Matching into Auctions

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Motivation

- On job search platforms, potential employers are looking for future workers
- Entrepreneurs search for VC funding among a large number of investors
- Employers/Investors receive relevant private information on the quality of the job applicants
- How are the employers/investors matched with the workers/entrepreneurs?

Matching with Frictions

- Maintained assumption: matching is with frictions
  - We are not designing the market to avoid the friction
- Matching friction: Urn-ball model
  - A single wage/rate offer per employer/investor to one worker/entrepreneur
- Given this friction, how should the market be organized? Formal auctions or informal bidding processes?

Limited market design

- We assume that the matching technology cannot be changed
  - Each seller is contacted by a random number of potential buyers
  - Contact $\leftrightarrow$ price offer
  - Buyers' search can be directed by signal observations (maybe with a higher probability of contacting high-signal sellers)
- Buyers are all identical and the sellers accept the highest price offer
  - Auction at the seller's location for the buyers contacting the seller: Known realized number of competing buyers
  - Direct price offers to the sellers: uncertainty about the number of competing offers

Related literature

Three connected literatures:

- Players decide entry into an auction after seeing a signal: Samuelson, 1985, Stegeman, 1996; Tan and Yilankaya, 2006; Roberts and Sweeting, 2013; Jehiel and Lamy, 2015; Murto and Välimäki, 2024
- Auctions with exogenous random participation: Lauermann and Speit, 2023 demonstrate existence problems stemming from non-monotonic winner's curse with common values FPA; Matthews, 1987, compares revenues between formal and informal FPA with affiliated private values
- Choosing Auction: Atakan and Ekmekci, 2021, Common values, buyers choose between two multi-unit common value auctions; Kim and Kircher, 2015, directed search with urn-ball matching of sellers with private reservation values to buyers

Setting

- A market place consisting of a unit mass of sellers (employees, entrepreneurs) $\ell \in [0, 1]$ with a single indivisible good
- A continuum of potential buyers (firms, investors) $k \in [0, K]$ with unit-demand
- A search friction: Urn-ball model of random matching of buyers to sellers
  - Buyers contact a single seller
  - Anonymous contacts: no coordination by player names
- Buyers receive a signal on seller's quality
- Sellers' reservation value r below buyers' willingness to pay for all seller qualities

Informational model: Affiliated values

- Common values with imperfect private signals
- $v(\omega_\ell)$ is the value of seller ℓ's product for seller's type $\omega_\ell \in \{\omega^0, \omega^1\}$, and $v(\omega^1) = v^1 > v(\omega^0) = v^0 > r$
- Prior on $\{\tilde{\omega}_\ell = \omega^1\} = q$ for all ℓ, and $\tilde{\omega}_\ell \perp \tilde{\omega}_{\ell'}$
- Each buyer k observes a signal $\theta_{k,\ell} \in \{\theta^0, ..., \theta^M\}$ from all $\ell \in [0, L]$
- Conditional on the quality of ℓ, the $\tilde{\theta}_{k,\ell}$ are i.i.d. across k
- Let $\alpha^{m,i} = \Pr\{\tilde{\theta}_{k,\ell} = \theta^m | \tilde{\omega}_\ell = \omega^i\}$, for $i \in \{0, 1\}$
- Order the signals by MLRP: $\frac{\Pr\{\tilde{\theta}^{k,\ell}=\theta^m|\tilde{\omega}_\ell=\omega^1\}}{\Pr\{\tilde{\theta}^{k,\ell}=\theta^m|\tilde{\omega}_\ell=\omega^0\}}$ increasing in m
- Posterior of k on ℓ after seeing signal m, $q^m_{k,\ell}$ increasing in m

Example: Binary everything

- Two sellers, two buyers
  - Let $q = \frac{1}{2}$, and assume that sellers of type ω^0 send always signal θ^0 while sellers of type ω^1 send θ^0 and θ^1 with equal probabilities
  - The relevant state variable:
    $(\tilde{\omega}_1, \tilde{\omega}_2)$ distributed on $\Omega = \Omega_1 \times \Omega_2 = \{(\omega^0, \omega^0), (\omega^0, \omega^1), (\omega^1, \omega^0), (\omega^1, \omega^1)\}$.
  - Posteriors $q^1_{k,\ell} = 1,\ q^0_{k,\ell} = \frac{1}{3}$ for $k, \ell \in \{1, 2\}$
  - Independent seller types $\implies$ k's beliefs on Ω given by the product

Example: Binary signals

Posterior on (ω_1, ω_2) given $\theta_k = (\theta_{k,1}, \theta_{k,2})$:

	(ω^0, ω^0)	(ω^0, ω^1)	(ω^1, ω^0)	(ω^1, ω^1)
(θ^0, θ^0)	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$
(θ^0, θ^1)	0	$\frac{2}{3}$	0	$\frac{1}{3}$
(θ^1, θ^0)	0	0	$\frac{2}{3}$	$\frac{1}{3}$
(θ^1, θ^1)	0	0	0	1

Example: Binary signals

Distribution of the θ'_k given state:

	(θ^0, θ^0)	(θ^0, θ^1)	(θ^1, θ^0)	(θ^1, θ^1)
(ω^0, ω^0)	1	0	0	0
(ω^0, ω^1)	$\frac{1}{2}$	$\frac{1}{2}$	0	0
(ω^1, ω^0)	$\frac{1}{2}$	0	$\frac{1}{2}$	0
(ω^1, ω^1)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Example: Binary signals

Distribution of the θ'_k given θ_k (obtained by matrix multiplication):

	(θ^0,θ^0)	(θ^0,θ^1)	(θ^1,θ^0)	(θ^1,θ^1)
(θ^0,θ^0)	$\frac{25}{36}$	$\frac{5}{36}$	$\frac{5}{36}$	$\frac{1}{36}$
(θ^0,θ^1)	$\frac{15}{36}$	$\frac{15}{36}$	$\frac{3}{36}$	$\frac{3}{36}$
(θ^1,θ^0)	$\frac{15}{36}$	$\frac{3}{36}$	$\frac{15}{36}$	$\frac{3}{36}$
(θ^1,θ^1)	$\frac{9}{36}$	$\frac{9}{36}$	$\frac{9}{36}$	$\frac{9}{36}$

Example: Anonymous surplus maximization

- A planner dictates to each buyer k an anonymous contacting probability $\pi_k : \{\theta^0, \theta^1\} \times \{\theta^0, \theta^1\} \to \Delta(\{1, 2\})$ to the buyers ℓ to maximize the expected total surplus:

$$\max_{\pi_1, \pi_2} \mathbb{E}_\omega \sum_{\ell \in \{1,2\}} [r + p_\ell(\pi)(v(\omega_\ell) - r)],$$

  where $p_\ell(\pi)$ is the probability of at least one buyer contacting seller ℓ under π
- By anonymity, π is non-trivial only when $\theta_{k,\ell} \neq \theta_{k,\ell'}$
- With common values, planner only wants to match high quality sellers with some buyer with high probability: no gain for having a second buyer

Example: How to maximize surplus?

- For symmetric π_k, congestion best avoided with $\pi_k = \frac{1}{2}$
- If values very different, search for quality would favor high $\pi(\theta^1, \theta^0)$
- With asymmetric π_k, it could be better to have buyer 1 contact the higher signal and buyer 2 contact the lower signal
- For surplus maximization, the choice of π makes a difference only if $\omega_1 \neq \omega_2$
- Optimal to choose $\hat{\pi}_k = 1$ if $\frac{v^1 - r}{v^0 - r}$ is large: search for quality

Example: The game with auctions

- Suppose that the seller organizes a second-price auction with reservation price r between all buyers that contact
- Buyers decide which seller to contact:

$$\pi_k : \{\theta^0, \theta^1\} \times \{\theta^0, \theta^1\} \to \Delta(\{1, 2\}).$$

- Anonymous strategies:

$$\pi_k(\theta^0, \theta^0) = \pi_k(\theta^1, \theta^1) = (1/2, 1/2).$$

$$\pi_k(\theta^0, \theta^1) = (1 - \pi_k, \pi_k), \;\; \pi_k(\theta^1, \theta^0) = (\pi_k, 1 - \pi_k).$$

- Bidding strategy at auction depends on the number of other bidders:

$$b_k : \{0, 1\} \times \{\theta^0, \theta^1\} \times \{\theta^0, \theta^1\} \to \mathbb{R}_+$$

Example: Payoffs

- Payoffs:

$$\sum_{\ell=1}^{2} \pi_k(\ell) \sum_{n_\ell=0}^{1} p(n_\ell) \Pr\{\text{win } \{\ell, n_\ell\} | b_k(n_\ell, \theta_{k,\ell})\} \cdot \mathbb{E}[v(\omega) - b_k(n_\ell, \theta_{k,\ell}) | b_k(n_\ell, \theta_{k,\ell})],$$

  where $p(n_\ell)$ is the probability that n_ℓ other bidders contact seller ℓ
- In $n_\ell = 0$, then expected payoff is: $\mathbb{E}_{\omega_\ell}[v(\omega_\ell) - r | \theta^m, n_\ell = 0]$ for $m \in \{0, 1\}$
- Aligned with increase in total surplus
- If $n_\ell = 1$,

$$b_k(1, \theta^0) = \mathbb{E}[v(\tilde{\omega}_\ell) \mid \theta_{k,1} = \theta_{k,2} = \theta^0]$$

$$b_k(1, \theta^1) = \mathbb{E}[v(\tilde{\omega}_k) \mid \theta_{k,1} = \theta_{k,2} = \theta^1]$$

- Since $\Pr\{\theta_{k',\ell} = \theta^0\} > 0$, payoff from contacting θ^1 exceeds increase in total surplus

Example: Equilibrium with auction

Proposition

The game has a symmetric equilibrium $(\pi^*,\ b^*)$*, where* $b(0, \theta^m) = r$ *for* $m \in \{0, 1\}$ *and* $b(1, \theta^m) = \mathbb{E}[v(\tilde{\omega})|\theta^m, \theta^m]$*. Furthermore* $\frac{1}{2} \leq \hat{\pi} \leq \pi^*$*. and the latter inequality is strict whenever* $\hat{\pi} < 1$*.*

- Unfortunately, the aggregate uncertainty (how many high types?) in the finite model makes the analysis quite complicated
- If the number of buyers and sellers is large, then aggregate uncertainty vanishes by the 'law or large numbers'
- With just two players, one could benefit from asymmetric buyer strategies, with large games, this is not possible (outcomes from asymmetric strategies can be replicated as outcomes from symmetric ones)

Large market

Doubly infinite array of signals

$$
\begin{array}{cccccc}
 & \vdots & \vdots & \vdots & \vdots & \iddots \\
 & \cdot & \cdot & \cdot & \cdot & \cdots \\
k & \theta_{k,1} & \cdot & \theta_{k,\ell} & \cdot & \cdots \\
 & \cdot & \cdot & \cdot & \cdot & \cdots \\
 & \cdot & \cdot & \cdot & \cdot & \cdots \\
 & \theta_{1,1} & \cdot & \theta_{1,\ell} & \cdot & \cdots \\
 & & & \ell & &
\end{array}
$$

Large market

- All sellers $\ell \in [0,1]$ send a signal to all buyers $k \in [0,K]$
- Fraction $\alpha^{m,i}$ of buyers see signal $\theta^m \in \{\theta^0, ..., \theta^M\}$ if seller is of quality ω^i
- By the 'law of large numbers', we can also view

$$q^m = \frac{\alpha^{m,1} q}{\alpha^{m,1} q + \alpha^{m,0} (1-q)}$$

as the fraction of sellers of quality $\omega = 1$ amongst the firms that sent signal θ^m

Large market: anonymous strategies

- All buyers see a positive measure of all signals θ^m
- Let $\pi_k = (\pi_k^0, ..., \pi_k^M) \in \Delta^M$ be the vector of probabilities π^m of contacting some seller that sent signal θ^m
- Anonymity requires equal probability of contacting any such seller
- We distinguish between two cases: formal auctions and informal bids to sellers
- In formal auctions, buyers that contact ℓ with $\theta_{k,\ell} = \theta^m$ bid based on the signal and the realized number n of contacts $F_k(n, m)$
- In informal bidding, $F_k(m)$

Large market: equilibrium matching

- Given any symmetric contact profile $\pi_k = \pi$ for all k, every seller is contacted by a random number of buyers that may have seen different signals
- The total mass on buyers that have decided to contact signal θ^m and meet a seller of type ω^1 is given by $K\pi^m q^m$
- The total mass of sellers of type ω^1 is q (prior)
- The expected number of contacts for a seller of type ω^1 is then:

$$\lambda^{m,1} := \frac{K\pi^m q^m}{q} = \frac{K\pi^m \alpha^{m,1}}{\alpha^{m,1} q + \alpha^{m,0}(1-q)}$$

- Similarly for a seller with true quality ω^0, the expected number is:

$$\lambda^{m,0} = \frac{K\pi^m (1-q^m)}{1-q} = \frac{K\pi^m \alpha^{m,0}}{\alpha^{m,1} q + \alpha^{m,0}(1-q)}. \tag{1}$$

Large market: equilibrium matching

- We assume (in accordance with urn-ball matching literature) that each seller ℓ with $\omega_\ell = \omega^i$ is contacted by $n^{m,i}$ buyers that have seen signal θ^m where $n^{m,i} \sim$ Poisson$(\lambda^{m,i})$
- This can be justified by looking at limits of finite markets with N sellers and KN buyers with $N \to \infty$
- We characterize the optimal anonymous total surplus maximizing vector $\hat{\boldsymbol{\pi}}$ and symmetric equilibria $(\boldsymbol{\pi}^a, F(n,m))$ and $(\boldsymbol{\pi}^i, F(m))$ for the auction model and for the informal bidding model respectively

Large market: anonymous total surplus maximization

- Consider the vector $\hat{\boldsymbol{\pi}}$ that maximizes total surplus in the market:

$$\max_{\pi^0,\ldots,\pi^M \geq 0} qv^1 \left(1 - e^{-\sum_{m=0}^{M} \lambda^{m,1}}\right) + (1-q)\, v^0 \left(1 - e^{\sum_{m=0}^{M} \lambda^{m,0}}\right),$$

$$\text{subject to: } \sum_{m=0}^{M} \pi^m = 1$$

- If $\hat{\pi}^m > 0$ for two (or more) m, then social benefit from a marginal addition to probability of contacting ω^0 and ω^1 sellers must be equalized
- If this equalization is possible, it can be done with $\hat{\boldsymbol{\pi}} = (\hat{\pi}^0, 0, ..., \hat{\pi}^M)$, if not, then $\hat{\pi}^M = 1$.

Large market: anonymous total surplus maximization

- At this solution, $\hat{\pi}^0 > 0$ if and only if $K > \bar{K}$, where:

$$\bar{K} := \frac{\alpha^{M,1} q + \alpha^{M,0} (1 - q)}{\alpha^{M,1} - \alpha^{M,0}} \log \left(\frac{v^1 - r}{v^0 - r} \right)$$

- The two extreme signals are more likely to be contacted if the quality difference is small, if there are lots of buyers, $\alpha^{M,0}$ is small so that few sellers of quality ω^0 are contacted at the corner solution
- There can be a multiplicity of solutions, but the mass of the two quality types $\{\omega^0, \omega^1\}$ contacted is uniquely determined

Observations on bidding in a formal auction

- In a *formal* first-price or second-price auction, the bidders learn the number of competing buyers that have contacted the same seller
- If $\hat{\pi}^M = 1$, then the unique symmetric equilibrium contact strategy is also $\pi_f^M = 1$, and the buyer k's expected payoff from contacting any ℓ is:

$$\Pr\{n_\ell = 0\}\mathbb{E}[v(\omega_\ell)|\theta_{k,\ell} = \theta^m]$$

- Only extreme types $\{\theta^0, \theta^M\}$ ever participate in formal auctions
- If $\hat{\pi}^M < 1$, then $\hat{\pi}^M < \pi_f^M < 1$ since expected payoff of a buyer k with $\theta_{k,\ell} = \theta^M$ earns more than the increase in total surplus, if $\theta_{k,\ell} = \theta^0$, these gains coincide
- Chi et al. (2019) show that formal SPA and FPA are revenue equivalent with binary bidder types

Equilibria in informal auctions with binary types

Proposition

Let F be an atomless bidding equilibrium for given contact rate profile $\pi = (\pi^0, \pi^M)$. Then the union of the bid supports of the buyers of the two types is an interval and the bid support of θ^0 is an interval that contains 0. If

$$\frac{1 - e^{-\lambda^{0,0}}}{1 - e^{-\lambda^{0,1}}} < \frac{v^1 - r}{v^0 - r},$$

then the bid supports are non-overlapping intervals with a single point in common. If

$$\frac{1 - e^{-\lambda^{0,0}}}{1 - e^{-\lambda^{0,1}}} > \frac{v^1 - r}{v^0 - r},$$

then the bid supports intersect and 0 is contained in the supports of both types.

Equilibrium bidding

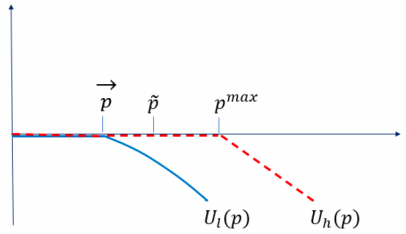

Figure 1: Bid supports for contacting *h* or *l*

Equilibrium bidding

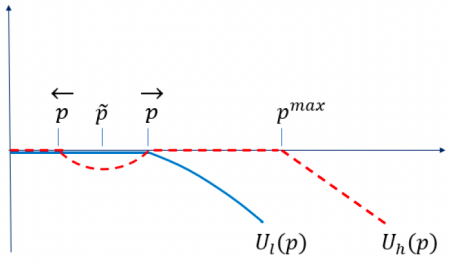

Figure 2: Bid supports for contacting *h* or *l*

Properties of equilibrium in informal model

- Translating into primitives of the model: If K is not too much higher than $\bar{K}$, and if
$$\frac{\alpha^{0,0}}{\alpha^{0,1}} > \frac{v^1 - r}{v^0 - r},$$
then the equilibrium contact rates coincide with the total surplus maximizing ones
- With the values in our binary example (with $K = 1$), efficiency holds whenever $\frac{v^1 - r}{v^0 - r} < 1.68$
- If the above inequality is violated, then both formal auctions and informal bids lead to distortions
- If K is not too large, the distortions in the informal bid model are smaller
- This requires an argument based on (the logic of) the linkage principle

Welfare comparisons

- Formal auctions benefit the buyers over informal ones whenever $\pi_i^M < \pi_f^M$
- Sellers of quality ω^1 also benefit from formal auctions
- Sellers of quality ω^0 are hurt
- If total surplus is larger for the formal auction, then K is large and both formats are almost efficient

Related literature

Three connected literatures:

- Players decide entry into an auction after seeing a signal (Samuelson, 1985, Stegeman, 1996, Tan and Yilankaya, 2006, Roberts and Sweeting, 2013, Jehiel and Lamy, 2015, Murto and Välimäki (2024)
- Auctions with exogenous random participation:
  - Common values FPA: Lauermann and Speit (2023) demonstrate existence problems stemming from non-monotonic winner's curse
  - Matthews (1987) compares revenues between formal and informal FPA with affiliated private values
- Choosing Auction: Atakan and Ekmekci (2021), Common values, buyers choose between two multi-unit common value auctions: Focus on information acquisition, Kim and Kircher (2015), directed search with urn-ball matching of sellers with private reservation values to buyers

Conclusions

- Form of competition matters for the selection of sellers
- This has consequences for total surplus and the distribution of surplus
- Large market model allows for a tractable analysis of the informal competition case
- Next steps:
  - Complete welfare comparisons for large K
  - Endogenize the probability of high quality (pre-matching investment in quality)
  - New rounds for unmatched buyers and sellers: dynamic incentives for wage offers