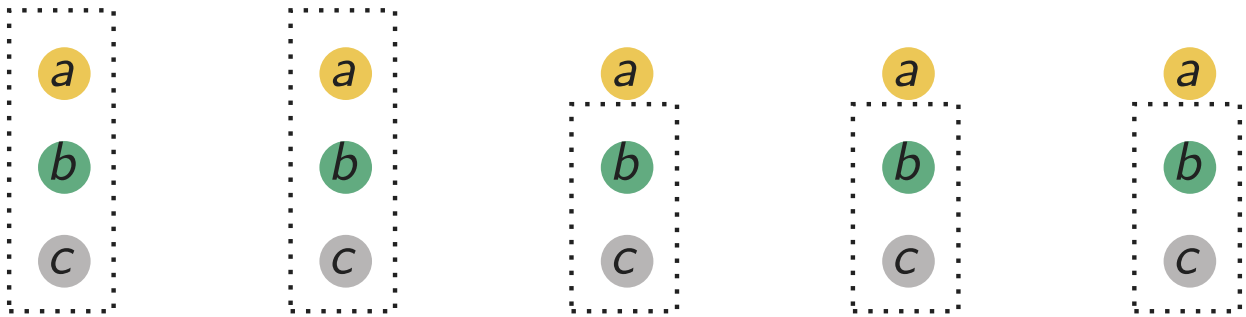






Negative Effects of Limited Energy

Suppose the alternatives are presented as: *b*, *c*, *a*.



- How much does limited energy harm collective outcomes?
- In what order should the alternatives be presented to achieve an outcome similar to the intrinsic one?

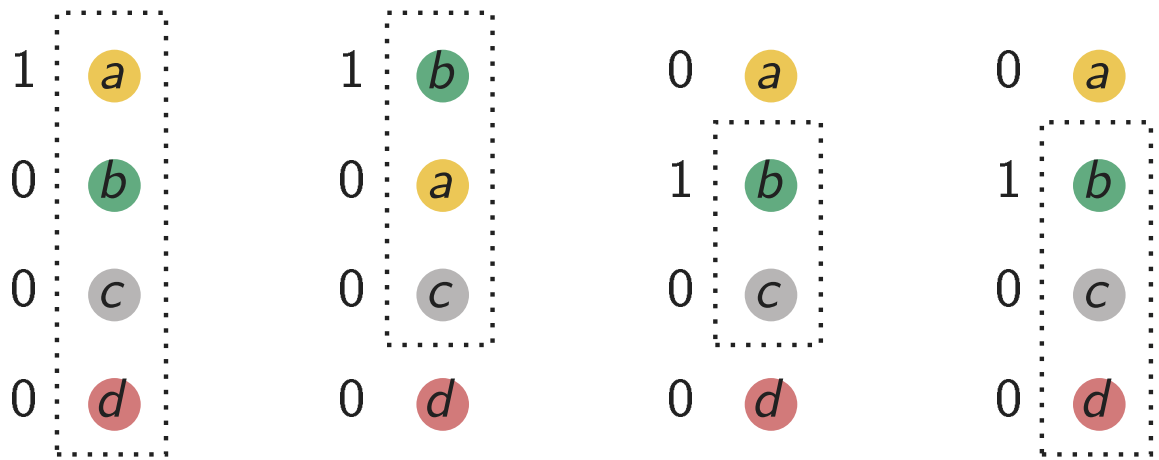
Our Model

- Agents $N = \{1, ..., n\}$.
- Alternatives $A = \{a, b...\}$.
- Energy function $e : N \to \{2, ..., |A|\}$
- Show order function $o : N \to \Pi(A)$, where $\Pi(A)$ the set of permutations on A.
- Every agent i holds an intrinsic preference $\succ_i$ over A.
- Every agent i only examines $A(e_i, o_i)$: the first e_i alternatives of o_i.
- Every agent i's reported preference is $\succ_i^{e,o}$ (a transitive relation):

$$x \succ_i^{e,o} y \text{ if and only if } x \succ_i y \text{ and } x, y \in A(e_i, o_i)$$

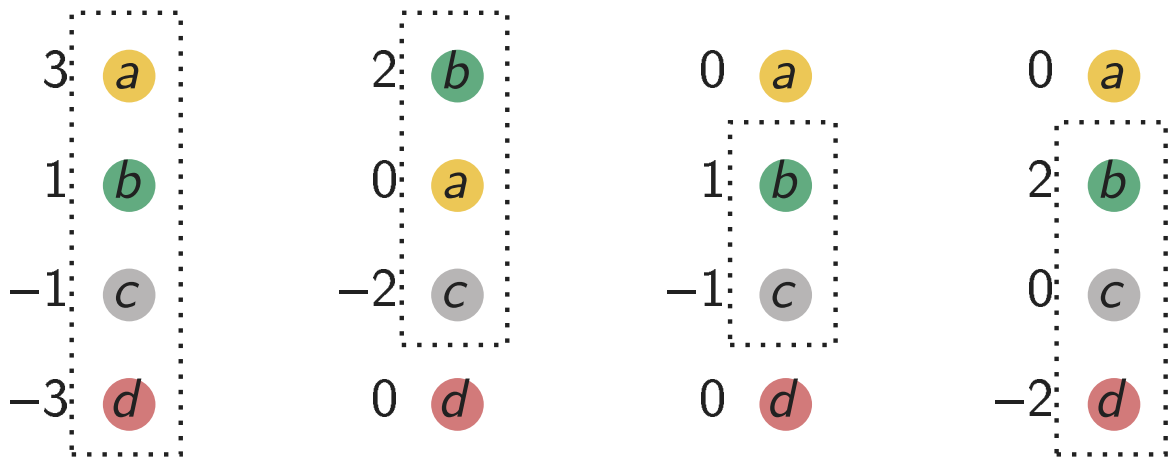
- A voting rule takes as input a profile of (possibly incomplete) preferences and outputs a winning alternative in A. $F : \succ^{\boldsymbol{e,o}} \mapsto x$.

Two Adjusted Voting Rules: Plurality



Winner is *b*!

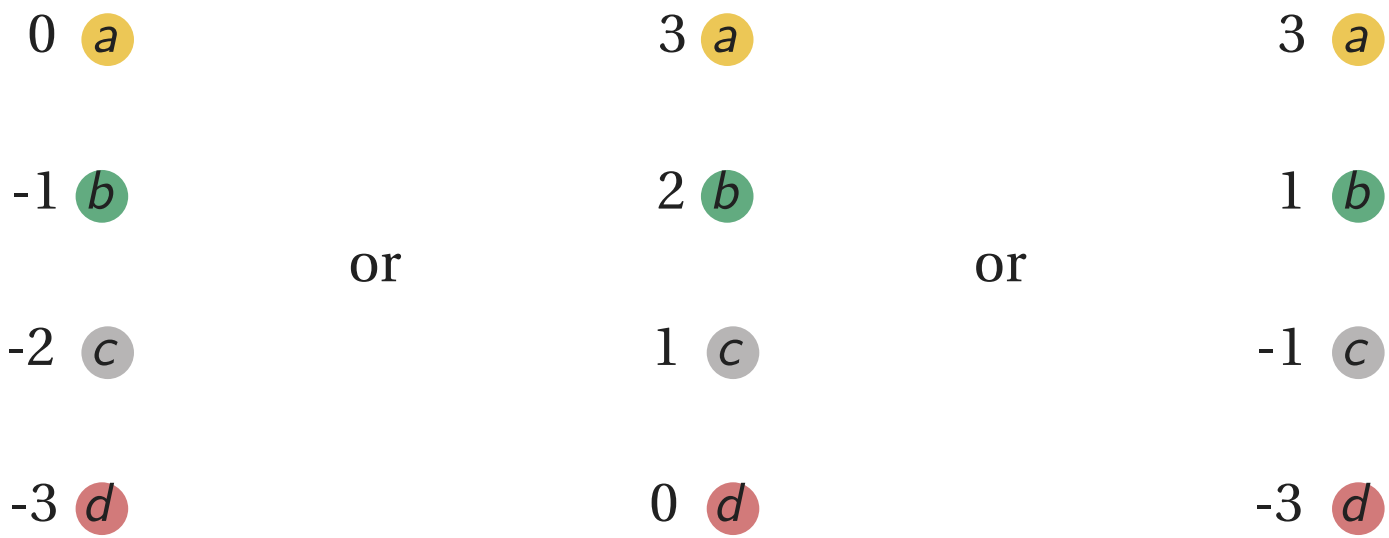
Two Adjusted Voting Rules: Borda



Winner is b !

Measuring Collective Consequences

When x wins, agent i gains welfare based on the position of x in $\succ_i$.
We use a regret-based notion of social welfare for $w_i(x)$:

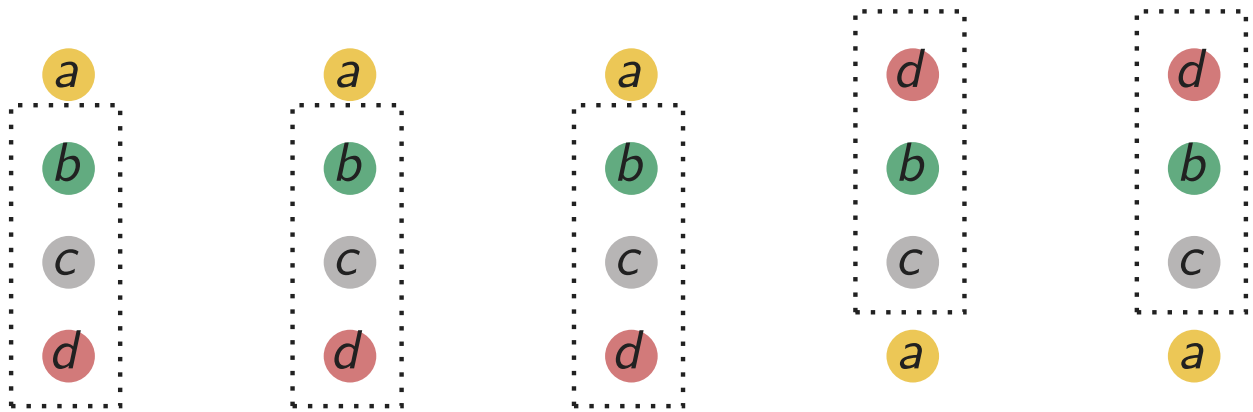


Indeed, this is equivalent to the Borda rule: $SW(x) = \sum_{i \in N} w_i(x)$

Limited and Full Energy: Comparison

$SW(Borda(\succ)) \geqslant SW(Borda(\succ^{e,o}))$
for **all** profiles $\succ$, energy and show order functions e, o.

$SW(Plurality(\succ^{e,o})) > SW(Plurality(\succ))$
for **some** profile $\succ$, energy and show order functions e, o.



Possibility of Preserving the Intrinsic Outcome

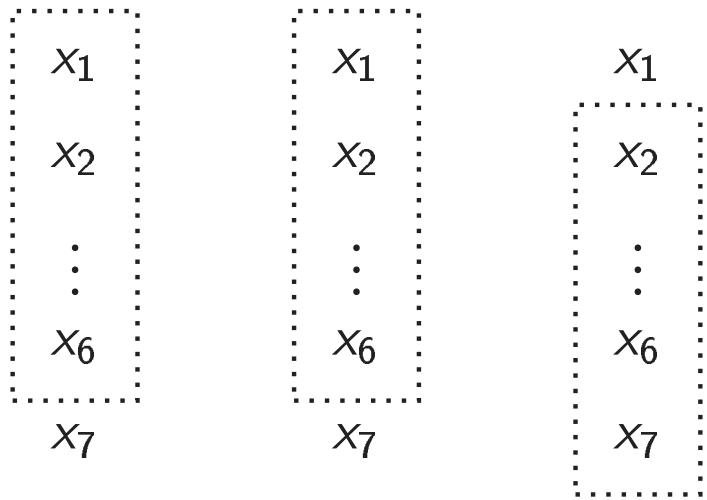
We need to choose the appropriate show order.

For **all** profiles $\succ^{e,o}$ and all energy functions e, it holds that $\mathit{Plurality}(\succ^{e,o}) = \mathit{Plurality}(\succ)$ for some show order o.

For **some** profile $\succ^{e,o}$ and energy function e, it holds that $\mathit{Borda}(\succ^{e,o}) \neq \mathit{Borda}(\succ)$ for all show orders o.

Comparing Two Rules under Limited Energy

$SW(Plurality(\succ^{e,o})) > SW(Borda(\succ^{e,o}))$
for **some** profile $\succ$, energy and show order functions e, o.



Note: Plurality is not so sensitive as Borda to small energy changes.

Worst-case Scenario

We define the Price of Limited Energy:

$$PLE(F, e) = \max_{\succ, o} \frac{SW(F(\succ))}{SW(F(\succ^{e,o}))}$$

Let F be the Plurality rule or the Borda rule. Then:

$$PLE(F, e) = \begin{cases} 1 & \text{if } e_i = m \text{ for all } i \in N \\ \alpha > 1 & \text{otherwise} \end{cases}$$

$PLE(Plurality, e) > PLE(Borda, e)$
for every non-trivial energy function e.

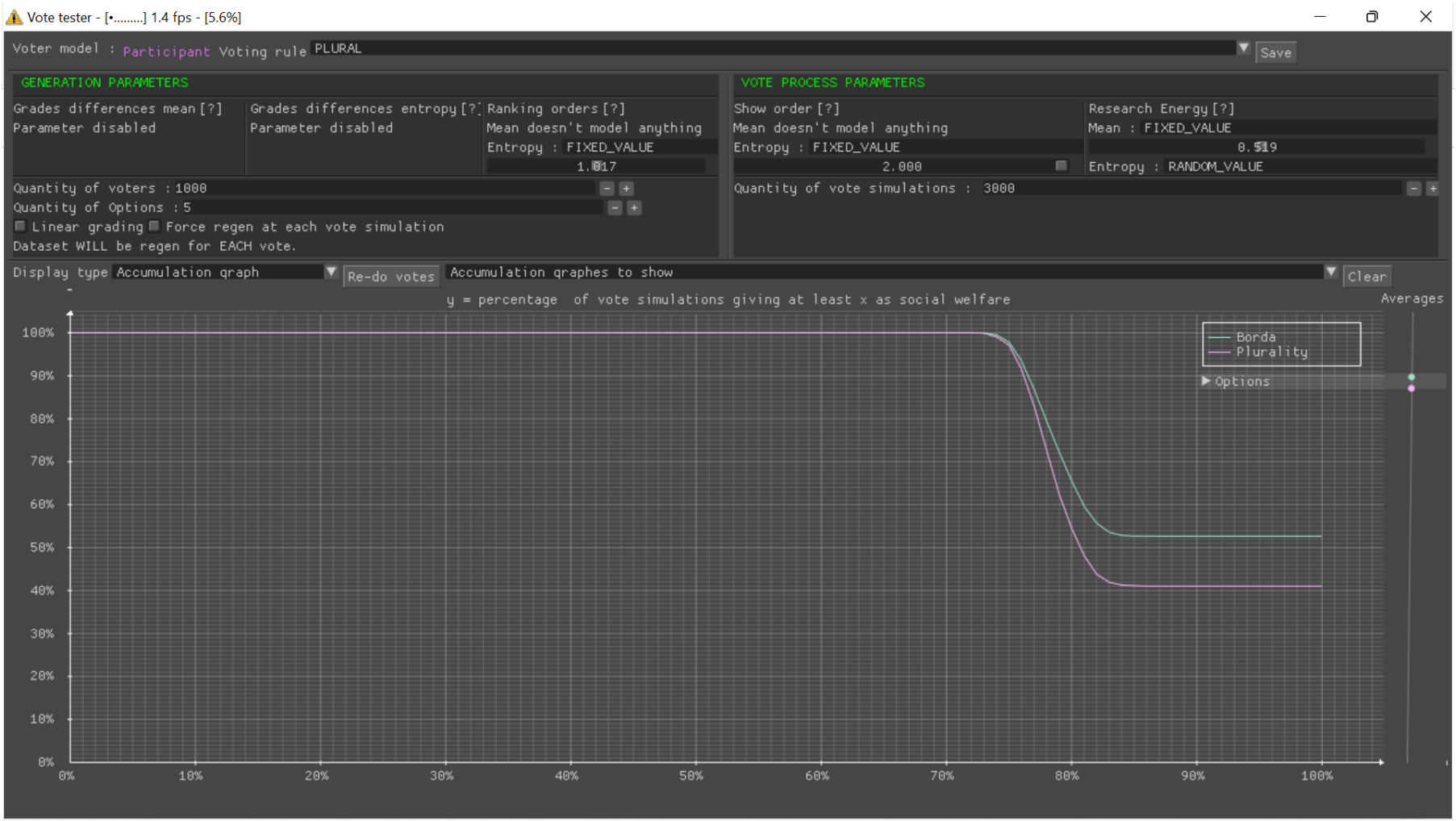
Numerical Simulations

We vary several parameters for the effects of limited energy in voting:

- The distribution of the intrinsic preference profiles (Coherent/Similar/Diverging).
- The voting rule to be applied (Plurality/Borda).
- The distribution of the energy limits of the agents (High/Medium/Low).
- The distribution of the show orders of the agents (Coherent/Similar/Diverging).

The App

Thanks to SpirOps-AI in Paris!



Summary of Experimental Results

- Choosing the show order entropy.
  - Similar is always better than Diverging for Plurality, and Similar is always better than Coherent for Borda.
  - High Energy or Coherent profiles: Similar is best for both rules.
  - Medium/Low Energy and Similar/Diverging profiles: Diverging is best for Borda and Coherent is best for Plurality.
- The Plurality rule can achieve (close to) optimal outcome for High energy, or Medium energy and Coherent profiles.
- Plurality > Borda for optimal orders (except Low Coher. Energy).
- Performance for different profile distributions. For both rules, High/Medium Energy performance is better in more coherent profiles. For low energy, not so clear.

Conclusion

- We need realistic assumptions in social choice theory: Incompleteness is one of them.
- Adjusting standard models and methods is not straightforward.
- Computational tools can give us valuable insights.
- Interdisciplinarity: Math, AI/Multiagent Learning, Cognitive Science/Psychology, Political Science, Economics

Further Questions

- Other rules, measures for social welfare.
- Computational complexity of determining the optimal order.
- Strategic manipulation under limited energy.
- ...